Fuzzy approaches to production research and information management: part two
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Preface: fuzzy approaches in production research and information management

Issue 4 in Volume 32 of JEIM was the first part of the special issue on fuzzy approaches in production research and information management. In this second part of the special issue, we present recent theoretical developments and real-world applications in the field of fuzzy systems for production research. Most papers in this part have been selected from the 16th Production Research Symposium that took place on October 12–14, 2016, in Istanbul, Turkey. All articles have been peer-reviewed by expert reviewers according to the journal’s standards.

The first paper proposes a two-phase methodology to select a new bread factory location in Istanbul. Every district of Istanbul is considered to be a suitable candidate for bread production. In the first phase, the authors use GIS for bread sales. They create bread sales density maps for each district in Istanbul. In the last phase, they recommend using an interval type-2 hesitant fuzzy-based MCDM method to analyze these four alternatives in detail.

The second paper develops interval type-2 fuzzy X-R control charts; type-2 fuzzy control limits are obtained and the control charts are graphically illustrated and interpreted. The data consist of 20 subgroups taken over periodic times from the production process of a packaged food.

The third paper applies Shingo’s single-minute exchange of die (SMED) methodology to the parts processed on a CNC machine where set-up time is reduced from 196 min to 87 min. A gain of 109 min or 55.61 percent is achieved and demonstrated an improvement in efficiency. However, there is a set-up activity that cannot be shortened with conventional SMED tools.

The fourth paper integrates a number of problem-specific heuristics with fix-and-optimize (FOPT) heuristic and tests their performances on different sizes of problems. The results of experiments show that utilizing problem-specific information improves the effectiveness of FOPT heuristic.

The fifth paper presents a structural competency model and remarks new personnel selection criteria. It presents an importance order and a causal relationship between personnel selection criteria by using one of the multi-criteria decision technique, Fuzzy DEMATEL, in a Turkish high-technology firm which has started to modify its processes according to Industry 4.0 and introduced a new department that is responsible for this transformation.

The sixth paper develops a hybrid algorithm that is the sequential use of the revised weighted fuzzy c-means clustering developed by Esnaf and Küçükdeniz (2013) and proposes derivative-free Nelder–Mead simplex algorithm. The proposed hybrid algorithm is applied to the generalized multisource Weber problem. The paper applies fuzzy C-means and Nelder–Mead algorithms to this type of Weber problems.

The seventh paper introduces a common time window to hybrid flow shop with multiprocessor task. A memetic algorithm (MA) is developed to provide good solutions to the problem. As the probability of finding quality solutions with MA relies upon the parameter choices such as crossover rate, mutation rate, population size, iteration size and parent selection rate, the full factorial design is implemented to decide the best parameter set for each problem type.

The eighth paper applies gray forecasting technique in healthcare sector. Small data are used for forecasting the demand on health sector with GM(1,1) and TFGM(1,1). It aims at indicating the accuracy of forecasting performance with gray model integrated triangular fuzzy numbers.
The ninth paper develops a new warehouse location selection model to contribute to the efficiency of the distribution network and to minimize their costs. Multi-criteria decision-making techniques help to choose the best alternative under multiple criteria are used to carry out the decision-making process.

We hope that this issue will provide a useful resource for ideas, techniques, and methods for fuzzy production research area. We are grateful to the referees whose valuable and highly appreciated works contributed to selecting the high-quality articles published in this issue. We are also very thankful to the Editor-in-Chief, Professor Zahir Irani, for his patience, support and efforts during the whole process.

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A GIS-based interval type-2 fuzzy set for public bread factory site selection

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Erzincan University, Erzincan, Turkey

Abstract

Purpose – The purpose of this paper is to present a new public bread factory location selection for Istanbul Metropolitan Municipality (IMM).

Design/methodology/approach – A two-stage methodology is proposed to determine the location for the public bread factory facility. This framework is based on both geographic information systems (GIS) and multi-criteria decision-making (MCDM) techniques. The first stage of the methodology aims to decrease the number of possible alternative locations to simplify the selection activity by applying GIS; the second stage utilises interval type-2 fuzzy MCDM approach to exactly determine the public bread factory site location.

Findings – In this study, the authors present weighted normalised-based interval type-2 hesitant fuzzy and interval type-2 hesitant fuzzy sets (IT2HFSs)-based compressed proportional assessment (COPRAS) methods to overcome facility location selection problem for a fourth public bread factory in Istanbul.

Practical implications – The results show that the proposed approach is practical and can be employed by the bakery industry.

Originality/value – In this study, the authors present a two-stage methodology for public bread factory site selection. In the first stage, the number of alternatives is reduced by the GIS. In the second stage, an interval type-2 fuzzy set is implemented for the evaluation of public bakery factory site alternatives. A new integrated approach based on COPRAS method and weighted normalised with IT2HFSs is proposed.

Keywords Multi-criteria decision-making, Bakery, COPRAS, Interval type-2 hesitant fuzzy sets, Location selection, Public bread factory

Paper type Research paper

1. Introduction

Strategic decisions are often evaluated as irreversible decisions that affect the future of a company (Reid and Sanders, 2011; Deveci et al., 2017). The reason behind this situation is the risky nature of strategic decisions. Many frameworks have been put forward for location selection, focusing on its different aspects (Deveci et al., 2018). Decision-makers (DMs) usually face uncertainty and vagueness because of subjective perceptions and experiences in the decision-making process. Fuzzy multi-criteria decision-making (MCDM) methods can help DMs to overcome vagueness resulting from subjective perceptions (Belbag et al., 2013; Demirel et al., 2016). The fuzzy MCDM methods have been studied in the literature within various areas for application. However, studies on bakery public factory facility location selection have not been conducted (see Tables I and II).

In recent years, the use of type-2 fuzzy sets and systems has increased. Uncertainty of inaccurate information by primary and secondary memberships was reflected more efficiently by interval type-2 fuzzy sets (IT2FSs) compared to interval type-1 fuzzy sets (ITFSs) (Chen and Lee, 2010).

The authors would like to thank Dr Çağatay Kalkancı from Istanbul Metropolitan Municipality for useful discussion and feedback.
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This study proposes a public bread factory location selection within Istanbul by explaining the potential benefits of the selection as well as underlying reasons for the selection. Istanbul is a metropolitan city in Turkey, which embodies a broad economic, cultural, and historical potential of two continents – Europe and Asia – with a population of 14.1m (Özkır et al., 2015). Bread is an important component of the daily life and is widely consumed by Turkish families. As of 2000, the country with the largest per capita consumption of bread is Turkey with 199.6 kg of bread consumption per person. Turkish people eat more than three times their own body weight in bread annually. With respect to bread consumption, Turkey is followed by Serbia and Montenegro with a consumption of 135 kg per person, and Bulgaria with a consumption of 133.1 kg per person (www.guinnessworldrecords.com).

While white bread consumption has recently decreased in Istanbul, the brown bread consumption has rapidly increased. Consumption of variety bread has increased depending on the increase in income level, age, and education level. In addition to white bread, the most consumed bread types are wholewheat bread, rye bread, corn bread, Trabzon bread, wholegrain bread, and sandwich bread, in terms of the amount of consumption (www.magazinebbm.com).

In this study, we present a new public bread factory location selection for Istanbul Metropolitan Municipality (IMM). This study consists of two stages: elimination (geographic information systems (GIS)) and application (MCDM). The first stage of the methodology aims to decrease the number of possible alternative locations to simplify the selection process by applying GIS, whereas the second stage utilises interval type-2 hesitant fuzzy-based MCDM approach to exactly determine the public bread factory site location within IMM.

The rest of the paper is organised as follows: The literature regarding this subject is reviewed in Section 2. Section 3 presents an overview of the public bread factories of Istanbul. Section 4 presents the fundamentals of interval type-2, hesitant fuzzy, and compressed proportional assessment (COPRAS) methods. Section 5 presents the problem description, GIS, and the proposed methodology steps. Section 6 presents computations and comparative analysis for the case study of Istanbul. Section 7 discusses the results. Finally, Section 8 concludes our paper.

2. Literature review

The GIS-based MCDM studies include the following: Sánchez-Lozano et al. (2013) presented a case study in Spain based on GIS and MCDM methods (technique for order preference by similarity to ideal solution (TOPSIS) and AHP) for assessing the location of solar farms. Özceylan et al. (2016) proposed a GIS-based MCDM model to assess the potential locations of freight villages. The proposed model consists of four successive stages. In the first stage, different geographical criteria are determined from the existing literature, and the data are collected using the GIS. The criterion is equal to the weights. In the second stage, alternative locations are determined by the GIS. In the third stage, the determined criteria are weighted by various stakeholders of potential freight villages using the ANP method. In the fourth stage, “TOPSIS” is applied to determine the best alternative, based on weighted criteria. Al Garni and Awasthi (2017) proposed evaluating and selecting the best location for useful scale solar PV projects, using GIS and an MCDM technique. The model deals with different aspects, such as the economic and technical factors to ensure maximum power success, while reducing project costs to a minimum. An AHP is used to calculate the land suitability index, for weighing the criteria and assessing potential areas. Asakereh et al. (2017) was held to prioritise the Khuzestan province of Iran to establish solar photovoltaic farms based on techno-economic and environmental directions. Fuzzy logic and fuzzy membership functions have been used to create criterion layers in the GIS environment to map the suitability of the land. The AHP technique has been used to measure techno-economic and environmental criteria, as well as to draw the
final map of the suitability of land for solar farms. Baseer et al. (2017) provided a wind farm compatibility analysis by a GBS-based MCDM approach. The AHP is used to determine appropriate weights according to their relative importance to the criteria. The developed model is then applied to the entire Kingdom of Saudi Arabia. Gigović et al. (2017) helped develop a reliable model for the identification of locations to establish wind farms that will provide significant support to planners in the strategy to develop and manage wind power. The proposed model is based on the combined application of GIS and MCDA, using DEMATEL, ANP, and MABAC. The fuzzy MCDM location selection problems are presented in Table I. GIS-based fuzzy MCDM location selection problems are presented in Table II.

3. Public bread factories of Istanbul Metropolitan Municipality
There are three public bread factories in IMM, which make production in two European and one Asian side factory. These factories are called Cebeci, Edirnekapı, and Cevizli, which consist of 883 mass consumption points, 29 chain markets (863 points), 395 dealers (sales point), 544 buffets. Density maps are shown in Figures 1 and 2 for some types of bread, depending on the sales amount:

- Cebeci public bread factory: Gaziosmanpaşa construction of the factory, with a closed area of 15,449 m² on 35,000 m² of land in Cebeci, was started in 1997, completed in 1998, and started production in 1999. The Cebeci factory was planned with five production lines; one line section is still empty. There are eight pieces of 50 tonnes of stainless steel hair, and a total 400 tonnes of flour silos in the factory.

- Edirnekapı public bread factory: Edirnekapı Centre factory is located on 14,960 m² of land and has a closed area of 11,363 m². There are six production lines in the factory. There also exists a production line for roll production (toast, sandwich, etc.). A part of these production lines up to the furnaces was partly renovated in 1998. Our headquarters has eight pieces of stainless steel weighing 50 tonnes, four of them are DKP (packet, rulo, shredded) hairy. There are 12 flour silos, with a total capacity of 520 tonnes, 30 tonnes each.

- Kartal Cevizli public bread factory: Kartal Cevizli has a closed area of 11,487 m² on 13,828 m² of land. The construction of the factory started in 1997, completed in 1998, and the production started in 1998. The Cevizli factory was planned with four production lines, where two production lines’ area is empty. In the factory, there are four pieces of 50 tonnes of stainless steel hair, a total of 200 tonnes of flour silos (www.ihe.istanbul/).

4. Preliminaries
In this section, some definitions and notions such as hesitant fuzzy sets (HFSs), COPRAS method, and IT2HFSs are examined.

4.1 Geographic information system (GIS)
GIS is designed to solve complex planning and management problems. It is a system consisting of hardware, software, and methods that cover the management, processing, analysis, modelling, and display of location-specific data in a location (Taylan and Damcayır, 2016).

One of the most commonly used deterministic models in spatial interpolation is the inverse-distance weighting (IDW) method. The use of spatial interpolation methods is becoming increasingly widespread in geophysical analysis. Currently, many software systems contain most of these methods, allowing for a more detailed study. Some of the
spatial interpolation methods are IDW, Natural Neighbour, Spline, and Kriging (Pavão et al., 2012). The IDW interpolation technique is often the preferred method of interpreting grids from sample point data. The IDW interpolation technique is based on the principle that adjacent points on the surface to be interpolated have more weight at...
This technique interpolates a surface according to the weighted average of the sample points, which reduces weight as it moves away from the point to be interpolated (Lu and Wong, 2008). Although there are several IDW methods, the most well-known is “Shepard’s (1968) method”.

Notes: (a) Rye bread; (b) sandwich bread
It assigns greater weights the points closer to the prediction location, than those farther away; hence, the name IDW (Johnston et al., 2003):

\[ f(p, r) = \sum_{n=1}^{m} w_n f_n, \]

where \( m \) is the number of scatter points, \( f_n \) denotes function values specified in the scatter points (e.g. data set values), and \( w_n \) denotes the weight function assigned to each scatter point. The classic form of the weight function is:

\[ w_n = \frac{d_n^{-s}}{\sum_{k=1}^{n} d_k^{-s}}, \]

where the power parameter is a random positive real number (typically \( s = 2 \)) and \( d_n \) denotes the distance from the distribution point to the interpolation point or:

\[ \beta_n = \sqrt{(p - p_n)^2 + (r - r_n)^2}, \]

where \((p, r)\) are the coordinates of the interpolation point and \((p_n, r_n)\) are the coordinates of each scatter point. The weight function changes from the \( \beta \)-value of the association, from the distribution point, to a value that approaches 0, as the distance from the scattering point increases. The weight functions are normalised so that the sum of the weights is 1.

### 4.2 Hesitant fuzzy set (HFS)

HFSs are powerful tools to manage simultaneous uncertainty sources (Zhang et al., 2017). Classical fuzzy sets are inadequate when DMs hesitate to make one preference. For this reason, Torra (2010) defined HFSs. Later, Rodriguez et al. (2012) examined those sets and proposed hesitant fuzzy linguistic term sets to enrich the content of linguistic terms.

HFSs are defined as a function returning a set of membership values for each element in the domain (Torra, 2010). Some of the related definitions are given by Torra (2010) and Torra and Narukawa (2009).

### 4.3 COPRAS method

The COPRAS method, which is an MCDM method (Zavadskas and Kaklauskas, 1996), was developed by researchers at Vilnius Gediminas Technical University in 1996. This method chooses the best decision alternatives, considering both beneficial and non-beneficial criteria (Das et al., 2012). The COPRAS method can be used for both the maximum and the minimum criterion values in a multi-criteria evaluation. This method can be also easily applied to complex criteria and problems involving various alternatives. Owing to these features, applications have been made in many different areas in literature (Sarıçali and Kundakçı, 2016).

The steps in the COPRAS method are given below (Das et al., 2012).

**Step 1.** The fuzzy decision-making is normalised as follows:

\[ j = 1, \ldots, n \text{ (set of alternatives)}; \quad i = 1, \ldots, m \text{ (set of criteria)}, \]

\[ \tilde{x}_{ij} = \begin{cases} \frac{x_{ij}}{\max x_{i}} & \text{if } iC_b \\ \frac{x_{ij}}{\min x_{i}} & \text{if } iC_n \end{cases} \]

where \( \tilde{x}_{ij} \) is the normalised value, \( C_b \) and \( C_n \) are the sets of benefit and cost criteria.
Step 2. The weighted normalised matrix is determined as follows:

\[ \tilde{V} = \left[ v_{ij} \right]_{mn} = \tilde{x}_{ij}(.)w_i , \]

where \( \tilde{V} \) is the normalised fuzzy decision matrix and \( w_i \) is the weight of the \( i \)th criteria.

Step 3. The sums \( B_j^+ \) and \( C_j^- \) of weighted normalised values are calculated for benefit and cost criteria. For beneficial criteria, a higher value is better and for non-beneficial criteria, a lower value is better to achieve the goal. These sums \( B_j^+ \) and \( C_j^- \) are calculated, respectively, as follows:

\[ B_j^+ = \sum_{i=1}^{k} v_{ij} \]
\[ C_j^- = \sum_{i=k+1}^{m} v_{ij}. \]

Step 4. The relative importance (\( R_j \)) of each alternative is calculated as follows:

\[ R_j = \frac{B_j^+ + \sum_{i=1}^{m} \frac{C_j^-}{C_j^-} \sum_{i=k+1}^{m} v_{ij}}{C_j^- \sum_{i=k+1}^{m} v_{ij}}. \]

Among the alternatives, the one with the highest degree of relative importance is the best choice.

Step 5. The performance index (\( P_j \)) of each alternative is calculated as follows:

\[ P_j = \left[ \frac{R_j}{R_{\text{max}}} \right] \times 100\%, \]

where \( R_{\text{max}} \) is the maximum relative importance value. The alternatives are ranked according to the decreasing values of \( P_j \).

4.4 Interval type-2 fuzzy set

Zadeh (1965) proposed theory of type-1 fuzzy sets. The notion of the type-2 fuzzy set was first presented as an extension and expanded version of the classical type-1 fuzzy sets. In this section, some concepts and arithmetic processes for fuzzy sets are defined as follows (Mendel et al., 2006; Lee and Chen, 2008):

**Definition 1.** \( A \) is a type-2 fuzzy set belonging to \( X \) universal set, and \( \mu_A(x, u) \) is a type-2 fuzzy membership function, expressed as in the following formula (Mendel et al., 2006):

\[ A = \left\{ \left( x, u, \mu_A(x, u) \right) \right\} \forall x \in X, \forall u \in f_x \subseteq [0, 1], 0 \leq \mu_A(x, u) \leq 1 \}

where \( f_x \) denotes \([0, 1]\) range. Besides, \( A \) type-2 fuzzy set is also expressed in a different way (Mendel et al., 2006); where \( f_x \subseteq [0, 1] \), \( \mathcal{J} \) expresses all acceptable combinations of \( x \) and \( u \) values.

**Definition 2.** Let \( \mu_A \) denotes a type-2 membership function defining \( A \) type-2 fuzzy set belonging to \( X \) universal set. In case that all \( \mu_A(x, u) = 1 \), the set \( A \) is called the interval type-2 fuzzy set. An IT2FS is regarded as a special
condition of \( \tilde{A} \) type-2 fuzzy set and expressed as in the following (Mendel et al., 2006) equation:

\[
\tilde{A} = \int_{x \in X} \int_{u \in U} 1/(x, u),
\]

where \( f_x \subseteq \{0, 1\} \).

**Definition 3.** (Mendel et al., 2006). The lower and upper membership functions of IT2FS are type-1 membership functions. Chen and Lee (2010) proposed a new method in their studies for the solutions of fuzzy multi-criteria group decision-making problems by using IT2FSs. According to this method, the reference points of IT2FSs, together with the upper and lower membership functions are used to characterise the fuzzy type-2 sets. A trapezoidal IT2FS is shown in Figure 3.

\[
\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = (a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), (d_{i1}^L, d_{i2}^L, d_{i3}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L))
\]

(Lee and Chen, 2008), where \( \tilde{A}_i^U \) and \( \tilde{A}_i^L \) denote type-1 fuzzy sets, \( a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; d_{i1}^L, d_{i2}^L, d_{i3}^L \) and \( a_{i4}^U \) denote the reference points of the trapezoidal IT2FS, \( H_j(\tilde{A}_i^U) \), as \( 1 \leq j \leq 2d_{i(j+1)}^U \) denotes the membership value for the of \( \tilde{A}_i \) upper trapezoidal membership function, \( H_j(\tilde{A}_i^L) \), as \( 1 \leq j \leq 2d_{i(j+1)}^L \) denotes the membership value for the of \( \tilde{A}_i \) lower trapezoidal membership function and \( 1 \leq j \leq 2, H_j(\tilde{A}_i^L) \):

\[
\tilde{A}_i^L, 1 \leq j \leq 2, H_1(\tilde{A}_i^U) \epsilon \{0, 1\}, H_2(\tilde{A}_i^U) \epsilon \{0, 1\}, H_1(\tilde{A}_i^L) \epsilon \{0, 1\}, H_2(\tilde{A}_i^L) \epsilon \{0, 1\}, \forall 1 \leq i \leq n.
\]

**Definition 4.** The addition of trapezoidal IT2FSs is indicated as in the following:

\[
\tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) = \left( \left( a_{11}^U, a_{12}^U, a_{13}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \right), \left( d_{11}^L, d_{12}^L, d_{13}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L) \right) \right)
\]

\[
\left( a_{11}^U, a_{12}^U, a_{13}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U) \right)
\]

Figure 3. The membership function for a trapezoidal interval type-2 fuzzy number.
Definition 5. The subtraction of trapezoidal IT2FSs is indicated as in the following:

\[ A_1 \oplus \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) \ominus (\tilde{A}_2^U, \tilde{A}_2^L) \]

and:

\[ \tilde{A}_2 = (\tilde{A}_2^U, \tilde{A}_2^L) = (a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), \]

\[ (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)) \]

\[ A_1 \oplus \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) \ominus (\tilde{A}_2^U, \tilde{A}_2^L) \]

\[ = \left( (a_{11}^U + a_{12}^U + a_{13}^U + a_{14}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))), \right. \]

\[ \left. \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U))), (a_{11}^L + a_{12}^L + a_{13}^L + a_{14}^L; \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L))), \right. \]

\[ \left. \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))) \right). \]

Definition 6. The multiplication of trapezoidal IT2FSs is indicated as in the following:

\[ \tilde{A}_1 \times \tilde{A}_2 = (\tilde{A}_1^U, \tilde{A}_1^L) \times (\tilde{A}_2^U, \tilde{A}_2^L) \]

\[ = \left( (a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \min(H_1(\tilde{A}_1^U), H_1(\tilde{A}_2^U))), \right. \]

\[ \left. \min(H_2(\tilde{A}_1^U), H_2(\tilde{A}_2^U)), (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \min(H_1(\tilde{A}_1^L), H_1(\tilde{A}_2^L))), \right. \]

\[ \left. \min(H_2(\tilde{A}_1^L), H_2(\tilde{A}_2^L))) \right). \]

Definition 7. Trapezoidal IT2FSs and the arithmetic processes between scaler k are indicated as in the following:

\[ \tilde{A}_1 = (\tilde{A}_1^U, \tilde{A}_1^L) \]

\[ = \left( (a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \right) \]

and the exact value of k is defined as the following:

\[ k \tilde{A}_1 = \left( \left( k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), \right. \right. \]

\[ \left. \left( k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \right) \right). \]
4.5 Interval type-2 hesitant fuzzy sets

In many real-life problems, there exist generally ambiguous or uncertain information, and DMs have a hard time in expressing their decisions with precise and accurate values. The IT2FSs modelling method is proposed by Hu et al. (2015). Some of the related definitions are given in the following (Xia and Xu, 2011; Hu et al., 2015):

**Definition 8.** Let X denote a fixed set. An IT2HFS on X is a function returning a subset of IT2FNs when it is applied for each x in the set X:

\[ E = \{ x, \tilde{h}_E(x) > |x X| \} \]

In other words, the IT2HFS is expressed with a mathematical symbol:

where \( \tilde{h}_E(x) \) is a set of some IT2FNs, being the possible membership degrees of the element \( x \in X \) for the set E. Briefly, it will be called \( \tilde{h}_E(x) = \tilde{h} = \{ \tilde{A}_1, \cdots, \tilde{A}_n \} \) (or \( \tilde{A}_1, \cdots, \tilde{A}_n \) : \( H_1(\tilde{A}_1), H_2(\tilde{A}_1) \)), an interval type-2 hesitant fuzzy element (IT2HFE).

**Definition 9.** Let \( \tilde{h}_1 = \{ \tilde{A}_1, \cdots, \tilde{A}_n \} = (a_{i_1}^U, a_{i_2}^U, a_{i_3}^U, a_{i_4}^U; H_1(\tilde{A}_1), H_2(\tilde{A}_1)), \) \( \tilde{h}_2 = (a_{i_1}^U, a_{i_2}^U, a_{i_3}^U, a_{i_4}^U; H_1(\tilde{A}_1), H_2(\tilde{A}_1)) \) and \( \tilde{h}_2 = (a_{i_1}^U, a_{i_2}^U, a_{i_3}^U, a_{i_4}^U; H_1(\tilde{A}_1), H_2(\tilde{A}_1)) \) be two IT2HFEs. The computation rules between \( \tilde{h}_1 \) and \( \tilde{h}_2 \) are ensured as in the following:

1. \( \tilde{h}_1 \oplus \tilde{h}_2 = \tilde{h}_2 \oplus \tilde{h}_1 \);  
2. \( \tilde{h}_1 \otimes \tilde{h}_2 = \tilde{h}_2 \otimes \tilde{h}_1 \);  
3. \( \lambda (\tilde{h}_1 \oplus \tilde{h}_2) = \lambda \tilde{h}_1 \oplus \lambda \tilde{h}_2 (\lambda > 0) \);  
4. \( (\tilde{h}_1 \otimes \tilde{h}_2)^* = \tilde{h}_1^* \otimes \tilde{h}_2^* (\lambda > 0) \);  
5. \( \lambda \tilde{h}_1 \oplus \lambda \tilde{h}_2 = (\lambda_1 + \lambda_2) \tilde{h}_1 (\lambda_1, \lambda_2 > 0) \); and  
6. \( \tilde{h}_1^{\lambda_1} \otimes \tilde{h}_1^{\lambda_2} = \tilde{h}_1^{\lambda_1 + \lambda_2} \).

**Definition 10.** Let \( \tilde{h} = (\tilde{A}_1, \cdots, \tilde{A}_n) = (a_{i_1}^U, a_{i_2}^U, a_{i_3}^U, a_{i_4}^U; H_1(\tilde{A}_1), H_2(\tilde{A}_1)), \) \( \tilde{h}_1^{\lambda_1}, \tilde{h}_2^{\lambda_2} \) be an IT2HFE. Then, the score function of \( \tilde{h} \) is defined as in the following:

\[
\text{score}(\tilde{h}) = \frac{1}{\# \tilde{h}} \sum_{\tilde{A}_h} \text{score}(\tilde{A}) = \frac{1}{\# \tilde{h}} \sum_{\tilde{A}_h} \left[ a_{i_1}^U + a_{i_2}^U + H_1(A^U) + H_2(A^U) + H_1(A^L) + H_2(A^L) \right] \times a_{i_1}^U + a_{i_2}^U + a_{i_3}^U + a_{i_4}^U + a_{i_1}^L + a_{i_2}^L + a_{i_3}^L + a_{i_4}^L.
\]
where \( \tilde{h} \) is an \( \tilde{A} \) number within IT2HFE, and score \( \tilde{h}_i \) is a crisp value in \([0, 1]\). For the two IT2HFSs, \( \tilde{h}_1 \) and \( \tilde{h}_2 \), if score \( \tilde{h}_1 \geq \) score \( \tilde{h}_2 \), then \( \tilde{h}_1 \geq \tilde{h}_2 \).

5. Proposed methodology

A two-stage methodology is proposed to determine the location for the public bread factory facility. This framework is based on both GIS and MCDM techniques. The first stage of the methodology aims to decrease the number of possible alternative locations to simplify the selection activity by applying GIS; the second stage utilises interval type-2 fuzzy MCDM approach to exactly determine the public bread factory site location. As seen in Figure 4, the first stage uses two factors, which are IDW method and bread sales amounts (Turkish Lira), to reduce the number of alternative locations from 39 to 4 alternatives. After reduction of the alternatives, the second stage including the selection activity among the alternatives is applied. In the proposed model, it is described as one of the IT2HFSs, based on MCDM techniques. However, this study prefers to utilise the weighted normalised-based interval type-2 hesitant fuzzy and the COPRAS approach. The details of both GIS and MCDM stages are explained as follows.

GIS technology is suitable for various uses such as environmental management, industry and commerce, tourism, health management, security and defence, and transportation planning (Özkır et al., 2015). It should be noted that the integration of GIS with MCDA techniques provides a powerful tool to facilitate any positioning process. GIS has unique features for automation and spatial analysis. In addition, MCDA is used to cope with the difficulties of handling complex information in large quantities, allowing a consistent assessment of potential areas based on a variety of criteria (Tavares et al., 2011).

5.1 Stage 1: GIS

We use the GIS-based IDW method to classify some alternatives through geographical information data and some relevant criteria. This stage of the methodology aims to reduce the number of alternatives (districts) to a reasonable number, with respect to the pre-determined factors, using a GIS-based IDW software. The factors enable us to reduce
the number of alternatives from 39 to 4 as shown in Figure 4. The IDW method is one of the more popular methods accepted by geoscientists and geographers, because it is partly applied in many GIS packages (Lu and Wong, 2008).

5.2 Stage 2: HFS-based interval type-2 fuzzy MCDM methods

In this study, an interval type-2 fuzzy MCDM method that integrates weighted normalised-based hesitant sets for the location selection of bread factory is proposed. Furthermore, IT2HFSs based on COPRAS and interval type-2 hesitant fuzzy method are applied to this problem. These methodologies are constitutively based on the works of Rodriguez et al. (2012) and Hu et al. (2015). The score function of each alternative is compared for deciding the public bread factory site. The proposed methodology consists of two stages, which are GIS and the MCDM process. The GIS application process is shown in Figure 5. The stages of the proposed MCDM methodology for the evaluation of potential alternatives are shown in Figure 6.

5.2.1 Weighted normalised-based IT2HFSs. The steps in the weighted normalised-based interval type-2 hesitant fuzzy method are constructed from inspiration by Rodriguez et al. (2012) and Hu et al. (2015).

Step 1. A committee comprising of expert DMs is formed and these DMs can assess the evaluation criteria and alternatives.

Step 2. The evaluation criteria $C = \{c_1, c_2, \ldots, c_n\}$ are identified and the possible alternative set $A = \{a_1, a_2, \ldots, a_n\}$ are composed.

Step 3. The linguistic term set, semantic, and linguistic expressions are determined. Suitable linguistic variables and translation standards are selected to transform linguistic evaluations into interval type-2 fuzzy number (IT2HFN), for each alternative and criterion (e.g. rating scale in Table III).

Step 4. The MCDM problem is formulated. $W = \{w_1, w_2, \ldots, w_n\}$ is the weighting vector of criterion $c_j$ ($j = 1, 2, \ldots, n$), where $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Step 5. DMs are asked to select suitable fuzzy numbers for criteria weights as well as the hesitant linguistic rating term, having the best representation for the evaluation of alternatives for each criterion.

Step 6. Optionally, the fuzzy linguistic term evaluations are transformed into interval type-2 hesitant fuzzy elements (IT2HFEs), obtain the rating, $\tilde{h}_{ij}$, for the alternative $A_i$ on criterion $c_j$, then obtain IT2HFEs matrix.

If criterion $c_j$ is a benefit criterion, then $\tilde{h}_{ij} = \tilde{h}_{ij}^{-}$; alternatively, if $c_j$ is a cost criterion, then $\tilde{h}_{ij} = \tilde{h}_{ij}^{+}$, where $\tilde{h}_{ij}^{-}$ is calculated according to the complementary definition of IT2HFEs.

Step 7. General preference values $h_i$ ($i = 1, 2, \ldots, m$) of alternative $A_i$ based on the interval type-2 hesitant fuzzy weighted averaging (IT2FWA) operators or the interval type-2 hesitant fuzzy weighted geometric (IT2FWG) aggregation operators are generated.

![Figure 5. Stage of the GIS application process](image-url)
Ranking the alternatives and determine the best alternative

**Figure 6.** Stages of the proposed methodology for public bread factory site location selection

**Table III.** Linguistic term set and their corresponding values

<table>
<thead>
<tr>
<th>Label</th>
<th>Linguistic terms</th>
<th>Corresponding IT2HFNs</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL</td>
<td>Absolutely low</td>
<td>(0.0, 0.0, 0.0, 0.0; 1, 1) (0.0, 0.0, 0.0, 0.0; 1, 1)</td>
</tr>
<tr>
<td>VL</td>
<td>Very low</td>
<td>(0.0075, 0.0075, 0.015, 0.0525; 0.8, 0.8) (0.0, 0.02, 0.07; 1.0, 1.0)</td>
</tr>
<tr>
<td>L</td>
<td>Low</td>
<td>(0.0875, 0.12, 0.16, 0.1825; 0.8, 0.8) (0.04, 0.10, 0.18, 0.23; 1.0, 1.0)</td>
</tr>
<tr>
<td>ML</td>
<td>Slightly low</td>
<td>(0.2325, 0.255, 0.325, 0.3575; 0.8, 0.8) (0.17, 0.22, 0.36, 0.42; 1.0, 1.0)</td>
</tr>
<tr>
<td>M</td>
<td>Middle</td>
<td>(0.4025, 0.4525, 0.5375, 0.5675; 0.8, 0.8) (0.32, 0.41, 0.58, 0.65; 1.0, 1.0)</td>
</tr>
<tr>
<td>MH</td>
<td>Slightly high</td>
<td>(0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8) (0.58, 0.63, 0.80, 0.86; 1.0, 1.0)</td>
</tr>
<tr>
<td>H</td>
<td>High</td>
<td>(0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8) (0.72, 0.78, 0.92, 0.97; 1.0, 1.0)</td>
</tr>
<tr>
<td>VH</td>
<td>Very high</td>
<td>(0.9475, 0.98, 0.9925, 0.9925; 0.8, 0.8) (0.93, 0.98, 1.0, 1.0) (1.0, 1.0, 1.0, 1.0)</td>
</tr>
<tr>
<td>AH</td>
<td>Absolutely high</td>
<td>(1.0, 1.0, 1.0, 1.0; 1.0, 1.0, 1.0, 1.0)</td>
</tr>
</tbody>
</table>

**Source:** Hu et al. (2015)
If a summation operator $\text{IT}_2\text{HFWA}$ is used, then we have:

$$\bar{h}_i = \text{IT}_2\text{HFWA}\left(\bar{h}_{i1}, \bar{h}_{i2}, \ldots, \bar{h}_{im}\right) = \sum_{j=1}^{m} w_j \bar{h}_{ij} (i = 1, 2, \ldots, m),$$

$$= \bigcup_{\bar{h}_{i1} \in \bar{h}_1, \bar{h}_{i2} \in \bar{h}_2, \ldots, \bar{h}_{im} \in \bar{h}_m} \left\{ \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^L_{ij}) \right) \right), \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^U_{ij}) \right) \right), \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^M_{ij}) \right) \right) \right\}$$

\[ \times \left\{ \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^L_{ij}) \right) \right), \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^U_{ij}) \right) \right), \left( I^{-1} \left( \sum_{j=1}^{m} w_j (d^M_{ij}) \right) \right) \right\} \right\} \]

Step 8. The hesitant fuzzy decision-making are normalised as follows.

Where $B$ and $C$ are the sets of benefit criteria and cost criteria, respectively, as in the following:

$$\tilde{r}_i = \left( \left( a^{U}_{i1}, a^{U}_{i2}, \ldots, a^{U}_{in}, H_1(\bar{A}^U_i), H_2(\bar{A}^L_i) \right), \left( a^{L}_{i1}, a^{L}_{i2}, \ldots, a^{L}_{in}, H_1(\bar{A}^L_i), H_2(\bar{A}^L_i) \right) \right),$$

$$\tilde{x}_i = \left( \left( a^{L}_{i1}, a^{L}_{i2}, \ldots, a^{L}_{in}, H_1(\bar{A}^L_i), H_2(\bar{A}^L_i) \right), \left( a^{U}_{i1}, a^{U}_{i2}, \ldots, a^{U}_{in}, H_1(\bar{A}^L_i), H_2(\bar{A}^L_i) \right) \right).$$

$$a^U_{ij} = \max_j a^U_{ij}, \text{ } j \in B, a^L_{ij} = \min_j a^L_{ij}, \text{ } j \in C; \quad \tilde{R}: \text{ normalised fuzzy decision matrix}; a^U_{ij}$: maximum value of the component in one column of the fuzzy decision matrix; $\tilde{r}_i$: normalised values obtained by dividing each value in fuzzy decision matrix into $c^T$ value.

Step 9. The weighted normalised matrix is determined as follows:

$$\tilde{V} = \left[ v_{ij} \right]_{max} = \tilde{r}_i(i)w_i.$$

Step 10. By using Definition 10, calculate the general scores $(\tilde{h}_{ij}) (i = 1, 2, \ldots, m)$ of $\text{IT}_2\text{HFEs}$. According to this definition, scores must be calculated for both optimistic and pessimistic values and, lastly, the average of pessimistic and optimistic values for each criterion and sub-criterion is identified as “the ultimate score”.

Step 11. The dominance matrix is structured considering the difference between the preference relations as shown below:

Let $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be interval utilities. The preference relation of $I_1$ over $I_2$ ($I_1 > I_2$) is calculated as:

$$R(I_1 > I_2) = \frac{\max \left( 0, a_1 - b_2 \right) - \max \left( 0, a_1 - b_2 \right)}{(b_1 - a_1) + (b_2 - a_2)}.$$
Similarly, preference relation of $I_2$ over $I_1$ ($I_2 > I_1$) can be calculated. Note that $R(I_1 > I_2) + R(I_2 > I_1) = 1$, and when $a_1 = a_2$ and $b_1 = b_2$, the interval utilities are equal ($I_1 = I_2$) and $R(I_1 > I_2) = R(I_2 > I_1) = 0.5$.

For example, if we want to compare the difference of $I_1$ with $I_2$, the following equation must be used:

$$DM_{12} = \max(0, (R(I_1 > I_2) - R(I_2 > I_1)))$$

Step 12. The non-dominance approach of Rodriguez et al. (2012) is adopted for other criteria and alternatives:

$$NDM_i = |\min((1-DM_1), (1-DM_2), \ldots, (1-DM_n))|$$

where $n\neq i$.

After normalisation, the alternatives are ranked as per $NDM_i = (NDM_i) / (\sum_{i=1}^{n} NDM_i)$. All alternatives $A_i$ and select the most suitable one ($\tilde{h}_i$) ($i = 1, 2, \ldots, m$). The greater the value of $s(\tilde{h}_i)$, the better the alternative $A_i$.

5.2.2 IT2HFS-based COPRAS method. The IT2HFS-based COPRAS method, that is proposed here, can be described in 15 steps. The first 12 steps in the IT2HFS-based COPRAS method are the same as those of the weighted normalised-based interval type-2 hesitant fuzzy method.

Step 13. The sums $B_{j+}$ and $C_{j-}$ of normalised score values are calculated for benefit and cost criteria. These sums $B_{j+}$ and $C_{j-}$ are calculated, respectively as follows:

$$B_{j+} = \sum_{i=1}^{k} v_{ij}$$

$$C_{j-} = \sum_{i=k+1}^{m} v_{ij}$$

Step 14. The relative importance ($R_j$) of each alternative is calculated as:

$$R_j = B_{j+} + \sum_{i=1}^{m} C_{j-} / C_{j-} \sum_{i=1}^{m} C_{j-}$$

Among the alternatives, one with the highest degree of relative importance is the best choice.

Step 15. The performance index ($P_j$) of each alternative is calculated as follows:

$$P_j = \left[ \frac{R_j}{R_{\max}} \right] \times 100\%,$$

where $R_{\max}$ is the maximum relative importance value. The alternatives are ranked according to the decreasing values of $P_j$.

6. Case study

The capacity and the number of public bread factories are insufficient for Istanbul. This framework considers demographic, environmental, social, and economic factors for public bread factory site selection of IMM, which are combined by the integration of HFSs based on interval type-2 MCDM methods. First, four alternative districts are selected with the help of GIS technique for this problem in Section 4.1. These alternatives are evaluated according to the bread sales amount. The four alternative locations are shown on the map in Figure 7. Three of the four alternatives are located in the European side (Fatih, Küçükçekmece, and Zeytinburnu) and one of them is located in the Asian side (Ümraniye).
The consumption rate of white bread is high in Fatih (A1) district. Zeytinburnu (A3) is located close to Fatih, and both are the main white bread consumption districts of Istanbul. These locations are also close to Edirnekapi public bread factory. Küçükçekmece (A2) is one of the places where white bread and other types of bread have high consumption. Umraniye (A4) is an important and densely populated area of Istanbul.

The criteria are determined by the DMs and literature review. These criteria are presented in Table IV. Second, the public bread factory site location is determined using these selected criteria and alternatives with help of MCDM techniques.

Figure 7 shows the bread sales density in 39 districts of Istanbul. Four alternative districts (location) are selected according to the density map. The main common characteristic of those locations is that they are all located where the sales density is the highest. Three of the four alternatives are located in the European side (Fatih, Küçükçekmece, and Zeytinburnu) and one of them is located in the Asian side (Umraniye).

6.1 Weighted normalised-based interval type-2 hesitant fuzzy method computation

Steps 1 and 2. DMs, criteria, and alternatives are determined.

Step 3. The linguistic variables with nine levels in Table III and the transformation standards are selected to transform linguistic rating term sets into IT2HFEs.

Steps 4 and 5. The fuzzy numbers of criteria weights are obtained as \( w_1 = w_2 = w_3 \) and hesitant fuzzy linguistic evaluations of alternatives.

Steps 6 and 7. The preference relationships \( (R^e) \) of DMs are summed up according to the number of DMs for each two criteria, sub-criteria, and alternatives, and here \( k \in \{1, 2, \ldots, k\} \) and express IT2HFLTS as \( (R^l, R^u) \) based on the lower and upper bounds. The summed preferences for the criteria are given in Table V. For example, when one compares the “costs” with “geographic and land characteristics”, the lower bound as the pessimistic preference is expressed as “H” and the upper bound as the optimistic preference is expressed as “VH”.

Figure 7. District clustering according to the sale amounts
Then, the numerical equivalents of pairwise comparison matrices are summed up. After obtaining the linguistic evaluations, they must be transformed into numerical intervals based on the interval type-2 hesitant fuzzy linguistic term set. Owing to page limit, we present only one example. The pairwise comparison of the evaluations for “costs” and “geographic and land characteristics” is defined as $[H, VH], [MH, AH], and [VH, AH]$. The table below shows the scores of pairwise comparison matrix according to three linguistic evaluation categories of decision-makers:

<table>
<thead>
<tr>
<th>Main criteria</th>
<th>Sub-criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>Investment cost</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>Infrastructure availability</td>
</tr>
<tr>
<td>Costs</td>
<td>Water and waste disposal facilities</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>Regional risks</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>Climatic conditions</td>
</tr>
<tr>
<td>Location</td>
<td>Potable water accessibility</td>
</tr>
<tr>
<td>Location</td>
<td>Proximity to suppliers and buffets</td>
</tr>
<tr>
<td>Location</td>
<td>Proximity to customers</td>
</tr>
<tr>
<td>Location</td>
<td>Traffic density</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>Proximity to workforce</td>
</tr>
<tr>
<td>Capacity</td>
<td>Contribution to regional economy</td>
</tr>
<tr>
<td>Capacity</td>
<td>Community attitude</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Costs</th>
<th>Geographic and land characteristics</th>
<th>Location</th>
<th>Environmental and social effects</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>$[VH, AH]$</td>
<td>$[MH, AH]$</td>
<td>$[VH, AH]$</td>
<td>$[ML, M]$</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>$[VH, AH]$</td>
<td>$[ML, H]$</td>
<td>$[VH, AH]$</td>
<td>$[VH, AH]$</td>
</tr>
<tr>
<td>Location</td>
<td>$[ML, MH]$</td>
<td>$[M, H]$</td>
<td>$[L, M]$</td>
<td>$[L, M]$</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>$[VL, L]$</td>
<td>$[ML, MH]$</td>
<td>$[VH, AH]$</td>
<td>$[ML, MH]$</td>
</tr>
<tr>
<td>Capacity</td>
<td>$[VH, AH]$</td>
<td>$[ML, H]$</td>
<td>$[MH, H]$</td>
<td>$[ML, MH]$</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>$[VH, AH]$</td>
<td>$[ML, H]$</td>
<td>$[ML, MH]$</td>
<td>$[VH, AH]$</td>
</tr>
<tr>
<td>Location</td>
<td>$[ML, MH]$</td>
<td>$[H, AH]$</td>
<td>$[LM, H]$</td>
<td>$[H, AH]$</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>$[AL, L]$</td>
<td>$[AL, VL]$</td>
<td>$[AL, VL]$</td>
<td>$[AL, L]$</td>
</tr>
<tr>
<td>Capacity</td>
<td>$[ML, H]$</td>
<td>$[VH, AH]$</td>
<td>$[H, AH]$</td>
<td>$[ML, H]$</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>$[VH, AH]$</td>
<td>$[ML, H]$</td>
<td>$[ML, MH]$</td>
<td>$[VH, AH]$</td>
</tr>
<tr>
<td>Location</td>
<td>$[ML, MH]$</td>
<td>$[H, AH]$</td>
<td>$[ML, H]$</td>
<td>$[H, AH]$</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>$[AL, L]$</td>
<td>$[VH, AH]$</td>
<td>$[H, AH]$</td>
<td>$[ML, H]$</td>
</tr>
<tr>
<td>Capacity</td>
<td>$[ML, H]$</td>
<td>$[VH, AH]$</td>
<td>$[H, AH]$</td>
<td>$[ML, H]$</td>
</tr>
</tbody>
</table>

Table V.
Scores of pairwise comparison matrix according to three linguistic evaluation categories of decision-makers.
The corresponding IT2HFE for \([H, VH]\) is calculated as in the following: \((0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8), (0.72, 0.78, 0.92, 0.97; 1.0, 1.0)\), \((0.9475, 0.985, 0.9925, 0.9975; 0.8, 0.8), (0.93, 0.98, 1.0, 1.0)\) and similarly, for \([H, AH]\) \((0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8), (0.72, 0.78, 0.92, 0.97; 1.0, 1.0), (1.0, 1.0, 1.0, 1.0; 1.0, 1.0)\) and similarly for \([MH, H]\) is determined as \((0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8), (0.58, 0.63, 0.80, 0.86; 1.1); (0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8), (0.72, 0.78, 0.92, 0.97; 1.0, 1.0)\).

Thereafter, the individual preferences are summed up, by using interval type-2 hesitant fuzzy weighted average (IT2HFWA) linguistic aggregate operator. For instance, the aggregated preference relationship for “costs” in terms of “geographic and land characteristics” is calculated below according to the summation operator:

\[
\hat{h}_{12} = \text{IT2HFWA}\left(\hat{h}_{11}, \hat{h}_{12}, \ldots, \hat{h}_{1m}\right) = \sum_{j=1}^{n} w_j \hat{h}_{ij}(i = 1, 2, \ldots, m)
\]

\[
= \frac{1}{3}(0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8), (0.72, 0.78, 0.92, 0.97; 1.0, 1.0),
\]

\[
(0.9475, 0.985, 0.9925, 0.9975; 0.8, 0.8), (0.93, 0.98, 1.0, 1.0; 1.0, 1.0)]
\]

\[
= \frac{1}{3}[0.65, 0.6725, 0.7575, 0.79; 0.8, 0.8)(0.58, 0.63, 0.80, 0.86; 1.1),
\]

\[
(0.7825, 0.815, 0.885, 0.9075; 0.8, 0.8)(0.72, 0.78, 0.92, 0.97; 1.0, 1.0)]
\]

\[
= [(0.745, 0.776, 0.853, 0.878; 0.8, 0.8)(0.679, 0.738, 0.891, 0.950; 1.0, 1.0),
\]

\[
(1.0, 1.0, 1.0, 1.0; 1.0, 1.0, 1.0, 1.0)]
\]

Step 8: The hesitant fuzzy decision matrix values obtained in the previous step are normalised.

Step 9. Similarly, normalised hesitant fuzzy values are calculated for the remaining criteria. Then, given the different importance weights for each criterion, weighted normalised fuzzy decision matrix is calculated.

Step 10. The scores \(s(h_{ij}) (i = 1, 2, \ldots, m)\) for \(h_i (i = 1, 2, \ldots, m)\) by using the score function definition. The pairwise comparison matrix scores are given in Table VI.

<table>
<thead>
<tr>
<th>Costs</th>
<th>Geographic and land characteristics</th>
<th>Location</th>
<th>Environmental and social effects</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>–</td>
<td>1.39319</td>
<td>0.87998</td>
<td>1.82968</td>
<td>0.90702</td>
</tr>
<tr>
<td>1.90000</td>
<td>1.66250</td>
<td>2.00000</td>
<td>1.70760</td>
<td></td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>0.05999</td>
<td>–</td>
<td>0.48935</td>
<td>1.68153</td>
</tr>
<tr>
<td>0.27992</td>
<td>1.06773</td>
<td>2.00000</td>
<td>1.47888</td>
<td></td>
</tr>
<tr>
<td>Location</td>
<td>0.34953</td>
<td>0.76707</td>
<td>–</td>
<td>1.47888</td>
</tr>
<tr>
<td>1.16234</td>
<td>1.47882</td>
<td></td>
<td>1.90000</td>
<td>1.16235</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>0.00665</td>
<td>0.45511</td>
<td>0.04324</td>
<td>–</td>
</tr>
<tr>
<td>0.14231</td>
<td>1.31503</td>
<td>0.29916</td>
<td>0.10083</td>
<td></td>
</tr>
<tr>
<td>Capacity</td>
<td>0.57356</td>
<td>0.50723</td>
<td>0.33543</td>
<td>1.77740</td>
</tr>
<tr>
<td>1.68153</td>
<td>1.10236</td>
<td>0.94979</td>
<td>2.00000</td>
<td></td>
</tr>
</tbody>
</table>

Table VI. Scores of pairwise comparison matrix for the main criteria
For example, score function for “costs” in terms of “geographic and land characteristics” is calculated as in the following:

Pessimistic score \( \bar{h}_{12} \) = \( \frac{1}{\#h} \sum_{A \in h} \text{score}(\bar{A}) \)

\[ \frac{1}{\#h} \sum_{A \in h} \left( \frac{0.745+0.776}{2} + \frac{0.8+0.8+1+1}{4} \right) \times \frac{0.745+0.776+0.853+0.878+0.679+0.738+0.891+0.950}{8} = 1.393. \]

Optimistic score \( \bar{h}_{12} \) = \( \frac{1}{\#h} \sum_{A \in h} \text{score}(\bar{A}) \)

\[ \frac{1}{\#h} \sum_{A \in h} \left( \frac{1+1+1+1+1+1+1+1}{2} \times \frac{1+1+1+1+1+1+1}{4} \right) \times \frac{1+1+1+1+1}{8} = 1.9. \]

Step 11. The dominance matrix considering the difference between preference relationships is built. The dominance matrix for main criteria is given in Table VII. The sample calculation is conducted for “location” and “costs” as follows:

\[ DM_{12} = \max \left( 0, (R(I_2 > I_4) - R(I_4 > I_2)) \right) \]

\[ = \max \left( 0; \left( \frac{(0.059+0.279)}{2} -(1.393+1.90)/2 \right) \right) = 1.477. \]

Step 12. The non-dominance rule of Rodriguez et al. (2012) is adapted. The calculation of non-dominance rule for “environmental and social effects” criterion is given below. The non-dominance rule results are in given in Table VIII:

\[ NDM_4 = \left| \min \left( (1-DM_1), (1-DM_2), (1-DM_3), \ldots, (1-DM_n) \right) \right| \]

\[ = \left| \min \left( (1-0.184), (1-0.956), (1-1.518), (1-1.838) \right) \right| = 0.840. \]

The alternatives are ranked as normalised \( NDM_i = \frac{NDM_i}{(\sum_{i=1}^{n} NDM_i)} \) after the normalisation process. The normalised weights of the main are given in Table IX:

\[ NDM_4 = \frac{0.840}{1.00+0.477+0.485+0.840+0.484} = 0.256. \]

The steps are both followed for sub-criteria and alternatives. The final scores are determined in Table X, yielding a final ranking of \( A_2 > A_4 > A_1 > A_3, A_2 \) is the best among the five alternatives because it has the largest score, whereas \( A_2 \) is the worst alternative.

<table>
<thead>
<tr>
<th></th>
<th>Costs</th>
<th>Geographic and land characteristics</th>
<th>Location</th>
<th>Environmental and social effects</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>–</td>
<td>1.477</td>
<td>0.515</td>
<td>1.840</td>
<td>0.180</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>0.000</td>
<td>–</td>
<td>0.000</td>
<td>0.956</td>
<td>0.516</td>
</tr>
<tr>
<td>Location</td>
<td>0.000</td>
<td>0.349</td>
<td>–</td>
<td>1.518</td>
<td>0.170</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>–</td>
<td>0.000</td>
</tr>
<tr>
<td>Capacity</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.838</td>
<td>–</td>
</tr>
</tbody>
</table>

Table VII. Dominance matrix for the main criteria
To prevent unnecessary repetition, the calculations, for the IT2HFSs of Hu et al. (2015) are shown in Table XI. According to this table, $A_2$ was selected as the best alternative.

6.2 Comparative analysis and discussion

A comparative analysis is carried out with the study of Hu et al. (2015) to test the validity of the proposed method in this study. To facilitate the comparison process, the analysis is based on the same decision-making problems and multiple criteria hesitant linguistic

<table>
<thead>
<tr>
<th>Costs</th>
<th>Geographic and land characteristics</th>
<th>Location</th>
<th>Environmental and social effects</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.477</td>
<td>0.485</td>
<td>0.840</td>
<td>0.484</td>
</tr>
</tbody>
</table>

Source: According to Rodríguez et al. (2013)

<table>
<thead>
<tr>
<th>Costs</th>
<th>Geographic and land characteristics</th>
<th>Location</th>
<th>Environmental and social effects</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.304</td>
<td>0.145</td>
<td>0.148</td>
<td>0.256</td>
<td>0.147</td>
</tr>
</tbody>
</table>

Table IX. Normalised weights of the main criteria

<table>
<thead>
<tr>
<th>Criteria weight</th>
<th>Sub-criteria weight</th>
<th>Global sub-criteria weight</th>
<th>Evaluation of alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>0.304</td>
<td>1.000</td>
<td>0.304 0.187 0.548 0.055 0.211</td>
</tr>
<tr>
<td>Investment cost</td>
<td>0.145</td>
<td>0.007</td>
<td>0.249 0.250 0.250 0.250</td>
</tr>
<tr>
<td>Geographic and land characteristics</td>
<td>0.148</td>
<td>0.196 0.028</td>
<td>0.247 0.257 0.225 0.271</td>
</tr>
<tr>
<td>Infrastructure availability</td>
<td>0.003</td>
<td>0.230 0.044</td>
<td>0.231 0.271 0.234 0.265</td>
</tr>
<tr>
<td>Water and waste disposal facilities</td>
<td>0.148</td>
<td>0.223 0.032</td>
<td>0.223 0.247 0.263 0.266</td>
</tr>
<tr>
<td>Regional risks</td>
<td>0.303</td>
<td>0.271</td>
<td>0.224 0.247 0.251 0.279</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>0.037</td>
<td>0.390 0.039</td>
<td>0.288 0.268 0.288 0.287</td>
</tr>
<tr>
<td>Potable water accessibility</td>
<td>0.021</td>
<td>0.339 0.050</td>
<td>0.253 0.281 0.210 0.256</td>
</tr>
<tr>
<td>Location</td>
<td>0.148</td>
<td>0.339</td>
<td>0.253 0.281 0.210 0.256</td>
</tr>
<tr>
<td>Proximity to suppliers and buffets</td>
<td>0.032</td>
<td>0.359 0.089</td>
<td>0.219 0.024 0.215 0.283</td>
</tr>
<tr>
<td>Proximity to customers</td>
<td>0.037</td>
<td>0.072 0.005</td>
<td>0.285 0.287 0.287 0.288</td>
</tr>
<tr>
<td>Traffic density</td>
<td>0.021</td>
<td>0.359</td>
<td>0.327 0.368 0.289 0.390</td>
</tr>
<tr>
<td>Proximity to workforce</td>
<td>0.147</td>
<td>0.100 0.256</td>
<td>0.332 0.131 0.147 0.391</td>
</tr>
<tr>
<td>Environmental and social effects</td>
<td>0.256</td>
<td>0.552 0.141</td>
<td>0.359 0.368 0.289 0.390</td>
</tr>
<tr>
<td>Contribution to regional economy</td>
<td>0.147</td>
<td>1.000 0.256</td>
<td>0.275 0.327 0.321 0.076</td>
</tr>
<tr>
<td>Community attitude</td>
<td>0.147</td>
<td>0.392 0.346</td>
<td>0.292 0.346 0.290 0.299</td>
</tr>
</tbody>
</table>

Table X. Defuzzified values of alternatives and main and sub criteria for proposed methodology
decision-making methods. The ranking shows that $A_2$ (Küçükçekmece) is the best alternative in Tables X (proposed method) and XI (interval type-2 hesitant fuzzy method).

In this study, interval type-2 hesitant fuzzy decision-making method and weighted normalised-based IT2HFSs are compared with interval IT2HFS-based COPRAS method. The results from the COPRAS-based approach are provided in Table XII. It does not give suitable comparison results for the solution of the problem. This is because of the differences in the calculation of benefit and non-benefit criteria between IT2HFS-based COPRAS and other MCDM methods. Table XIII presents the comparison of alternative rankings for one fuzzy MCDM and the proposed methodologies.

The COHF method presents a different calculation method for the benefit and non-benefit criteria of the weighted normalised values, and thus yields difference results.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$S_+ $</th>
<th>$S_-$</th>
<th>$Q_i$</th>
<th>$P_i$ (%)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>2.666671</td>
<td>0.702517</td>
<td>2.7410</td>
<td>97.25</td>
<td>2</td>
</tr>
<tr>
<td>$A_2$</td>
<td>2.419945</td>
<td>1.05289</td>
<td>2.4672</td>
<td>87.53</td>
<td>4</td>
</tr>
<tr>
<td>$A_3$</td>
<td>2.459192</td>
<td>0.575408</td>
<td>2.5499</td>
<td>90.47</td>
<td>3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>2.750205</td>
<td>0.763363</td>
<td>2.8186</td>
<td>100.00</td>
<td>1</td>
</tr>
</tbody>
</table>

Table XIII
Comparison of the alternative rankings for one fuzzy MCDM in the literature and the two proposed methodologies

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Interval type-2 hesitant fuzzy set (Hu et al’s, 2015 study)</th>
<th>Weighted normalised-based IT2HFS</th>
<th>Weighted normalised-based IT2HFS COPRAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$A_2$</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$A_3$</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$A_4$</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
7. Results and discussion

The best site for fourth bread factory in Istanbul is determined as $A_2$ (Küçükçekmece). $A_2$ is a very important district because of its bread sales intensity and demographic characteristics. It is also a district in the centre of Istanbul. The results show that a very good decision has been taken.

Consumer demand is gradually changing from regular bread to traditional (artisan) sourdough/whole grain/enriched products and their sliced, small-sized, and packaged versions. These products are freshly shipped as well as sold half-baked/frozen. These products are brought together at the supermarket and boutique points of sale, and by the end consumer through collective consumption channels.

With the establishment of the new public bread factory (IPB, 2017):

- The need for a large area for sourdough, long-fermented bread production, the establishment of frozen and roll bread production systems/equipment. The fact that the breads produced as part of contract manufacturing can be produced more controlled and healthier (with sourdough), in our own operation, the need for a new factory with the latest technology suitable for today’s conditions will replace our 40-year-old Edirnekapi public bread factory, which has an economic lifespan/production capability of production lines and is resistant to natural disasters.

- It is aimed at recognising the possibility of transferring new products to the market that provide high value added industrial yeast through R&D studies with sourdough and frozen systems. In the Kartal and Cebeci factories, a fermentation system (1,500 m$^3$) is needed to produce sourdough, long-fermented, and frozen bread as fermentation rooms allow for only 30 minutes (175 m$^3$) of hot fermentation.

8. Conclusion

In this study, we proposed a two-phase methodology to select a new bread factory location in Istanbul. Every district of Istanbul is considered to be a suitable candidate for bread production. In the first phase, we preferred to use GIS for bread sales intensive and IDW. This phase determined suitable alternatives such as Fatih, Küçükçekmece, Zeytinburnu, and Ümraniye according to bread sales intensity. Spatial analyst tools programmed ArcGIS®10.5 using the interpolation tools available at ESRI; we created bread sales density maps for each district in Istanbul. In the last phase, we recommended using an interval type-2 hesitant fuzzy-based MCDM method to analyse these four alternatives in detail. We presented weighted normalised-based IT2HFS and IT2HFS-based COPRAS methods to overcome facility location selection problem for a fourth public bread factory in Istanbul. Performances of the proposed methodologies were compared with the methodology by Hu et al. (2015) for the site selection of public bread factory. As can be seen from the results, the proposed methodology could work more effectively.

The methodology developed in this study can be transferred to other cities for facility location selection or other problems to determine the best location or selection. Different fuzzy decision-making techniques such as interval hesitant or intuitionistic fuzzy sets in selection of car-sharing stations can also be examined in future studies. Furthermore, other multi-criteria decision methods (TOPSIS, AHP/ANP, VIKOR, PROMETHEE, etc.) can be used to solve location selection problems.

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Further reading


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Different methods to fuzzy $\bar{X}$-R control charts used in production

Interval type-2 fuzzy set example

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Abstract

Purpose – The control charts are used in many production areas because they give an idea about the quality characteristic(s) of a product. The control limits are calculated and the data are examined whether the quality characteristic(s) is/are within these limits. At this point, it may be confusing to comment, especially if it is slightly below or above the limit values. In order to overcome this situation, it is suitable to use fuzzy numbers instead of crisp numbers. The purpose of this paper is to demonstrate how to create control limits of $\bar{X}$-R control charts for a specified data set of interval type-2 fuzzy sets.

Design/methodology/approach – There are methods in the literature, such as defuzzification, distance, ranking and likelihood, which may be applicable for interval type-2 fuzzy sets. This study is the first that these methods are adapted to the $\bar{X}$-R control charts. This methodology enables interval type-2 fuzzy sets to be used in $\bar{X}$-R control charts.

Findings – It is demonstrated that the methods – such as defuzzification, distance, ranking and likelihood for interval type-2 fuzzy sets – could be applied to the $\bar{X}$-R control charts. The fuzzy control charts created using the methods provide similar results in terms of in/out control situations. On the other hand, the sample points depicted on charts show similar pattern, even though the calculations are different based on their own structures. Finally, the control charts obtained with interval type-2 fuzzy sets and the control charts obtained with crisp numbers are compared.

Research limitations/implications – Based on the related literature, research works on interval type-2 fuzzy control charts seem to be very limited. This study shows the applicability of different interval type-2 fuzzy methods on $\bar{X}$-R control charts. For the future study, different interval type-2 fuzzy methods may be considered for $\bar{X}$-R control charts.

Originality/value – The unique contribution of this research to the relevant literature is that interval type-2 fuzzy numbers for quantitative control charts, such as $\bar{X}$-R control charts, is used for the first time in this context. Since the research is the first adaptation of interval type-2 fuzzy sets on $\bar{X}$-R control charts, the authors believe that this study will lead and encourage the people who work on this topic.

Keywords Production, Interval type-2 fuzzy sets, Fuzzy control charts, Interval type-2 fuzzy sets methods, $\bar{X}$-R control charts

Paper type Research paper

1. Introduction

Control chart is one of the statistical process control methods that describe the state of the product or production process. It was originally developed at Bell Laboratories (Shewhart, 1931). Control charts are easy and useful for observing variation on the critical quality characteristic(s) of product or production process.

In the basic level, the upper control limit (UCL) and lower control limit (LCL) are calculated after determining the center line, and data are compared regarding with the control limits. By this, abnormal conditions are observed in the production process or in the product and regulations are made accordingly.
The fuzzy set theory was first introduced by Zadeh (1965). In general terms, the fuzzy set theory is a theorem in which there is not only one value for each number but also adjacent number values can represent the number. This theory, put forward in 1965, is called ordinary fuzzy numbers, and in 1975 this theory which fuzzifies both the numbers and the membership degrees at the same time was suggested by Zadeh, calling this type-2 fuzzy numbers.

The research works of Wang and Raz (1990) and Raz and Wang (1990) are the first studies showing that fuzzy sets can be used in control charts. Then, Kanagawa et al. (1993) used fuzzy probability and membership expressions in their own research works. Over the next few years, some research works in the literature discussed that fuzzy control charts can be applied to categorical data. Following these studies, fuzzy control charts have been studied for both qualitative and quantitative data (Asai, 1995; Laviolette et al., 1995; Woodall et al., 1997).


Fuzzy X-R charts with fuzzy mean and variance used to improve experts’ decisions (Shu and Hsien, 2011). Demirli et al. (2010) developed a fuzzy inference system to recognize unnatural situations in X-R control charts. Similarly, Saricicek and Cimen (2011) proposed a fuzzy inference system for X-R control charts in terms of assessing out of control patterns. Moameni et al. (2012) considered fuzzy numbers for problems that can be caused by measurement errors, and these fuzzy numbers taken into account for X-R control charts. Kaya and Kahraman (2010) described process capability indexes as triangular and trapezoidal fuzzy numbers.

All of the above-mentioned studies used ordinary (type-1) fuzzy sets. Şentürk and Antuchieviciene (2017) obtained c-control charts using defuzzification method for interval type-2 fuzzy sets. Ercan-Teksen and Anagün (2017a) used the methods of defuzzification and likelihood for c-control charts. In addition, Ercan-Teksen and Anagün (2017b) examined using interval type-2 fuzzy sets ranking methods for c-control charts. Furthermore, Ercan-Teksen and Anagün (2018) adapted the type reduction and defuzzification methods used for interval type-2 fuzzy sets to c-control charts.

Within the scope of this study, X-R control charts are basically formed with interval type-2 fuzzy sets. Different fuzzy control charts are drawn, and comparisons are made for different methods – such as defuzzification, ranking, distance and likelihood. The current study using interval type-2 fuzzy sets for X-R control charts based on different methods is the first research in the accessed literature.
In the following sections, these subjects will be briefly discussed. In Section 2, interval type-2 fuzzy sets and operations performed with these numbers will be clarified. The methods used in Section 3 will be explained. Section 4 will describe how the limits of the X-R control charts are formed with interval type-2 fuzzy sets. Section 5 will provide illustrative example. Finally, information on what is generally done in the study will be given in conclusion section.

2. Interval type-2 fuzzy sets and operations

This section gives some fundamental information about interval type-2 fuzzy sets and provides some arithmetic operations. Since the membership functions and the values of type-2 fuzzy numbers are both fuzzy, the numbers are different from type-1 fuzzy numbers (Zadeh, 1975). Therefore, this study is formed as using interval type-2 fuzzy sets, and no comparisons are made with type-1 fuzzy sets.

The type-2 fuzzy numbers obtained are shown as follow:

\[ \tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u) \mid \forall x \in X, \forall u \in J_x \leq [0, 1], 0 \leq \mu_{\tilde{A}}(x, u) \leq 1\}, \]

where \( J_x \) symbolizes an interval \([0,1]\). If all \( \mu_{\tilde{A}} = (x, u) = 1, \tilde{A} \) is called an interval type-2 fuzzy sets (Buckley, 1985).

\[ \tilde{A} = ((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U, H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U, H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U))) \]

is the illustration of trapezoidal interval type-2 fuzzy sets where \( a_{ik}^U \) is the reference point of the interval type-2 fuzzy sets \( \tilde{A}_i, k = 1, 2, 3, 4, m = U, L \) (U defines upper membership function and L defines lower membership function) and \( 1 \leq i \leq n \). \( H_j(A_i^U) \in [0,1] \) denotes the membership value of the element \( a_{ij}^U, j = 1, 2, m = U, L \) and \( 1 \leq i \leq n \). Figure 1 shows the illustration of trapezoidal interval type-2 fuzzy sets.

Some arithmetic operations related to trapezoidal interval type-2 fuzzy sets are given in following.

The addition operation between trapezoidal interval type-2 fuzzy sets \( \tilde{A}_1 \) and \( \tilde{A}_2 \) is defined as follow:

\[ \tilde{A}_1 + \tilde{A}_2 = \left( \begin{array}{l}
\min \left( H_2 \left( \tilde{A}_1^U \right) ; H_2 \left( \tilde{A}_2^U \right) \right), (a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \right)
\end{array} \right) \]

\[ \min \left( H_1 \left( \tilde{A}_1^U \right) ; H_1 \left( \tilde{A}_2^U \right) \right), (a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \right) \]

\[ \min \left( H_2 \left( \tilde{A}_1^U \right) ; H_2 \left( \tilde{A}_2^U \right) \right), \min \left( H_2 \left( \tilde{A}_1^U \right) ; H_2 \left( \tilde{A}_2^U \right) \right) \right) \]  

(1)

Figure 1.
Illustration of trapezoidal interval type-2 fuzzy sets.
The subtraction operation between trapezoidal interval type-2 fuzzy sets \( \tilde{A}_1 \) and \( \tilde{A}_2 \) is given below:

\[
\tilde{A}_1 - \tilde{A}_2 = \left( \left( a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U; \min \left( H_1 \left( \tilde{A}_1^U \right); H_1 \left( \tilde{A}_2^U \right) \right) \right), \right.
\]

\[
\left. \left( \left( a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L; \min \left( H_1 \left( \tilde{A}_1^L \right); H_1 \left( \tilde{A}_2^L \right) \right) \right) \right) \right) \right).
\]

(2)

The arithmetic operations between trapezoidal interval type-2 fuzzy sets \( \tilde{A}_1 \) and the crisp value \( k \) are defined in Equation (3) and the multiplication operation between trapezoidal interval type-2 fuzzy sets \( \tilde{A}_1 \) and \( \tilde{A}_2 \) is shown in Equation (4):

\[
k \times \tilde{A}_1 = \left( k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1 \left( \tilde{A}_1^U \right), H_2 \left( \tilde{A}_1^U \right) \right),
\]

\[
\left( k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1 \left( \tilde{A}_1^L \right), H_2 \left( \tilde{A}_1^L \right) \right) \right).
\]

(3)

\[
\tilde{A}_1 \times \tilde{A}_2 = \left( \left( a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \min \left( H_1 \left( -\tilde{A}_1^U \right); H_1 \left( -\tilde{A}_2^U \right) \right) \right),
\]

\[
\left. \left( \left( a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \min \left( H_1 \left( -\tilde{A}_1^L \right); H_1 \left( -\tilde{A}_2^L \right) \right) \right) \right) \right) \right).
\]

(4)

3. Methods applied on interval type-2 fuzzy sets

It is quite difficult to operate with type-2 fuzzy sets. For this reason, some comparison methods for interval type-2 fuzzy sets have been implemented. These methods often define type-2 fuzzy set as two type-1 fuzzy sets and use one of the comparison methods for type-1 fuzzy sets then.

This section discusses four different methods mentioned in the accessed literature, i.e., Kahraman et al.'s (2014) defuzzification method, Qin and Liu's (2015) ranking method, Chen's (2013) distance method and Chen and Lee's (2010) likelihood method. The comparisons of the interval type-2 fuzzy sets are given in the following subsections.

3.1 Kahraman et al.'s defuzzification method

The defuzzification equation appeared in the below was proposed by Kahraman et al. (2014) for trapezoidal interval type-2 fuzzy sets:

\[
D_{\text{TraT}} = \frac{\left( a_{11}^U - a_{12}^U + a_{21}^U - a_{22}^U \right) + \left( a_{21}^L - a_{22}^L + a_{11}^L - a_{12}^L \right)}{2} + \frac{\left( a_{11}^U - a_{12}^U + a_{21}^U - a_{22}^U \right) + \left( a_{21}^L - a_{22}^L + a_{11}^L - a_{12}^L \right)}{2},
\]

(5)

where \( D_{\text{TraT}} \) is abbreviation for the trapezoidal interval type-2 fuzzy sets.
3.2 Qin and Liu’s ranking method

Qin and Liu (2015) developed ranking method for interval type-2 fuzzy sets. Ranking of A was shown in the following equation:

\[
\text{Rank}(A) = \sum_{i=1}^{3} (M_i(A^U) + M_i(A^L)) - \frac{1}{4} \sum_{i=1}^{3} (S_i(A^U) + S_i(A^L) + H_i(A^U) + H_i(A^L)),
\]

where \( M_i(A') = (a_{1p}^i + a_{1p+1}^i)/2 \) and \( S(A') = \sqrt{\sum_{k=1}^{n+1} ((a_{1k}^i - (1/2) \sum_{k=1}^{n+1} a_{1k}^i))^2} \), \( i = 1, 2, 3 \).

3.3 Chen’s distance method

Chen (2013) defined a formula that finds the distance between constant interval type-2 fuzzy sets which point to \( I_1 \) and interval type-2 fuzzy sets, it is given in the following form:

\[
d(A, \bar{A}_1) = \frac{1}{8} (a_1^2 + a_2^2 + a_3^2 + a_4^2 + 4a_1^U + 2a_2^U + 2a_3^U + 4a_4^U + 3(a_2^U + a_3^U - a_4^U)(H_i^U - H_i^L)/H_A^2 - 16).
\]

3.4 Chen and Lee’s likelihood method

Chen and Lee (2010) proposed likelihood method for interval type-2 fuzzy sets. Likelihood of \( \bar{A}^U \geq \bar{A}^L \) is shown as follow:

\[
P(\bar{A}^U \geq \bar{A}^L) = \max \left( 1 - \max \left( \frac{\sum_{i=1}^{n} \max(a_{1i}^U - a_{1i}^L, 0) + (a_{1i}^U - a_{1i}^L)}{\sum_{i=1}^{n} |a_{1i}^U - a_{1i}^L| + (a_{1i}^U - a_{1i}^L) + \sum_{i=1}^{n} H_i(\bar{A}_i^U) - H_i(\bar{A}_i^L)}, 0 \right), 0 \right).
\]

Equation (8) is for likelihood value of upper membership function, and it applies for likelihood value of lower membership function. Then the ranking values for upper and lower membership functions are defined as follows:

\[
\text{Rank}(\bar{A}_i^L) = \frac{1}{n(n-1)} \sum_{k=1}^{n} p(\bar{A}_s^L \geq \bar{A}_i^L) + \frac{n-1}{2}, \quad (9)
\]

where \( m = U, L \) and \( n \) is a number of sets.

Finally, the ranking values of the interval type-2 fuzzy set \( \bar{A}_i \) can be calculated by following equation:

\[
\text{Rank}(\bar{A}_i) = \frac{\text{Rank}(\bar{A}_i^U) + \text{Rank}(\bar{A}_i^L)}{2}.
\]

4. \( \bar{X} \)-R fuzzy control charts with interval type-2 fuzzy sets

In general, it can be said that fuzzy numbers are preferred because of their flexibility. Therefore, even if the data gathered from real life are correct, fuzzy sets and operations may give more appropriate results.
\(\bar{X}\)-R control charts, mostly preferred in quantitative control charts, may be applied to inspect the variation on quality characteristic(s) such as weight, length, diameter, etc., but it is very difficult to say that every measured value is the right value. The person making the measurement, the precision of the measuring instrument, the ambient conditions may vary, so daily life data may be different. For this reason, fuzzy numbers are preferred. In this study, interval type-2 fuzzy sets are preferred because the membership scores of the numbers may not be obvious. The main purpose of this study is to demonstrate how to create control limits of \(\bar{X}\)-R control charts for a specified data set of interval type-2 fuzzy sets. Since it is the first attempt for using the \(\bar{X}\)-R control charts with interval type-2 fuzzy sets in the accessed literature, it is thought that the contribution made be considered valuable and important for future studies.

Interval type-2 fuzzy operations will be used, taking advantage of the equations created for the classical \(\bar{X}\)-R control charts. After these operations, interval type-2 fuzzy \(\bar{X}\)-R control chart limits will be determined. First of all, the limits of the classical \(\bar{X}\)-R control charts are needed to be calculated as below:

\[
CL = \bar{X} = \frac{\sum_{i=1}^{m} X_i}{m}, \tag{11}
\]

\[
LCL = \bar{X} - A_2 \bar{R}, \tag{12}
\]

\[
UCL = \bar{X} + A_2 \bar{R}, \tag{13}
\]

\[
CL_R = R = \frac{\sum_{i=1}^{m} R_i}{m}, \tag{14}
\]

\[
LCL_R = D_3 \bar{R}, \tag{15}
\]

\[
UCL_R = D_4 \bar{R}, \tag{16}
\]

where \(m, \bar{X}_i\) and \(R_i\) are defined as sample number, mean of the \(i\)th sample and the range of the \(i\)th sample, respectively. \(A_2, D_3\) and \(D_4\) are the coefficients obtained from a table for the \(\bar{X}\)-R control charts.

Interval type-2 fuzzy \(\bar{X}\)-R control limits may then be calculated using the above equations are given as follows:

\[
\bar{CL} = \left( \left( \frac{\bar{a}_1}{a_1'}, \frac{\bar{a}_2}{a_2'}, \frac{\bar{a}_3}{a_3'}, \frac{\bar{a}_4}{a_4'} , \min \left( H_1 \left( \bar{A}_1' \right) \right) , \min \left( H_2 \left( \bar{A}_1' \right) \right) \right) , \left( \frac{\bar{a}_1}{a_1'}, \frac{\bar{a}_2}{a_2'}, \frac{\bar{a}_3}{a_3'}, \frac{\bar{a}_4}{a_4'} ; \right) \right) \\
\min \left( H_1 \left( A_1' \right) \right) , \min \left( H_2 \left( A_1' \right) \right)
\right), \tag{17}
\]

\[
\bar{UCL} = \left( \left( \frac{\bar{a}_1}{a_1'}, \frac{\bar{a}_2}{a_2'}, \frac{\bar{a}_3}{a_3'}, \frac{\bar{a}_4}{a_4'} , \min \left( H_1 \left( \bar{A}_1' \right) \right) , \min \left( H_2 \left( \bar{A}_1' \right) \right) \right) , \left( \frac{\bar{a}_1}{a_1'}, \frac{\bar{a}_2}{a_2'}, \frac{\bar{a}_3}{a_3'}, \frac{\bar{a}_4}{a_4'} ; \right) \right) \\
\min \left( H_1 \left( A_1' \right) \right) , \min \left( H_2 \left( A_1' \right) \right)
\right), \tag{17}
\]
rU
rU
rU
rL
aU
aU
aL
aU
aL
aL
aU
aU
aL
54x558
854
31,6
JEIM
control limits are obtained from these numbers. As a result of the calculations, the interval
the range of each subgroup are shown in Tables I and II, respectively.
implementation. The data consist of 20 subgroups taken over periodic times from the
5. Illustrative example
Once the control limits are calculated for interval type-2 fuzzy sets, some methods
may be applied for making comparisons. The methods used in this study are explained
in the previous section. These methods are used for control limits. Then, in the
classical method, the data are evaluated whether they were within the specified control
limits or not.

\[
\bar{R} = \left( \left( r_1^U, r_2^U, r_3^U, r_4^U; \min \left( H_1 \left( A_1^U \right) \right) \right), \right.
\]
\[
\min \left( H_2 \left( A_1^U \right) \right), \left( r_1^U, r_2^U, r_3^U, r_4^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right)
\]
\[
\left( \sum_{m} r_i^U \sum_{m} r_j^U \sum_{m} r_k^U \sum_{m} r_l^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right)
\]
\[
\left( \left( \sum_{m} r_i^U \sum_{m} r_j^U \sum_{m} r_k^U \sum_{m} r_l^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right) \right), \quad (18)
\]
\[
LCL = \left( \left( a_1^U - a_2^U, a_3^U - a_4^U, a_5^U - a_6^U, a_7^U - a_8^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right), \min \left( H_2 \left( A_1^U \right) \right), \quad (19)
\]
\[
UCL = \left( \left( a_1^U + a_2^U, a_3^U + a_4^U, a_5^U + a_6^U, a_7^U + a_8^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right), \min \left( H_2 \left( A_1^U \right) \right), \quad (20)
\]
\[
\bar{R}_{LCL} = \left( \left( D_1^U, D_2^U, D_3^U, D_4^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right), \min \left( H_2 \left( A_1^U \right) \right), \quad (21)
\]
\[
\bar{R}_{UCL} = \left( \left( D_1^U, D_2^U, D_3^U, D_4^U; \min \left( H_1 \left( A_1^U \right) \right), \min \left( H_2 \left( A_1^U \right) \right) \right) \right), \min \left( H_2 \left( A_1^U \right) \right), \quad (22)
\]

In this part of the study, an illustrative example is given for a better understanding of the
implementation. The data consist of 20 subgroups taken over periodic times from the
production process of a packaged food. Each subgroup has five units. The mean value and
the range of each subgroup are shown in Tables I and II, respectively.

Interval type-2 fuzzy sets derived from crisp numbers have been calculated. Fuzzy
control limits are obtained from these numbers. As a result of the calculations, the interval
type-2 fuzzy \( \bar{X} \)–R control limits are found as follows. These values are calculated
using Equations (17)-(22). The table values of $A_3$, $D_3$ and $D_4$ in these equations were selected as 0.577, 0 and 2.115, respectively, for the sample size of 5:

\[
\bar{CL} = ((38.05, 38.28, 38.61, 38.82; 0.66, 0.64), (38.17, 38.38, 38.49, 38.72; 0.6, 0.59)),
\]

\[
\bar{R} = ((9.62, 10.10, 10.72, 38, 11.17; 0.66, 0.64), (9.89, 10.32, 10.50, 10.98; 0.6, 0.59)),
\]

\[
\bar{UCL} = ((43.60, 44.11, 44.79, 45.26; 0.66, 0.64), (43.87, 44.33, 44.55, 45.05; 0.6, 0.59)),
\]

\[
\bar{LCL} = ((31.6, 32.10, 32.78, 33.27; 0.66, 0.64), (31.83, 32.32, 32.54, 33.01; 0.6, 0.59)),
\]

\[
\bar{R}_{UCL} = ((20.36, 21.37, 22.67, 23.63; 0.66, 0.64), (20.92, 21.82, 22.21, 23.23; 0.6, 0.59)),
\]

\[
\bar{R}_{LCL} = ((0, 0, 0, 0; 0.66, 0.64), (0, 0, 0, 0; 0.6, 0.59)).
\]

First, the $X$-R control charts drawn using the crisp numbers are obtained using Minitab and the charts are depicted in Figure 2.

As seen in Figure 2, the sample points 2 and 15 of $X$ control chart are out of the control limits, the remaining are within the control limits. There are no abnormal situations for the $R$ control chart.

Now, the methods used for interval type-2 fuzzy sets can be applied to the interval type-2 fuzzy sets obtained from these data. First, defuzzification method is applied. The control limits as well as the data have been defuzzified and the limits have been calculated. The defuzzification values for $X$ and $R$ data are given in Tables III and IV, respectively.

The control limits calculated by the defuzzification for $X$ are: $\bar{\bar{CL}} = 31.18$, $\bar{\bar{LCL}} = 26.31$, $\bar{\bar{R}} = 36.06$, and for $R$ are: $\bar{\bar{CL}} = 8.45$, $\bar{\bar{LCL}} = 0$, $\bar{\bar{R}} = 17.87$.

The control charts drawn according to these values are shown in Figures 3 and 4, respectively. The horizontal axis indicates sample points, the vertical axis indicates the defuzzification values.

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**Table I.** Crisp data for $X$

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**Table II.** Crisp data for $R$
As shown in Figure 3, the sample points of 2 and 15 are out of control, the remaining are within the control limits for \( \bar{X} \) control chart. There are no out of control sample points for R control chart as seen in Figure 4.

Tables V and VI are obtained using the ranking method. There is also a ranking value for the control limits. If the ranking value of the data is higher than the ranking value of UCL or less than the ranking value of LCL, this sample point is considered as out of control.

The control charts drawn according to these values are shown in Figures 5 and 6, respectively. The horizontal axis indicates sample points, the vertical axis indicates the ranking values.

The control limits calculated by the ranking method for \( \bar{X} \) are; \( \text{Rank}_{\bar{X}} = 232.96 \), \( \text{Rank}_{\bar{X} \text{ LCL}} = 196.72 \), \( \text{Rank}_{\bar{X} \text{ UCL}} = 269.15 \), and for R are; \( \text{Rank}_{R} = 64.64 \), \( \text{Rank}_{R \text{ LCL}} = 2.49 \), \( \text{Rank}_{R \text{ UCL}} = 133.94 \).
Different methods to fuzzy $X$-$R$ control charts

Table V. Ranking values for $X$-$R$ control charts

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Figure 3. $X$ chart with defuzzification

Figure 4. $R$ chart with defuzzification
While the points 2 and 15 are outside the control limits for $\bar{X}$ control chart, it can be said that there is no abnormality in the direction of $R$ control chart (see Figures 5 and 6).

It is thought that fuzzy control charts can be obtained using the distance method. This distance value is also calculated for the control limits while the distance to a certain point is determined, and the distance values that are less than the distance from LCL are interpreted as being outside LCL. Similarly, the value at the greater distance from UCL is also interpreted as outside UCL. The distance values obtained for data using Equation (7) are shown in Tables VII and VIII, respectively.

The control limits calculated by the distance method for $\bar{X}$ are; $\text{Dis}_{CL} = 599.06$, $\text{Dis}_{LCL} = 502.93$, $\text{Dis}_{UCL} = 695.82$, for $R$ are; $\text{Dis}_{CL} = 150.60$, $\text{Dis}_{LCL} = -16$, $\text{Dis}_{UCL} = 336.36$ and The control charts drawn according to these values are depicted in Figures 7 and 8. Again, the horizontal axis indicates sample points, the vertical axis indicates the distance values.

By the distance method, the sample points of 2 and 15 are also appeared to be out of control for $\bar{X}$ control chart. On the other hand, all sample points are shown up between the control limits without having any abnormal pattern for the $R$ control chart.

By considering the likelihood method, the likelihood values are also calculated for the control limits using Equations (8)-(10). The method examines uncontrolled situations while designing the probability approach. Then the rules have been developed accordingly. In other words, the likelihood of data greater than the UCL and the likelihood of the LCL

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Table VI. Ranking values for $R$

![Figure 5. X chart with ranking](image-url)
greater than the data are calculated using the Equations (8)-(10). After that $P_T$ which is the sum of these two values is determined. A decision process is employed according to $P_T$ values. A flow chart regarding with the decision rules is depicted in Figure 9.

After determining the decision rules, the calculations are made using Equation (8) and Tables IX-XII are obtained. Tables IX and X show the results that are greater than the UCL and smaller than the LCL for $X$ chart, respectively. In addition, Tables XI and XII show the results that are greater than the UCL and smaller than the LCL for $R$ chart, respectively. In all tables, column, which is expressed as the average likelihood ($P$), shows the calculation of Equation (6) where $n = 2$.

Based on the values given in Tables IX and X, and the charts in Figures 10 and 11, the sample point 2 is below the LCL, and the sample point 15 is above the UCL for $X$ chart.
On the other hand, all sample points are in control for R chart by considering the values given in Tables IX and X, and charts in Figures 12 and 13, respectively. The horizontal axis indicates sample points, the vertical axis indicates the likelihood values.

When the R control charts obtained by the likelihood method are examined, the uncontrolled situation for the range is not seen. On the other hand, the X charts using the likelihood method show that the 2nd and 15th data are out of control. For this example, the charts generated by the likelihood method yielded values of 0 and 1. According to these results, flexible decisions that express “rather in control” or “rather out of control” have not been achieved. However, the flexibility could be provided with other data.
6. Conclusion

Control charts are one of the statistical methods that enable us to follow the production process or the critical quality characteristic(s) of the product. Traditionally, the classical control charts are created with crisp numbers. However, in real-life situations, we cannot always be sure of the accuracy of whether the sample point is outside of the control limits or not. For instance, it is unreasonable to express the sample point in a single number when the competence of the person collecting data and the sensitivity of the tools that measure data are taken into consideration. At this point, the fuzzy set theory may be the required method.

In this study, interval type-2 fuzzy sets based on different methods are considered for \( \bar{X} \)-R control charts; on the other hand, there is no study found with interval type-2 fuzzy \( \bar{X} \)-R control charts according to the accessed literature. Methods of interval type-2 fuzzy sets are introduced for fuzzy control charts, and comparisons are made among the methods.

Calculate \( P_{UCL} \) and \( P_{LCL} \)

Rather in control

Rather out of control

Out of control

In control

\( P_F \) \( \geq 0.7 \)

\( P_F \) \( \geq 0.5 \)

\( P_F \) \( \geq 0.3 \)

\( P_F \) \( \geq 0.7 \)

\( P_F \) \( \geq 0.5 \)

\( P_F \) \( \geq 0.3 \)

Figure 9.

A flow chart regarding decision rules for likelihood method

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Likelihood for R data according to UCL

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Table XII.
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Figure 10.
Likelihood for $\bar{X}$ according to UCL
For interval type-2 fuzzy control charts, not only interval type-2 fuzzy control charts are introduced, but also the formulas for such control charts are created. Moreover, the defuzzification, ranking, distance and likelihood methods for interval type-2 fuzzy sets are applied to same data for comparisons. Subsequently, values for both the fuzzy values and the control limits are obtained using these methods, and the control charts are drawn and interpreted based on these data.

In order to understand this study better, an illustrative example is given, and the results obtained from each method are discussed. First, $\bar{X}$-R control chart of the crisp data are drawn.
Then the interval type-2 fuzzy sets for \( \bar{X}-R \) control charts, derived from these crisp numbers, are calculated. After calculating mean values, upper and LCLs for \( \bar{X} \) and \( R \) are computed as interval type-2 fuzzy sets. The limits and the data are reduced to single values by means of various methods of interval type-2 fuzzy set, such as defuzzification, ranking, distance and likelihood. The control charts generated with all of the methods showed similarities to the classical control chart.

Finally, the major contribution of this study may be explained as follow: \( \bar{X}-R \) control charts for interval type-2 fuzzy sets were successfully used for the first time. In addition, the methods, previously used for decision-making processes, were adapted to control charts.

As for the future studies, the possibility of different interval type-2 fuzzy sets methods may be discussed to \( \bar{X}-R \) control charts, and the applicability of new interval type-2 fuzzy control charts may be investigated using methods available in the related literature.

References


Different methods to fuzzy $\bar{X}$-R control charts


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SMED methodology based on fuzzy Taguchi method
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Abstract
Purpose – There can be activities that cannot reduce times by conventional single minute exchange of die (SMED) tools. In this case more advanced tools are needed. The purpose of this paper is to integrate the fuzzy Taguchi method into the SMED method in order to improve the setup time. The reason for using fuzzy logic is the subjective evaluation of factor’s levels assessment by experts. Subjective assessment contains a certain degree of uncertainty and is vagueness. The fuzzy Taguchi method provides to determining optimal setup time parameters in an activity of SMED. So it is possible to reduce time more than the conventional SMED method.

Design/methodology/approach – In this study, the SMED method and the fuzzy Taguchi method are used.

Findings – In this study, it has been shown that the setup time is reduced (from 196 to 75 min) and the optimum value can be given at the intermediate value by the fuzzy Taguchi method.

Originality/value – In this limited literature research, the authors have not found a study using the fuzzy Taguchi method in the SMED method.

Keywords Fuzzy Taguchi method, Reducing setup time, Setup observation and analysis form, SMED methodology, Single minute exchange of die (SMED)

Paper type Research paper

1. Introduction
Today, companies have been implementing lean and customer-based production to meet all customer demands. Therefore, they are forced to produce smaller lots. However, producing products at smaller batch sizes results in more changeovers. Both setup and changeover have been defined as the process of switching from the production of one product or part number to another in a machine by changing parts, dies or fixtures (Benjamin et al., 2013). Thus, a short setup time is critical for being able to produce small quantities of large diversity of products, the basis of a lean manufacturing. The reduction of setup times of a machine is important in application of flexible and lean manufacturing (Gung and Studel, 1990). Productive time for a machine increases when its setup time is reduced. It is not possible to completely remove setup time, but there are activities which reduce time, such as improving die transportation mechanization, standardizing activities, implementing parallel operations and improving die transportation function (Singh and Khanduja, 2010).

Most applications for reducing setup time have been associated with Shingo’s single minute exchange of die (SMED) methodology. The SMED methodology makes it possible to perform equipment setup (changeover) operations in fewer than 10 min, i.e. a number of minutes expressed by single digit (Shingo, 1985). The SMED methodology is also one of the tools of lean manufacturing. The SMED methodology provides to reduce waste and improve flexibility in manufacturing processes. Thus, it helps in reducing lot size and improving manufacturing flow. The SMED methodology reduces the nonproductive time by streamlining and standardizing the setup activities, using simple techniques and easy applications. However, the process does not give the specific actions to implement which can result in overlooking improvements (Desai, 2012).

There is little awareness about the quantitative techniques that can be used to calculate requirements of having shorter setup times (Singh and Khanduja, 2010). Advanced methods to reduce setup time are needed (Karasu et al., 2014). Alternative methods to shorten internal
setup times can be searched in detail. Therefore, in this study, a new integrated SMED method is developed using the conventional SMED and fuzzy Taguchi methods.

In the conventional SMED method, setup activities are mostly performed by improvements on machines; however, not only machines but also workers take part in the setup process. An important lack of the SMED methodology was the consideration and motivation of human factor (Cakmakci and Karasu, 2007).

This study aimed at further improvement of total setup time after the application of SMED methods. Special methods to shorten times are needed, such as the Taguchi method (Karasu et al., 2014). It is proposed that time of measuring and assessment of first produced product after a changeover operation can be reduced by the Taguchi method. This time of activity is non-value-added time and can be minimized. The Taguchi method is performed to determine factors (parameters of process) and their levels. In this study, the Taguchi method based fuzzy logic is applied. The reason for using fuzzy logic is the subjective evaluation of factor’s levels assessment by experts. Subjective assessment contains a certain degree of uncertainty and vagueness, and does not ensure that the setup time in optimal. For this reason there is no certainty, on the contrary, there is fuzziness and vagueness. Perhaps the intermediate values can be better than the defined levels of the parameters. This allows fuzzy logic to determine these levels that will give more optimal process parameter values. For example, the factor A has two levels (low and high) and these are 10 and 20, respectively. The appropriate level will be 10 or 20 at the end of the experiment. However, for example, 12 will be a better level of factor A in terms of setup time. The Taguchi method with fuzzy logic is used as an efficient approach to determine the optimal setup time parameters in activity of SMED, so that it can provide more setup time reduction.

The proposed integrated SMED model has been applied in an aluminum profile manufacturing company. The setup time was reduced from 126 to 75 min, resulting in a total improvement of 121 min.

2. Literature review

The SMED methodology has been extensively implemented in various industries such as automotive (Desai, 2012; Cakmakci, 2009; Deros et al., 2011; Joshi and Naik, 2012), electronics assembly (Trovinger and Bohn, 2005), aluminum profiles extrusion (Assaf and Haddad, 2014) and electrical power control (Ribeiro et al., 2011). There are also different studies in literature. Trovinger and Bohn (2005) showed that modern information technology can be used in SMED concepts in some complex situations. Cakmakci and Karasu (2007) achieved new integrated methodology containing SMED and MTM. This integration provides both further detailed analysis by motion study and standardization of the optimal changeover procedure that is achieved by SMED and MTM analysis. Cakmakci (2009) used the process capability analysis technique, index $C_{pb}$, to investigate the relation between SMED method and equipment design. Almomani et al. (2013) studied different issues about SMED. They used analytical hierarchical process, preference selection index and technique for order preference by similarity to ideal solution that are multiple criteria decision-making techniques (MCDM) to select the best setup technique among the available alternatives. Other factors such as cost, energy, facility layout, safety, life, quality and maintenance were considered that affect the decision-making process. Stadnicka (2015) studied to reduce a setup time and the risk of problems which can appear during the setup. In this study, the tools such as Pareto analysis, statistical analysis and FMEA were used together with the SMED methodology. Braglia et al. (2016) introduced an integrated approach consisting of conventional SMED and duplication strategy. It follows a top-down functional decomposition of machines, identifying all those items impacting on the changeover process and developing intervention strategies.
Fuzzy Taguchi methods are commonly used in industry (Tang et al., 2000; Hsiang et al., 2012; Lin and Kuo, 2011).
A study using together SMED and Taguchi methods was performed by Karasu et al. (2014). They applied the Taguchi method in SMED to achieve the optimal level set of setup parameters. In our limited literature research, we have not found a study using the fuzzy Taguchi method in the SMED method.

3. Methods
3.1 SMED methodology
SMED methodology was first defined by Shingo as “a scientific approach to setup time reduction that can be applied in any factory to any machine.”

The SMED methodology makes it possible to perform equipment setup (changeover) operations in fewer than 10 min. It provides rapid way of converting a manufacturing process from processing the current product to processing the next product (Mcintosh et al., 1996). The SMED methodology was originally developed to improve die press and machine tool setup, but its principles are applied to changeovers in all types of machines and process. A setup operation is preparation or adjustment that is performed once before each lot is processed (Shingo, 1985). The SMED methodology also provides eliminating waste in a manufacturing process. Thus, it is an important lean production tool.

In the SMED methodology, setup activities are divided into internal and external activities. External activities can be carried out during the normal operation of machine, when it is still running, for example, getting the equipment ready for the setup operation can be done before the machine is shut down. Internal activities can be performed only when the machine is shut down, for example, attaching or removing the dies. The internal and external setup activities contain different operations such as preparation, after-process adjustment, checking of materials, mounting and removing tools, settings and calibrations, measurements, trial runs, adjustments, etc. (Ferradás and Salonitis, 2013). Some methods can be applied in order to reduce duration of activities. Some options such as using functional clamps, implementing parallel operations, reducing adjustments to minimum and designing effective tools are suggested.

First, a video recording of different runs of the machine is analyzed. The activities and their times can be determined using video recording and routing diagrams. There are steps which are used to reduce the setup time using the SMED methodology (Shingo, 1985):

- Separation of the internal and external activities.
- Conversion of internal activities into external activities.
- Streamlining the external and internal activities. Focusing on streamlining all aspects of the setup operation. Specific principles are applied to shorten the setup times.
- Standardization: it is more difficult to sustain new changeover activity unless standardized. All the activities must be standardized to reduce setup time in other machines.

The SMED methodology provides optimizing machine utilization, enabling small lot sizes, reducing production times, reducing machine adjustment times and reducing stocks, reducing setup scrap, decreasing setup labor and manufacturing cost, reducing product lead time and enhancing productivity and utilization of assets (Pannesi, 1995). It also improves safety and health of workers in the workplace during the setup activities (Deros et al., 2011).
3.2 Taguchi method

The Taguchi method is a powerful design of experiments method (Taguchi, 1990). It provides a simple, efficient and systematic approach to optimizing designs for performance, quality and cost (Ross, 1988; Bendell et al., 1989). The Taguchi method can optimize performance characteristics (output or responses) through the settings of process parameters (factors) and reduce the sensitivity of the system performance to sources of variation. Responses, factors and factors’ levels can be chosen based on the earlier experts’ opinions and research works. An orthogonal array (OA) is a major tool used in the Taguchi design. It is subset of selected combinations of multiple factors at multiple levels. Suitable OA is selected according to number of factors and their levels. To select an appropriate OA for the experiment, the total degrees of freedom need to be computed. The degrees of freedom are defined as the number of comparison between process parameters. The degrees of freedom for the OA should be greater than or at least equal to those for the process parameters. The signal-noise (S/N) ratio can be used to measure the deviation of the performance characteristics from the desired values. There are three categories of the responses in the analysis of the S/N ratio, that is, the smaller-the-better, larger-the-better and nominal-the-best and their equations are as follows:

\[ S/N = -10 \log \left( \frac{y_i^2/n}{C_0} \right), \]  
(1)

\[ S/N = -10 \log \left( \frac{1/y_i^2/n}{C_0} \right), \]  
(2)

\[ S/N = -10 \log (s^2), \]  
(3)

where \( y_i \) is \( i \)th value of \( y \) and \( s^2 \) is sample variance.

The obtained results of experiment are analyzed by using the ANOVA method formed to identify the factors (process parameters) that are statistically significant. Levels of important factors are determined. Confirmation experiments are repeated several times. Predicted mean is calculated based on the results of experiments:

\[ \hat{\mu} = \bar{T} + \sum_{i=1}^{p} \left[ (T_{i,j})_{\text{max}} - \bar{T} \right], \]  
(4)

where \( \bar{T} \) is the total mean, \( (T_{i,j})_{\text{max}} \) is the mean at the optimal level, and \( p \) is the number of the factors that significantly affect the responses.

The confidence interval (CI) value for the optimum factor level combination is determined. CI is given by the following equation:

\[ CI = \pm \sqrt{\frac{F(\alpha, 1, \nu) \times MSE}{N_e}}. \]  
(5)

A confirmatory runs based on the optimal factor settings are carried out and average is computed. If the value of average obtained from the experiment is between the estimated \( \hat{\mu} \pm \text{CI value} \) then it is accepted set of factors and their levels.

3.3 Fuzzy logic system and fuzzy-based Taguchi method

The theory of fuzzy logics, initiated by Zadeh (1965), has proven to be useful for dealing with uncertain and vague information. A fuzzy logic system consists of a fuzzifier, membership functions, a fuzzy rule base, an inference engine and a defuzzifier (Figure 1).
Through fuzzification, the entered crisp value is converted into suitable linguistic fuzzy information. Next, the inference engine performs a fuzzy reasoning on fuzzy rules which is an if-then expression and a conditional descriptive sentence. Rules describe the relationship between input and output. In this paper, the type of inference engine developed by Mamdani and Assilian (1975) was used by employing a compositional minimum operator. If there are two inputs, $x_1$ and $x_2$ and one output $y$, the fuzzy rules of Mamdani is as follows:

\begin{align*}
\text{Rule 1: } & \text{ if } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } B_1 \text{ then } y \text{ is } C_1 \\
\text{Rule 2: } & \text{ if } x_1 \text{ is } A_2 \text{ and } x_2 \text{ is } B_2 \text{ then } y \text{ is } C_2 \\
& \ldots \quad \ldots \quad \ldots \\
\text{Rule n: } & \text{ if } x_1 \text{ is } A_n \text{ and } x_2 \text{ is } B_n \text{ then } y \text{ is } C_n
\end{align*}

$A_i$, $B_i$ and $C_i$ are fuzzy subsets and they are defined by the corresponding membership functions, i.e., $\mu_{A_1}$, $\mu_{B_1}$ and $\mu_{C_1}$. There are different forms of membership functions such as triangular, trapezoidal, sigmoidal and Gaussian, etc., for input and output variables. Triangular membership shape function is specified by three parameters \{a, b, c\} as follows:

$$\text{Triangle}(x: a, b, c) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases}. \quad (7)$$

Trapezoidal membership function depends on four parameters $a, b, c$ and $d$, as given by the following equation:

$$\text{Trapesoidal} (x: a, b, c, d) = \begin{cases} 0 & x < a \\ \frac{(x-a)(b-a)}{(b-a)(a-b)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{(d-x)(d-c)}{(d-c)(d-a)} & c \leq x \leq d \\ 0 & x \geq d \end{cases}. \quad (8)$$

For the two input values of the fuzzy logic unit, the membership function of the output of fuzzy reasoning can be expressed as the following equation:

$$\mu_{C_0}(y) = (\mu_{A_1}(x_1) \wedge \mu_{B_1}(x_2)) \vee (\mu_{A_2}(x_1) \wedge \mu_{B_2}(x_2)) \ldots (\mu_{A_n}(x_1) \wedge \mu_{B_n}(x_2)), \quad (9)$$

where $\wedge$ is the minimum operation and $\vee$ is the maximum operation.

The defuzzifier converts the fuzzy value into crisp value. There are few methods of defuzzification described in the literature (Yen and Langari, 1999). The center of gravity is
most commonly used (Rezaeia and Dowlotshahi, 2010). This technique calculates the center of the area of the combined membership function as the following equation:

$$y_0 = \frac{\sum y\mu_{C_0}(y)}{\sum \mu_{C_0}(y)}, \quad (10)$$

where $\mu_{C_0}(y)$ is degree of membership of $y$ to fuzzy set $C_0$ and $y$ is the crisp value of the output.

4. Proposed integrated SMED methodology

The tools utilized in the conventional SMED methodology implementation may not be sufficient to reduction the setup time. It is clear that more sophisticated methods are needed. A model that integrates the Taguchi method with the traditional SMED method was developed. In this model, it is aimed to reduce the setup time by the Taguchi method, which is defined as internal activity and cannot be converted to external activity by the SMED application. The proposed model consists of three main parts (Figure 2). In the first step, the setup times are reduced by applying the traditional SMED methodology. In the second step, the Taguchi method is applied to determine the factors which are effective in reducing time. In the third step, the factor levels of the effective factors determined by the fuzzy Taguchi method are determined. In this study, the Taguchi method was combined with the fuzzy logic model to find the combination of process parameters that optimize time of setup activity. The application of fuzzy logic is due to the fact that factors and levels are determined by experts with limited information in uncertain environments. In the conventional Taguchi method, the optimal set of factor levels consists of one of the levels initially defined for each factor. But often the intermediate values of the levels give more optimal values. The fuzzy logic provides to determine intermediate values of levels.

![Flow chart of the proposed integrated SMED method](image-url)
5. Implementation of proposed integrated SMED model

It was decided to apply new integrated SMED method to reduce the setup time in XYZ Company in Turkey. The company produces aluminum profiles. The setup operations of all the machines chosen were observed before SMED was implemented and the time taken for setup of each machine was noted. The CNCx machine was chosen that takes longer setup times compared to other machines’ setup times. The details of the setup process of product on the CNCx were recorded on a video camera. It was determined that part no. 1111 has longest total setup time.

5.1 Implementing SMED method

Total setup time of part was broken down into 19 setup activities. Activities’ numbers, times and descriptions were recorded on setup observation and analysis form, respectively. In current practice, all of the activities of SMED were determined as internal activities. Internal was written for all activities in internal/external column on the form. Improvement proposal for conversion of internal activities into external activities and reducing time of activity has been developed.

New time has been measured for each improved activity since the suggestions were implemented (Table I).

5.2 Implementing Taguchi method

The Taguchi experimental design method was applied to activity 15 (control of first produced part by an operator) which is internal activity. In this activity, the part is removed from the CNCx machine and moved to the control by operator. The chosen factors accepted as important and their levels are summarized in Table II. Since operator’s qualification is effective on time, two different operators (O₁ and O₂) were evaluated. Similarly, it was assumed that winch speed, screwdriver speed and caliper tool are effective on time. Each factor has two levels.

In addition, interactions among factors were not taken into consideration. The S/N ratio was determined as smaller-the-better for activity time. Hereby, \( L_8 \) orthogonal array was chosen according to total degree of freedom (Table III). All the eight experiments were randomly repeated twice.

The graphs of the factor levels according to the S/N ratio are as shown in Figure 3.

ANOVA analysis was applied according to the S/N ratio of the test results in order to determine the effective factors on the setup time. Minitab 16 was used for ANOVA analysis and the results obtained are as in Table IV.

Factors A, B and C are effective but factor D is not effective as shown in Table IV.

5.3 Implementing fuzzy Taguchi method

Each effective factor was considered as an input variable for fuzzy logic. The time of activity was taken as the output factor. The membership function of factor A was determined as trapezoidal, and factors B and C as triangular. Factor A was divided into two ranges of values, Factors B and C were divided into three ranges of values: slow, medium and fast. The activity time was evaluated in five ranges of very low, low, medium, high and very high. A total of 18 (2 × 3 × 3) fuzzy rules were derived. The values of the output were obtained by the determined rules. The 18 rules required to initiate the fuzzy system are similar to the following example:

- If the operator is O₁ AND the winch speed is slow AND the screwdriver speed is slow then the time is very high.
<table>
<thead>
<tr>
<th>No.</th>
<th>Description</th>
<th>Before activity</th>
<th>After activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Time (min)</td>
<td>Suggestions for time reduction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Internal/external</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Bringing the overhead crane to in front of the CNC machine</td>
<td>8 Internal</td>
<td>Bringing overhead crane in front of the CNC machine before the end of the previous production</td>
</tr>
<tr>
<td>2</td>
<td>Taking the fixture from the CNC machine</td>
<td>7 Internal</td>
<td>Not</td>
</tr>
<tr>
<td>3</td>
<td>Taking left diffuser from the CNC machine</td>
<td>5 Internal</td>
<td>Not</td>
</tr>
<tr>
<td>4</td>
<td>Putting the fixture which is received from the CNC machine, on the shelf of fixture</td>
<td>8 Internal</td>
<td>Moving the fixture near the CNC machine and then carrying to the shelf of fixture while processing the second piece</td>
</tr>
<tr>
<td>5</td>
<td>New fixture placed near the CNC machine after taking the shelf of fixture</td>
<td>8 Internal</td>
<td>Bringing new fixture to the CNC machine before the end of the previous production</td>
</tr>
<tr>
<td>6</td>
<td>Cleaning the CNC machine</td>
<td>4 Internal</td>
<td>Not</td>
</tr>
<tr>
<td>7</td>
<td>Putting the fixture on the platform</td>
<td>9 Internal</td>
<td>The fixture is placed on the tray slides for easier settlement and housing to the point that its locks were opened</td>
</tr>
<tr>
<td>8</td>
<td>Determining the position of fixture on the platform and fixing</td>
<td>14 Internal</td>
<td>Adjusting process was designed to be a reference part of the plane. This plane has provided faster and easier settlement of fixture</td>
</tr>
<tr>
<td>9</td>
<td>Creating plane of reference for datum point in X-axis</td>
<td>11 Internal</td>
<td>It was determined to be the datum point within the same plane of reference, which is designing activity number 8, in X-axis</td>
</tr>
<tr>
<td>10</td>
<td>Adapting initial processing of the pistons on the fixture</td>
<td>6 Internal</td>
<td>It was decided pistons that take place in the liquid used in the production and located on the CNC machine whether to add liquid and do necessary fixes or not to be ready when producing the second piece</td>
</tr>
<tr>
<td>11</td>
<td>Controlling of cutting tools on the CNC machine and inserting into the magazine</td>
<td>17 Internal</td>
<td>Keeping the tools close to the CNC machine</td>
</tr>
<tr>
<td>12</td>
<td>Taking datum point in X-axis and preparing computer program</td>
<td>4 Internal</td>
<td>It was decided to plane of reference piece which is eighth designing step used in this process</td>
</tr>
<tr>
<td>13</td>
<td>Setting the piece on the fixture</td>
<td>6 Internal</td>
<td>Not</td>
</tr>
<tr>
<td>14</td>
<td>Production of the first piece</td>
<td>25 Internal</td>
<td>Not</td>
</tr>
<tr>
<td>15</td>
<td>Moving the first produced piece to control panel by the operator and control</td>
<td>21 Internal</td>
<td>Fuzzy Taguchi method can be applied to reduce time of the activity</td>
</tr>
<tr>
<td>16</td>
<td>Taking the first produced piece to 3D inspection</td>
<td>6 Internal</td>
<td>Taking the first produced piece away from 3D inspection by operator when processing the second piece in the CNC machine</td>
</tr>
<tr>
<td>17</td>
<td>3D inspection</td>
<td>25 Internal</td>
<td>It was decided that the operator would make the necessary preparations for the next production instead of waiting for the track to be controlled</td>
</tr>
</tbody>
</table>

Table I.  
Setup observation and analysis form  
(continued)
Before activity | After activity
--- | ---
No. | Description | Time | Internal/external | Suggestions for time reduction | Time | Internal/external
18 | Getting back the piece to the workshop | 6 | Internal | Taking controlled piece away from 3D inspection by operator and bringing back to workshop when processing the second piece in the CNC machine | 0 | External
19 | Retouching in the computer software for the second piece that will be processed | 6 | Internal | During the 3D examination of this piece of the process, 17th step was proposed | 6 | Internal

Table I.

<table>
<thead>
<tr>
<th>Factors</th>
<th>Symbols</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator</td>
<td>A</td>
<td>$O_1$</td>
<td>$O_2$</td>
</tr>
<tr>
<td>Winch speed (m/min)</td>
<td>B</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Screwdriver speed (rev/min)</td>
<td>C</td>
<td>90</td>
<td>2200</td>
</tr>
<tr>
<td>Caliper tool</td>
<td>D</td>
<td>Vernier</td>
<td>Digital</td>
</tr>
</tbody>
</table>

Table II.

<table>
<thead>
<tr>
<th>Experiment no.</th>
<th>Operator</th>
<th>Winch speed</th>
<th>Screwdriver speed</th>
<th>Caliper tool</th>
<th>Average time</th>
<th>S/N time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$O_1$</td>
<td>8</td>
<td>90</td>
<td>Vernier</td>
<td>21.75</td>
<td>$-26.74$</td>
</tr>
<tr>
<td>2</td>
<td>$O_1$</td>
<td>8</td>
<td>2200</td>
<td>Digital</td>
<td>19.25</td>
<td>$-25.68$</td>
</tr>
<tr>
<td>3</td>
<td>$O_1$</td>
<td>9</td>
<td>90</td>
<td>Digital</td>
<td>19.50</td>
<td>$-25.80$</td>
</tr>
<tr>
<td>4</td>
<td>$O_1$</td>
<td>9</td>
<td>2200</td>
<td>Vernier</td>
<td>17.25</td>
<td>$-24.73$</td>
</tr>
<tr>
<td>5</td>
<td>$O_2$</td>
<td>8</td>
<td>90</td>
<td>Digital</td>
<td>14.25</td>
<td>$-23.07$</td>
</tr>
<tr>
<td>6</td>
<td>$O_2$</td>
<td>8</td>
<td>2200</td>
<td>Vernier</td>
<td>12.75</td>
<td>$-21.11$</td>
</tr>
<tr>
<td>7</td>
<td>$O_2$</td>
<td>9</td>
<td>90</td>
<td>Vernier</td>
<td>12.25</td>
<td>$-21.76$</td>
</tr>
<tr>
<td>8</td>
<td>$O_2$</td>
<td>9</td>
<td>2200</td>
<td>Digital</td>
<td>9.25</td>
<td>$-19.32$</td>
</tr>
</tbody>
</table>

Table III.

Figure 3.

Factor levels according to the signal-to-noise ratio
The rules were derived from Matlab R2013 program. The minimum time was 9 min when the operator was number 2, winch speed was 13 m/min and screwdriver speed was 2,000 rev/min. The minimum time of 9 min was also found with the traditional Taguchi method. The fuzzy logic gave this time with smaller winch speed and screwdriver speed. This gives a cost advantage.

6. Results and conclusion

It is possible to shorten the setup times by the conventional SMED method. In this study, the SMED method was applied to part 1111 processed on the CNCx machine and setup time was reduced from 196 to 87 min. A gain of 109 min or 55.61 percent was achieved, but there were setup activities that cannot be shortened with conventional SMED tools. It was continued to study on activity 15 that remained as the internal activity after SMED and the time could be reduced. More advanced methods are needed to determine the parameters that will give the optimum setup time. It takes time to apply trial and error method to determine factors of setup activities time and their levels. For this reason, the Taguchi method was used. The conventional Taguchi method was used to determine the factors that were effective in the time of the activity. It was determined that three factors (A, B and C) were effective and one factor (D) was ineffective. In all, 9 min with the optimum set of factor levels, $A_2B_2C_2$, were obtained. Thus, activity time of 21 min could be reduced to 9 min. The fuzzy logic was applied to determine the levels of predefined effective factors that would give better optimum time. In this study, optimum time achieved fuzzy logic was determined the same as the one predetermined. But the level values were lower for the two factors. The optimum activity time was determined as 9 min when factor $A$ was $O_2$, factor $B$ was 13 m/min and factor $C$ was 2,000 rev/min. The lower the speed, the lower the cost, so the same optimum result can be handled at a lower cost. The improvements provided by the methods are as in Table V.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Before Time (min)</th>
<th>After Time (min)</th>
<th>Factors levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMED</td>
<td>196</td>
<td>87</td>
<td>$A_1B_1C_1D_1$</td>
</tr>
<tr>
<td>Taguchi method</td>
<td>21</td>
<td>9</td>
<td>$A_1B_2C_2$</td>
</tr>
<tr>
<td>Fuzzy Logic</td>
<td>21</td>
<td>9</td>
<td>$A$ (operator 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$B$ (13)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$C$ (2000)</td>
</tr>
</tbody>
</table>

Table V. Results

<table>
<thead>
<tr>
<th>Factor</th>
<th>Degree of freedom</th>
<th>Sum of squares</th>
<th>Mean of square</th>
<th>$F$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operator (A)</td>
<td>1</td>
<td>69.742</td>
<td>69.742</td>
<td>354.220</td>
<td>0.000</td>
</tr>
<tr>
<td>Winch speed (B)</td>
<td>1</td>
<td>9.007</td>
<td>9.007</td>
<td>45.750</td>
<td>0.000</td>
</tr>
<tr>
<td>Screwdriver speed (C)</td>
<td>1</td>
<td>7.649</td>
<td>7.649</td>
<td>38.850</td>
<td>0.000</td>
</tr>
<tr>
<td>Caliper tool (D)</td>
<td>1</td>
<td>0.540</td>
<td>0.540</td>
<td>2.740</td>
<td>0.126</td>
</tr>
<tr>
<td>Error</td>
<td>3</td>
<td>2.166</td>
<td>0.197</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>89.103</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table IV. ANOVA results for S/N ratio

Note: SD = 0.4437; $R^2 = 97.57$ percent; Adj. $R^2 = 96.69$ percent
References


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Fix-and-optimize heuristics for capacitated lot sizing with setup carryover and backordering

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Abstract

Purpose – The capacitated lot sizing problem (CLSP) is one of the most important production planning problems which has been widely studied in lot sizing literature. The CLSP is the extension of the Wagner-Whitin problem where there is one product and no capacity constraints. The CLSP involves determining lot sizes for multiple products on a single machine with limited capacity that may change for each planning period. Determining the right lot sizes has a critical importance on the productivity and success of organizations. The paper aims to discuss these issues.

Design/methodology/approach – This study focuses on the CLSP with setup carryover and backordering. The literature focusing on this problem is rather limited. To fill this gap, a number of problem-specific heuristics have been integrated with fix-and-optimize (FOPT) heuristic in this study. The authors have compared the performances of the proposed approaches to that of the commercial solver and recent results in literature. The obtained results have stated that the proposed approaches are efficient in solving this problem.

Findings – The computational experiments have shown that the proposed approaches are efficient in solving this problem.

Originality/value – To address the solution of the CLSP with setup carryover and backordering, a number of heuristic approaches consisting of FOPT heuristic are proposed in this paper.

Keywords Heuristic approaches, Production planning, Backordering, Capacitated lot sizing problem, Setup carryover

Paper type Research paper

Introduction

Lot sizing problems are production planning problems with the objective of determining the periods where production should take place and the quantities to be produced. Determining the right lot sizes is important and critical in order to minimize the overall cost since lot sizing decisions have important effects on the efficiency of the production and inventory systems. The literature on lot sizing is quite rich and problems with different features have been studied by various authors. Among these problems, the capacitated lot sizing problem (CLSP) has been the most extensively studied one. The CLSP consists of determining the production quantity and timing for several items on a single facility over a finite number of periods so that the demand and capacity constraints can be satisfied at a minimum cost. Recently, a new feature called setup carryover has been added to the problem and a new problem type has arisen. The CLSP with setup carryover, which is a combination of big bucket and small bucket models, allows producing more than one product per period and also carrying a setup of at most one product from one period to the other. The first study focusing on the CLSP with setup carryover belongs to Sox and Gao (1999) where the authors use a shortest route representation to reformulate the mathematical programming model for the problem. The resulting reformulated problem is then solved using a Lagrangean decomposition heuristic. Gopalakrishnan et al. (2001) propose a Tabu Search (TS) heuristic which consists of five basic move types – three for the sequencing and two for the
lot-sizing decisions. An extended formulation and valid inequalities to solve this problem under the assumption that a setup state for the same product can be conserved over two consecutive periods have been presented by Suerie and Stadtler (2003). The authors also propose a time decomposition heuristic to solve the same problem both for single and multi-level case. Sequence-dependent setup costs and times have been added to the CLSP with setup carryover in Gupta and Magnusson (2005). Almada-Lobo et al. (2007) present two novel mixed integer programming (MIP) formulations for the real-world CLSP with setup carryover, sequence dependent setup times and costs in the glass container industry. A GRASP meta-heuristic is proposed in Nascimento and Toledo (2008) to solve the CLSP with setup carryover in a multi-plant setting. Another extension to this problem is to permit backordering of unsatisfied demand. Two formulations for the uncapacitated and CLSP with backordering are presented in Pochet and Wolsey (1998). Millar and Yang (1994) develop Lagrangean heuristics for solving the problem. The authors also present extensive computational results assessing the performances of the proposed heuristics. Cheng et al. (2001) deal with the CLSP with backorder consideration under the various types of production capacities such as regular time, overtime and subcontracting. To solve this problem, the authors propose a heuristic which is based on the special structure of fixed charge transportation problem. Absi and Kedad-Sidhoum (2007) study this problem under the assumption that demand cannot be backordered, rather it is totally or partially lost. This study focuses on the CLSP with setup carryover and backordering abbreviated to CLSP⁺. During the survey of current relevant literature, only a few study has been noted in this area (Karimi and Ghomi, 2002; Karimi et al., 2006; Quadt and Kuhn, 2009). This could be due to the complexity of the problem which makes it difficult to produce effective heuristics. Moreover, in practical problems, usually it is very difficult to estimate accurately the values of the unit backorder costs (Millar and Yang, 1994). Karimi and Ghomi (2002) propose a greedy heuristic consisting of four stages to deal with this problem. In another study (Karimi et al., 2006), the authors propose a TS-based approach and use this greedy heuristic to find a feasible initial solution. However, possibly to reduce the complexity of the problem the authors do not take into account the setup times. Motivated by a real-life example, Quadt and Kuhn (2009) propose a new solution procedure to deal with the CLSP⁺ with parallel machines. The proposed method aggregates the lot sizing and scheduling parts of the model, which uses integers instead of binary variables. Recently, Goren Guler and Tunah (2016) propose heuristic approaches consisting of Genetic Algorithms (GAs) and fix-and-optimize (FOPT) heuristic for solving the CLSP⁺. The authors also investigate the sensitivity of two best performing approaches to changes in problem-specific parameters including backorder costs, setup times, setup costs, capacity utilization and demand variability. Considering the features of the problem, commercial solvers are not able to solve large size lot sizing problems in reasonable computational time (Goren Guler and Tunah, 2016). The other alternative is to use heuristic methods which cannot assure the optimality of the solution found but can find good solutions in a very short time. In recent years, researchers have widely used the FOPT heuristic in solving lot sizing problems with different modeling features (Sahling et al., 2009; Helber and Sahling, 2010; Lang and Shen, 2011; Goren Guler et al., 2012; Seanner et al., 2013; Xiao et al., 2013; Goren Guler and Tunah, 2015, 2016; C.L. Chen, 2015). FOPT is a MIP based heuristic which decomposes the large problem into smaller ones to obtain a solution. However, during this survey study, only a few number of studies (H. Chen, 2015; Goren Guler and Tunah, 2016) utilizing FOPT heuristic has been noted to solve CLSP⁺. To fill this gap and address the solution of the CLSP⁺, in this study, we integrated a number of problem specific heuristics with FOPT heuristic and tested their performances on different size of problems. The results of experiments show that utilizing problem specific information improves the effectiveness of FOPT heuristic.
The rest of the study is organized as follows. Background information about the problem studied is given in the second section. The third section explains the proposed approaches in detail. The computational results are given in the fourth section. The fifth section concludes the study with some future research directions.

**Background information**

The objective in the CLSP\(^+\) is to determine the production quantities and timing of the products along with the semi-sequencing (i.e. first and last product produced in a period) in a period. If the demand cannot be satisfied in a period, it is backordered meaning that it is satisfied by production in the following periods during the planning horizon. In solving the CLSP\(^+\), the model proposed by Suerie and Stadtler (2003) has been modified and backordering costs and constraints are added to the model.

Throughout the paper, the indices for a product, and period are denoted by \(j = 1, 2, 3, ..., K\) and \(t = 1, 2, 3, ..., P\), respectively. \(X_{jt}\) is defined as the quantity of product \(j\) produced in period \(t\). The variables \(I^+_{jt}\) and \(I^-_{jt}\) denote inventory and backorder levels for product \(j\) at the end of period \(t\), respectively. The binary variable \(Y_{jt}\) is equal to 1 if a setup for product \(j\) is performed in period \(t\). The setup carryover variable, which is a binary variable, is denoted by \(W_{jt}\). This variable is set to 1 if a setup state for product \(j\) is carried from period \((t-1)\) to \(t\). The binary variable \(Q_t\) is the single product variable which equals one if the resource is occupied solely by product \(i\) in period \(t\). Moreover, \(sc, h_j, C_t, a_j, s_j, d_{jt}, b_j\) denote the setup cost for product \(j\), the unit holding cost for product \(j\) in period \(t\), the amount of resource available in period \(t\), the time to process one unit of product \(j\), the setup time of product \(j\), the demand for product \(j\) in period \(t\), and the backordering cost of product \(j\), respectively. Finally, \(M\) is a sufficiently large number:

\[
\text{Min} \sum_{j=1}^{K} \sum_{t=1}^{P} \left( sc \cdot Y_{jt} + h_j \cdot I^+_{jt} + b_j \cdot I^-_{jt} \right)
\]

s.t:

\[
I^+_{jt-1} + X_{jt} - I^+_{jt} - I^-_{jt-1} + I^-_{jt} = d_{jt} \quad \forall j \in K; \quad \forall t \in P
\]

\[
\sum_{j \in K} (a_j \cdot X_{jt} + s_j \cdot Y_{jt}) \leq C_t \quad \forall t \in P
\]

\[
\sum_{j \in K} W_{jt} \leq 1 \quad \forall t \in \{2, ..., P\}
\]

\[
W_{jt} \leq Y_{jt-1} + W_{jt-1} \quad \forall j \in K, \quad \forall t \in \{2, ..., P\}
\]

\[
W_{jt+1} + W_{jt} \leq 1 + Q_t \quad \forall t \in \{1, ..., P-1\}
\]

\[
Y_{jt} + Q_t \leq 1 \quad \forall t \in \{1, ..., P-1\}
\]

\[
X_{jt} \leq M \cdot (Y_{jt} + W_{jt}) \quad \forall j \in K, \quad \forall t \in \{1, ..., P\}
\]

\[
Q_t \geq 0 \quad \forall t \in \{1, ..., P-1\}
\]
The objective function (1) minimizes the total inventory, backorder and setup costs. Inventory balance Equation (2) describes the evolution of the inventory and backorder over time and require that the demand is satisfied either by production, inventory or backorder. There is a limited capacity available in each period, and both production and setup times are taken into account (Constraint (3)). At most, one setup state can be carried over from one period to the next on the resource (Constraint (4)). Constraints (5) impose that a setup can be carried over to period \( t \) only if either the item is setup in the previous period or if the setup state is already carried from period \( t-2 \) to \( t-1 \). A setup state for a specific item can only be preserved over two bucket boundaries if that product is the only one produced in the middle period (Constraint (6)), which is only possible if there is no setup in the middle period (Constraint (7)). Constraints (8) enforce either a setup or a carryover if there is strictly positive production for a specific item. Further, the non-negativity and binary conditions are shown in Constraints (9)–(11). There are no setup carryovers in the first period (Constraint (10)) and all demand should be satisfied during the planning horizon (Constraint (12)).

**The proposed approaches**

The proposed approaches in this study consist of FOPT heuristic. FOPT heuristic is a MIP-based heuristic in which a sequence of MIP models is solved over all real-valued decision variables and a subset of the binary variables. It should be noted that the numerical effort required to solve the MIP model is determined by the number of binary setup and setup carryover variables (Gören Güner and Tunalı, 2015). In recent years, the FOPT heuristic has been widely used in lot sizing literature. FOPT heuristic is applied to solve the multi-level CLSP with setup times and setup carryover in Sahling et al. (2009). The authors test product, resource and process oriented decomposition strategies. Lang and Shen (2011) develop FOPT and relax-and-fix heuristics to address the single machine CLSP with item substitution options. Another heuristic approach combining the principles of Variable Neighborhood Decomposition Search and FOPT heuristic is presented in Seanner et al. (2013) to solve the multi-level lot sizing and scheduling problems. New FOPT-based heuristics are presented in Xiao et al. (2013) to deal with the CLSP with sequence-dependent setup times, time windows, machine eligibility and preference constraints. H. Chen (2015) propose a FOPT heuristic for two dynamic multi-level CLSP with setup times and setup carryover which yields superior results when compared to those obtained in Helber and Sahling (2010). Recently, Gören Güner and Tunalı (2016) apply FOPT heuristic to improve the performance of GAs in solving the CLSP+

The idea in the FOPT heuristic is to systematically solve a series of sub-problems that are derived from the model stated in the previous section. In each iteration of the algorithm, one sub-problem is solved by setting some of the binary setup and carryover variables to fixed values based on the decomposition schemes. This reduction leads to a limited number of non-fixed binary variables which are optimized in a given sub-problem. Then the problem is solved using a standard MIP solver (Gören Güner and Tunalı, 2015). In the next iteration, a new sub-problem with a different subset of fixed binary variables is solved. The solution with respect to the binary variables is a fixed parameter for the next MIPs that optimize other binary variables (Helber and Sahling, 2010). Therefore, in each sub-problem, the complete set of real-valued decision variables is considered. The optimization of the model is done within the MIP solver and therefore the approach is flexible (Sahling et al., 2009). For more information, the reader can refer to Pochet and Wolsey (2003).
**Decomposition schemes**

In this study, the sub-problems in FOPT are built using time and product decompositions. Simply, a sub-problem can be derived from the model given in the second section by adding the following constraints (Gören Güner et al., 2012):

\[ Y_{jt} = \overline{Y}_{jt} \quad \forall (j,t) \in KP_{Y,s}^{fix} \]  
\[ W_{jt} = \overline{W}_{jt} \quad \forall (j,t) \in KP_{W,s}^{fix} \]  

The explanations of the parameters used above can be found in Table I. FOPT heuristic starts with an initial solution. This initial solution yields an initial objective value which is shown by \( Z_{new} \) (see Figure 1). After initialization, the algorithm iterates through the ordered set of sub-problems according to time or product decompositions either once (\( l = 1 \)) or until it reaches a local optimum or predetermined number of iterations (\( l_{max} \)). It should be noted that a capacity infeasible solution is never considered as a candidate for the best solution.

The decomposition types used in this study are explained in the following.

### Sets

- \( (j,t) \in KP \): Set of all product-period combinations
- \( KP_{Y,s}^{opt} \subseteq KP \): Set of product-period combinations in which binary setup variables \( Y_{jt} \) are optimized
- \( KP_{W,s}^{opt} \subseteq KP \): Set of product-period combinations in which binary setup carryover \( W_{jt} \) variables are optimized
- \( KP_{Y,s}^{fix} \subseteq KP \): Set of product-period combinations in which binary setup variables \( Y_{jt} \) are fixed
- \( KP_{W,s}^{fix} \subseteq KP \): Set of product-period combinations in which binary setup carryover variables \( W_{jt} \) are fixed

### Parameters

- \( \overline{Y}_{jt} \): Exogenous value of the fixed setup variable \( Y_{jt} \)
- \( \overline{W}_{jt} \): Exogenous value of the fixed setup carryover variable \( W_{jt} \)

### Table I.

**Additional notation for the definition of a sub-problem**

<table>
<thead>
<tr>
<th>Source: Gören Güner and Tunali (2015)</th>
</tr>
</thead>
</table>

**Figure 1.**

The outline of the fix-and-optimize heuristic

<table>
<thead>
<tr>
<th>Capacitated lot sizing</th>
<th>Initial Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve the sub problem and determine the objective value ( Z_{new} = Z )</td>
<td></td>
</tr>
</tbody>
</table>

Repeat

\[ l = l + 1 \]

\[ Z_{old} = Z_{new} \]

For each sub problem by time decomposition do

Solve the sub problem and determine the objective value \( Z \)

IF \( Z < Z_{old} \)

THEN

\[ \overline{Y}_{kt} = Y_{kt} \quad \forall (k,t) \in PT_{Y,s}^{opt} \]

\[ \overline{W}_{kt} = W_{kt} \quad \forall (k,t) \in PT_{W,s}^{opt} \]

\[ Z_{new} = Z \]

ENDIF

Until

\( l = l_{max} \) or \( Z_{new} \geq Z_{old} \)
Time decomposition: to decompose the problem into smaller ones, the whole planning horizon \(P\) is divided into time windows and for each time window a sub-problem is formed. In each iteration of the algorithm, the variables related to one sub-problem are optimized while the other variables are fixed to the best values obtained so far. Starting from the first period, the proposed time decomposition algorithm optimizes the binary setup and setup carryover variables in each time window (Gören Güner et al., 2012).

Product decomposition: to form the sub-problems, this decomposition scheme focuses on products instead of time periods. Particularly, based on some product specific information such as setup costs and holding costs, first a priority is assigned to each product, and sub-problems are ordered according to these priorities. Starting with high priority product the relevant binary setup and setup carryover variables are optimized while the rest is set to the fixed values based on the best solution obtained so far. Similar to our previous study (Gören Güner and Tunali, 2016), the setup cost has been considered in forming the sub-problems using product decomposition.

The initial solution
An initial solution is needed to start the search in FOPT as indicated above. The initial solution is obtained by solving the linear programming (LP) relaxation of the CLSP in this study. Based on the values of the relaxation, two heuristics have emerged.

Initial Heuristic 1 (IH1): this heuristic rounds the fractional values of the LP relaxation solution to the nearest integer. For example, 0.4 becomes 0 whereas 0.8 becomes 1.

Initial Heuristic 2 (IH2): different from IH1, this heuristic rounds up the fractional values of the LP relaxation. All fractional values greater than 0 become 1 with this heuristic.

With different decomposition criteria, the FOPT heuristic is employed in eight different ways as follows:
- FOPT_1: IH1 and then FOPT heuristic with time decomposition;
- FOPT_2: IH2 and then FOPT heuristic with time decomposition;
- FOPT_3: IH1 and then FOPT heuristic with product decomposition;
- FOPT_4: IH2 and then FOPT heuristic with product decomposition;
- FOPT_5: IH1 and then FOPT heuristic with time decomposition followed by product decomposition;
- FOPT_6: IH2 and then FOPT heuristic with time decomposition followed by product decomposition;
- FOPT_7: IH1 and then FOPT heuristic with product decomposition followed by time decomposition;
- FOPT_8: IH2 and then FOPT heuristic with product decomposition followed by time decomposition.

The control logic of the proposed heuristics is given in Figure 2.

Computational results
Since published test problems with backorder costs were not available to evaluate the performances of our proposed hybrid approaches, the problem instances given in Trigeiro et al. (1989) were modified by adding the backorder costs. Specifically, to introduce backordering, the demands were modified and multiplied by 1.1, so that demand could not always be satisfied without backorders and backorder cost was defined as a linear function of the holding cost \(b = fh\), where \(f = 2\) as in Millar and Yang (1994). Table II presents the features of the data instances studied.
Since there is no study in the literature to which the performance of the proposed approaches can be compared, the lower bounds obtained from the simple plant location formulation of the MIP model were used in comparisons. The solution quality of the proposed approaches was measured by computing the deviation as follows:

$$\text{Gap} = 100 \times \frac{\text{heuristic solution} - \text{lower bound}}{\text{lower bound}}$$  \hspace{1cm} (15)$$

As a result of some preliminary tests, the computational time to solve each problem using FOPT heuristic was limited to 2 seconds. All computations were carried out on a PC with Dual Core, 2 GHz microprocessor and 2 GB RAM. All proposed approaches were coded in Visual C++ 2008 Express Edition and all sub-problems were solved using Concert Technology of Cplex 11.2.

In order to obtain compatible results, all the proposed heuristic approaches were run under the same computational time. The deviations from lower bound are stated in Table III. Based on preliminary experiments, descending order of the setup costs is used in the product decomposition employed in FOPT3, FOPT4, FOPT5, FOPT6, FOPT7 and FOPT8.

As seen in Table III, the FOPT heuristic cannot find feasible solution to all problem instances in all problem classes (i.e. FOPT2, FOPT4, FOPT6, FOPT8). Interestingly, the FOPT heuristic with the IH2 gives much better results than the FOPT with the IH1 in most of the problem classes. This could be attributed to the features of the initial heuristic used. While the implementation of IH1 requires the values obtained by the LP relaxation to round up to the nearest integer (i.e. 0.45 is rounded to 0), the implementation of IH2 requires just to round up. The value of 0.45 becomes 1 in this case and in overall IH2 results in greater

<table>
<thead>
<tr>
<th>Problem class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of products</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>6</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Number of periods</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Number of instances</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Table II. Classification of data instances
number of solutions with value of 1. However, with the IH1, since the initial solution might have more 0's the chance for the FOPT heuristic to start with an infeasible initial solution may increase and in turn the deviations may go up.

Comparison with the results in current relevant literature and research synthesis
This section compares the performance of the FOPT heuristics to the recent results reported in literature (Gören Güner and Tunahi, 2016) and the commercial solver (CPLEX). It should be noted that the same computational times given in Table III are used in order to obtain compatible results. Gören Güner and Tunahi (2016) propose eight different hybrid approaches for solving the CSLP*. These approaches consist of GAs and FOPT heuristics using these decomposition schemes which are named as GA-based approaches in the rest of the paper. The authors classify these heuristics in two groups as sequential and embedded approaches. In the sequential approaches, GAs and FOPT heuristics are run sequentially. The best solution obtained by running GAs for a predetermined number of generations, becomes the initial solution for the FOPT. On the other hand, in the embedded approaches, FOPT is embedded into the main loop of GAs. In every generation, a randomly selected solution becomes the initial solution for the FOPT where it is improved throughout the iterations. Then, the improved or at least the same solution is given to the GAs again. The control logic of these approaches can be seen in Figures 3 and 4. The interested reader can refer to Gören Güner and Tunahi (2016) for further information about these approaches.

![Figure 3. Sequential approaches in Goren Guner and Tunahi (2016)](image-url)

Table III. The results of FOPT heuristics

<table>
<thead>
<tr>
<th>Problem Class</th>
<th>FOPT1 (IH1)</th>
<th>FOPT2 (IH2)</th>
<th>FOPT3 (IH1)</th>
<th>FOPT4 (IH2)</th>
<th>FOPT5 (IH1)</th>
<th>FOPT6 (IH2)</th>
<th>FOPT7 (IH1)</th>
<th>FOPT8 (IH2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>0.21</td>
<td>0.18*</td>
<td>0.30</td>
<td>0.25*</td>
<td>0.21</td>
<td>0.17*</td>
<td>0.21</td>
<td>0.17*</td>
</tr>
<tr>
<td>Class 2</td>
<td>0.11</td>
<td>0.104</td>
<td>0.15</td>
<td>0.16*</td>
<td>0.099</td>
<td>0.096</td>
<td>0.15</td>
<td>0.098</td>
</tr>
<tr>
<td>Class 3</td>
<td>0.054</td>
<td>0.05</td>
<td>0.062</td>
<td>0.061*</td>
<td>0.048</td>
<td>0.051</td>
<td>0.062</td>
<td>0.046</td>
</tr>
<tr>
<td>Class 4</td>
<td>0.254</td>
<td>0.27*</td>
<td>0.363</td>
<td>0.35</td>
<td>0.263</td>
<td>0.27</td>
<td>0.363</td>
<td>0.26</td>
</tr>
<tr>
<td>Class 5</td>
<td>0.10</td>
<td>0.098</td>
<td>0.139</td>
<td>0.15</td>
<td>0.111</td>
<td>0.103</td>
<td>0.139</td>
<td>0.103</td>
</tr>
<tr>
<td>Class 6</td>
<td>0.065</td>
<td>0.066</td>
<td>0.069</td>
<td>0.067*</td>
<td>0.056</td>
<td>0.054*</td>
<td>0.069</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Note: *Indicates an infeasible solution found in this problem class with this heuristic.
The overall comparison of the proposed approaches to the recent results in literature and CPLEX are given in Table IV. Table IV reveals that the performance of the Pure GAs (PGA) is the worst and hybridizing GAs with FOPT improves the performance (Gören Güner and Tunali, 2016). Based on the results presented in Table IV, it is seen that the performances of the FOPT6 and FOPT8 are better than the performances of the GA-based approaches in classes 1, 2 and 3. However, these heuristics could not find feasible solutions to all instances in these classes whereas the GA-based approaches and CPLEX could find feasible solutions in the same computational time (i.e. FOPT8 for instances in Class 1). However, the performance of CPLEX is superior to both FOPT heuristics and the GA-based approaches in classes 1, 2 and 3. On the other hand, all GA-based approaches (i.e. H1, H2, H3, H4, H5, H6, H7 and H8) outperform CPLEX and FOPT heuristics in problem classes 4 and 5. The deviations obtained by the GA-based approaches are approximately half of the deviations obtained by CPLEX and FOPT heuristics. In class 6,
which is the largest problem size, H3, H4, FOPT5, FOPT6 and CPLEX have similar performances but performances of FOPT6 is slightly better than both hybrid approaches (H3, H4) and CPLEX.

The general conclusion which can be drawn from this comparative experimental study is that the proposed FOPT approaches outperform our earlier proposed approaches in solving the CLSP especially in small size problems. Even they could not find feasible solutions to all instances, the solution quality is quite good. Moreover, it has been observed that the performances of these approaches are affected by both the type of the decomposition scheme and also the sequence in which these decomposition schemes are used. In summary, the proposed FOPT6 and FOPT8 approaches using both decompositions schemes have better performances than other approaches.

**Conclusion**

Lot sizing is one of the most well-known optimization problem in production planning. Most of the previous studies in this field focus on solving the CLSP which is known to be NP-Hard. Including the setup carryover and backordering to the model makes it more complex so exact algorithms are not capable of producing good quality solutions in reasonable computational time. This study presents a number of heuristic approaches for the CLSP which are shown to be efficient for solving this problem. These heuristic approaches are based on the FOPT heuristic. Time and product decomposition schemes are used in order to decompose the problem. Therefore, eight different heuristic approaches are obtained and their performances are investigated in this paper. Moreover, the performances of the proposed approaches are compared to the recent results in literature and a commercial solver. Based on the results, it can be stated that the performance of the proposed approaches are quite good. Future research directions in this area can be listed as follows:

- the performance of proposed FOPT heuristics can be improved so that a feasible solution can be found to all instances;
- the CLSP can be extended to include the issues of sequence dependent set-up times and costs on parallel machines and the performance of the approaches can be investigated on this extended problem;
- the performance of the proposed approaches can be tested on multi-level CLSPs; and
- the multi-objective lot sizing problems including routing, scheduling and lot sizing decisions can be solved by using the proposed approaches.

**References**


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Analyzing Workforce 4.0 in the Fourth Industrial Revolution and proposing a road map from operations management perspective with fuzzy DEMATEL

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Abstract
Purpose – The purpose of this paper is threefold: first, to present a structural competency model; second, to remark new criteria for personnel selection in Industry 4.0 environment; and third, to contribute to the operations management literature by focusing on recruitment process in Industry 4.0 environment and supporting human resources activities with Industry 4.0 related criteria and point out a new research field in Industry 4.0.
Design/methodology/approach – Fuzzy DEMATEL has been used in the implementation. The study is conducted in a high-tech firm, which has started to modify its processes according to Industry 4.0, and introduces a new specific department that is responsible of this transformation. In total, 11 personnel selection criteria were presented and then assessed by experts through a fuzzy linguistic scale. Both importance order and causal relation between criteria are presented at the end of the study.
Findings – According to the results, the most important criteria in the selected firm are the ability of dealing with complexity and problem solving, thinking in overlapping process, and flexibility to adapt new roles and work environments. While cause group includes criteria such as knowledge on IT and production technologies, awareness of IT security and data protection, and ability of fault and error recovery, effect group includes flexibility to adapt new roles and work environments, organizational and processual understanding, and the ability to interact with modern interfaces.
Practical implications – Analytical thinking and system approach are the key topics for new supporting personnel selection criteria, which lead to the need for the skills and qualifications in decision making and process management. Results of the cause group criteria also indicate the importance of technical abilities such as coding, IT security and human-machine interfaces. On the other hand, effect group of the study emphasizes on the flexibility and interdisciplinary working structure that suggests the suitability of matrix organization in the companies which follow the Industry 4.0 trends. Moreover, team work comes forward as another key concept for organizations transforming to Industry 4.0.
Originality/value – The originality of this study appears on modeling of a competency structural model for Workforce 4.0 which is proposed as a road map, including the suggested set of related criteria and the fuzzy MCDM-based methodology for companies which alter their organizations according to Industry 4.0.
Keywords Fuzzy DEMATEL, Industry 4.0, Personnel selection, Workforce 4.0
Paper type Research paper

1. Introduction
Industrial revolutions were always triggered by technological developments in history. First, using steam and water power in industry, and then the use of electricity and moving to mass production, after that the use of IT and automation. All these developments in industry were the key concepts of first three industrial revolutions. Within the last decade, following technologies, such as cyber-physical systems (CPS) and Internet of Things (IoT), has led the new industrial revolution which was named as Industry 4.0 in Hannover Fair Event in 2011. Industry 4.0 has started to transform manufacturing environment completely in developed countries and is expected to spread all around the world during the next decades. This transformation process does not only affect the manufacturing systems but also there is a
significant effect on the nature of work, which alters the expectations from employees in industry as well. Especially, in the departments related to operations management, personnel selection criteria are expected to change due to changing job profiles. Therefore, it is important to state new criteria for firms and ways to evaluate them. However, even the research works related to technical aspects of Industry 4.0 are common in the literature, there are only few studies which focus on the changes in workforce and none of them is related to new personnel selection criteria for Industry 4.0 needs. Therefore, the goals of this study are as follows: to present a structural competency model and remark new personnel selection criteria and present an importance order and a causal relationship between personnel selection criteria by using one of the multi-criteria decision techniques, Fuzzy DEMATEL, in a Turkish high-technology firm which has started to modify its processes according to Industry 4.0 and introduce a new department which is responsible for this transformation. The selected method, DEMATEL, is useful for analyzing complex relations by using matrix operations, whereas the vagueness inherent in the evaluations of the experts is handled with fuzzy logic. The main features of fuzzy DEMATEL are to present the prioritization of the criteria and demonstrate causal relations, which give the opportunity to evaluate criteria in terms of their interrelations. Since the aim of this study is not only to prioritize the personnel selection criteria but also to evaluate their interdependencies that may affect the recruitment processes in Industry 4.0, fuzzy DEMATEL method is preferred by the authors. By doing this, research works aim to contribute to the operations management literature by focusing on recruitment process in Industry 4.0 environment and supporting human resources activities with Industry 4.0 related criteria.

2. Literature review
In this part of the study, overview of related research is presented. First, cores of Industry 4.0 and common concepts are discussed according the previous studies. After that, changes in position of employees in Industry 4.0 environments are presented under the heading of Workforce 4.0.

2.1 Industry 4.0 concepts
Since the beginning of industrialization, technological transformations have led to paradigm shifts, in other words industrial revolutions (Lasi et al., 2014). At the end of the 18th century, introducing mechanical production by using steam and water power has led the First Industrial Revolution. Mass production by using electricity was the key concept of Second Industrial Revolution at the end of 19th century. In 1990s, with the use of IT systems and automation, the Third Industrial Revolution occurred and today with the centralization of CPS the Fourth Industrial Revolution or in other words Industry 4.0 has started to change economy. Industrial revolutions in history have been summarized with their keywords in Figure 1.

Figure 1. Industrial revolutions and key concepts
Research works related to the Fourth Industrial Revolution are rapidly increasing in the literature and most of the studies are directly related to the manufacturing environment. Since the Industry 4.0 is at the beginning of its existence, most of the studies are based on creating conceptual models (Schuh et al., 2014; Toro et al., 2015; Kolberg and Zühlke, 2015). Fundamental concepts of Industry 4.0 were summarized as: smart factory, CPS, self-organization, new systems in distribution and procurement adaptation to human needs and corporate social responsibility (Lasi et al., 2014).

Main trends of Industry 4.0 are IoT, machine to machine technologies and CPS (Mosterman and Zander, 2016). Especially, research models and implementations of CPS in manufacturing environment are highly popular in the literature (Lee, 2008; Lee et al., 2015). CPS refers to transformative technologies for managing interconnected systems between its physical and computational assets. Lee et al. (2015) suggested a five-level architecture for CPS. These levels are as follows: smart connection level, data to information conversation level, cyber level, cognition level and configuration level. This study is significant since it can be used as a guideline while implementing CPS in a manufacturing environment. IoT is another important feature in Industry 4.0, and refers to the interconnection of physical objects, by equipping them with sensors, actuators and a means to connect to the internet (Dijkman et al., 2015). It is an older concept than Industry 4.0; IoT has become more popular by the new industrial revolution. Moreover, the research fields related to Industry 4.0 are summarized as individualized production, horizontal integration in collaborative networks and end-to-end digital integration (Brettel et al., 2014).

Furthermore, Qin et al. (2016) focused on Industry 4.0 concepts in manufacturing visions under four headings including, factory, business, products and customer. According to this study, future factory is going to be intelligent and control itself fully and manage the factory systems by itself. On the other hand, business network in Industry 4.0 is going to be self-organizing and transmit the real-time responses. Moreover, products are going to be smart by including sensors and identifiable components, and processors to carry information about customers and transmit them to manufacturing systems. Finally, Industry 4.0 is expected to give customers the opportunity of ordering fully customized products according to their needs (Qin et al., 2016).

All these changes, explained above, are expected to alter the job profiles of employees in different kinds of ways, and it is essential to focus that area as well. In the following section, changes in workforce in Industry 4.0 are explained with related research works.

2.2 Workforce 4.0

Industry 4.0 will not only affect the manufacturing systems, but also there will be a significant effect on workforce. All these changes in industrial environment directly affect the position of the workers on the field. Need for man power in a traditional way decreases due to high-tech machines which can communicate between each other and control themselves. This situation leads a significant transformation in job profiles. One kind of transformation is new industrial revolution allows new types of interactions between employees and machines (Romero et al., 2016).

In Figure 1, changes of job profile and competences (Dombrowski and Wagner, 2014) are given as a model. According to this model, decreasing executive production tasks and less specific work tasks are some of changes in job profiles. In Industry 4.0 environment, interdisciplinary cooperation and focusing on error recovery are some of important aspects that workers need to have. Self-organization, complexity and thinking in overlapping processes are other features. It is also expected that instead of detailed thinking on single processes or having specific technical capabilities, it is important to be a problem solver and a multitask worker in complex situations (Figure 2).

According to BCG September 2015 Report (Lorenz et al., 2015), Big data driven quality control, robot assisted production, self-driving logistics vehicles, production line...
simulation, smart supply network, predictive maintenance, machines as a service, self-organizing production, additive manufacturing of complex parts and augmented work, maintenance and service are some of example cases that are already used in manufacturing environment, thanks to Industry 4.0. All these changes transform the expectations towards workers.

Gehrke et al. (2015) summarized expectations from Industry 4.0 employees in Figure 3. According to the figure, first tier, tools and technologies indicate to any type of tools and technologies that the skilled employee utilizes and gets affected with, and second tier refers to tasks that Industry 4.0 employee needs to perform. On the other hand, third tier is the core of this model and refers to qualifications and skills that Industry 4.0 employee needs to have in order to use proper tools and technologies and complete the given tasks. In this study, focused area is the first tier of the model and includes skills and qualifications. Skills and qualifications of the new kind of employee in Industry 4.0 play the most important role in personnel selection criteria.

Recruitment in Industry 4.0 is a challenge for the companies due to significant changes in working environments. Therefore, new approaches need to be considered by companies to choose right employee.

Dombrowski and Wagner (2014) summarized the main differences between Industries 3.0 and 4.0 as technology-oriented work tasks in Industry 3.0 are changed to process-oriented tasks in Industry 4.0 and computer integrated manufacturing concept of
Industry 3.0 leaves its place to combination of automated processes and manual tasks in hybrid systems by focusing on human in the working system. This variation in production processes leads the need of employees who can execute complex manual tasks to control and manage machines and processes; in other words, human–machine interaction is very significant in Industry 4.0.

Focusing on capabilities instead of qualifications is suggested by Lorenz et al. (2015). Due to a high variety of tasks in Industry 4.0, there will be requirements for many new and emerging skills which may not be given in employees’ former education and even some of these former skills may become obsolete. Therefore, it is important to focus on proper skills for new roles in Industry 4.0 employees. Because of the changes outlined above, personnel selection criteria are expected to alter as well. Personnel selection by using MCDM methods is a popular topic in the related literature (e.g. Kelemenis and Askounis, 2010; Kabak et al., 2012; Dursun and Karsak, 2010; Baležentis et al., 2012; Liu et al., 2015). Some of traditional personnel selection criteria, which are used commonly, are strategic decision making, adaptability to change, interpersonal skills, leadership, emotional steadiness, self-confidence, past experience, etc. However, Industry 4.0 requires the employees to have hard and soft skills such as combining know-how related to each specific job or process, IT knowledge and abilities, organizational and processual understanding, ability to interact with modern interfaces, trust in new technologies, awareness of its security and data protection, flexibility to adapt new roles and work environments, continual interdisciplinary learning, interdisciplinary cooperation, ability of dealing with complexity and abstraction (Dombrowski and Wagner, 2014; Gehrke et al., 2015; Lorenz et al., 2015). These relatively new criteria can be used in Industry 4.0 recruitment process to support traditional personnel selection criteria.

3. Proposed competency structural model
The literature of Industry 4.0 and its aspects is developing rapidly; however, there is still a gap in the literature which is related to changes in job profiles and expectations from future employees, in other words features of Workforce 4.0. Even though the new and expected properties of employees in Industry 4.0 are presented in different sources (Dombrowski and Wagner, 2014; Gehrke et al., 2015; Lorenz et al., 2015; Romero et al., 2016), there is no published reports from studies focusing on new personnel selection criteria based on operations management in Industry 4.0. Therefore, this study supports human resources specifically in recruitment process from operations management perspective.

Due to increasing complexity and intelligence in Industry 4.0, there is a need for employees who have multidimensional aspects. Therefore, it is essential to present a road map for companies to select personnel selection criteria according to their needs. These criteria can assist the HR activities by focusing on unique needs of companies due to Industry 4.0 changes. Therefore, in Figure 4, proposed structural competency model of this study is presented.

Since there is a need for employees who have multidimensional aspects, due to increasing complexity and intelligence in Industry 4.0, it is essential to present a road map for companies to select personnel selection criteria according to their needs. These criteria can assist the HR activities by focusing on unique needs of companies due to Industry 4.0 changes. Therefore, in Figure 4, proposed structural competency model of this study is presented.

According to the model, companies need to identify the needs of Industry 4.0 in order to start the transformation process. Therefore, it is important to understand the core of the new industrial revolution. Second, it is necessary to identify the needs from Workforce 4.0. This part of the model represents understanding of changes in job profiles due to transformation of Industry 4.0.
At the third step, new phase of the model starts. Defining criteria for workforce 4.0 is the key part of the model because it includes analyzing company expectations and Industry 4.0 changes and integrating them in order to present most suitable criteria for the company.

Fuzzy DEMATEL method is selected for this model due to its multidirectional results. Comparing to other MCDM methods, fuzzy DEMATEL gives the opportunity to examine both prominence of criteria and the causal relation between them. This gives a wider chance to analyze the criteria and make a more reliable decision. Fuzzy logic also helps to represent and handle vagueness in this decision-making process. Results of the fuzzy DEMATEL method give both cause-effect groups and prioritization of criteria and these results can be used to support HR during the recruitment process.

This model can be seen as a road map for companies, which makes transformations in their organizations according to the industrial revolution.

4. Research methodology
In this study, one MCDM method, fuzzy DEMATEL, is used. In this part of the research, brief explanation of the fuzzy logic and details of fuzzy DEMATEL are given.

4.1 Fuzzy Logic
Fuzzy set theory was introduced by Zadeh in 1965 to solve fuzziness problem. It is a mathematical way to represent and handle vagueness in decision making. In fuzzy logic, each number between 0 and 1 indicates a partial truth, whereas crisp sets correspond to binary logic: 0 or 1. Therefore, fuzzy logic can express and handle vague or imprecise judgments mathematically. According to Zadeh (1965), a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one (Zadeh, 1965).

In the literature, it is seen that the most widely used fuzzy numbers are triangular and trapezoidal ones. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one. A tilde "~" will be placed above a symbol if the symbol represents a fuzzy set (Kahraman et al., 2003).
A triangular fuzzy number $\tilde{N}$ can be defined as a triplet $(l, m, r)$, and the membership function is defined as:

$$
\mu_{\tilde{N}}(x) = \begin{cases} 
0, & x < l \\
\frac{x - l}{m - l}, & l \leq x \leq m \\
\frac{r - x}{r - m}, & m \leq x \leq r \\
0, & x > r
\end{cases}
$$

(1)

where $l$, $m$ and $r$ are real number and $l \leq m \leq r$.

4.2 Fuzzy DEMATEL method

Decision Making Trial and Evaluation Laboratory (DEMATEL) is a suitable MCDM method to analyze complicated relationships with certain scores by using matrix operation. The DEMATEL method gives the opportunity to analyze the cause-effect relationship. This is also the main feature of the DEMATEL method. The DEMATEL method, developed by the Science and Human Affairs Program of the Battelle Memorial Institute of Geneva between 1972 and 1976, can convert the relationship between the causes and effects of criteria into an intelligible structural model of the system. In recent years, it has become more popular among researchers and has been used widely (Wu and Lee, 2007; Yang and Tzeng, 2011). Details of the DEMATEL method are shown below:

**Definition 1.** The pair-wise comparison scale may be designated five levels, where the scores of 0, 1, 2, 3 and 4 represent “No influence,” “Very Low Influence,” “Low influence,” “High influence,” and “Very High Influence,” respectively.

**Definition 2.** The initial direct-relation matrix $Z$ is a $n \times n$ matrix obtained by pair-wise comparisons in terms of influences and directions between criteria, in which $z_{ij}$ is denoted as the degree to which the criterion $i$ affects the criterion $j$, i.e. $Z = [z_{ij}]_{n \times n}$.

**Definition 3.** The normalized direct-relation matrix $X$, i.e. $X = [x_{ij}]_{n \times n}$ and $0 \leq x_{ij} \leq 1$ can be obtained through formulas (1) and (2), in which all principal diagonal elements are equal to zero:

$$
X = s \cdot Z,
$$

(2)

$$
s = \frac{1}{\max_{i,j} z_{ij} \sum_{j=1}^{n} z_{ij}},
$$

(3)

**Definition 4.** The total-relation matrix $T$ can be acquired by using formula (3), in which the $I$ is denoted as the identity matrix:

$$
T = X(I-X)^{-1}.
$$

(4)

**Definition 5.** The sum of rows and the sum of columns are separately denoted as $D$ and $R$ within the total-relation matrix $T$ through formulas (4)–(6):

$$
T = t_{ij}, \ i,j = 1,2,\ldots,n
$$

(5)
where $D$ and $R$ denote the sum of rows and the sum of columns, respectively.

In reality, most of decisions include imprecision because aims, constraints and possible actions are not known exactly (Bellman and Zadeh, 1970). While making decisions in this fuzzy environment, the result of decision making is highly affected by subjective judgments that are vague and inexact. The sources of imprecision include: unquantifiable information, incomplete information, non-obtainable information and partial ignorance (Chen and Hwang, 1992).

There are several useful defuzzification methods that may be divided into two classes by considering either the vertical or the horizontal representation of possibility distribution (Qussalah, 2002). In this study, Converting Fuzzy data into Crisp Scores (CFCS) defuzzification method is used. The CFCS method proposed by Opricovic and Tzeng (2003) is based on the procedure of determining the left and right scores by fuzzy min and fuzzy max, and the total score is determined as a weighted average according to the membership functions. Let $z_{ij}^k = (l_{ij}^k, m_{ij}^k, r_{ij}^k)$ indicate the fuzzy assessments of evaluator $k$ ($k = 1, 2, \ldots, p$) about the degree to which the criterion $i$ affects the criterion $j$. The CFCS method includes five-step algorithms described as follows:

1. **Normalization:**
   \[ x_{lk}^i = \frac{l_{ij}^k - \min l_{ij}^k}{\Delta_{\min}}, \]
   \[ x_{mk}^i = \frac{m_{ij}^k - \min l_{ij}^k}{\Delta_{\max}}, \]
   \[ x_{rk}^i = \frac{r_{ij}^k - \min l_{ij}^k}{\Delta_{\max}}, \]
   where $\Delta_{\max} = \max r_{ij}^k - \min l_{ij}^k$.

2. **Compute left (ls) and right (rs) normalized values:**
   \[ xls_{ij}^k = x_{mk}^i / \left(1 + x_{mk}^i - xls_{ij}^k\right), \]
   \[ xrs_{ij}^k = x_{rk}^i / \left(1 + x_{rk}^i - xrs_{ij}^k\right). \]

3. **Compute total normalized crisp value:**
   \[ x_{ij}^k = \left[ xls_{ij}^k \left(1 - xls_{ij}^k\right) + xrs_{ij}^k xrs_{ij}^k \right] / \left[1 - xls_{ij}^k + xrs_{ij}^k\right]. \]
(4) Compute crisp values:

\[ z^k_{ij} = \min l^k_{ij} + x^k_{ij} \Delta_{\text{max}}. \]  

(14)

(5) Integrate crisp values:

\[ z^k_{ij} = \frac{1}{p} \left( z^1_{ij} + z^2_{ij} + \ldots + z^p_{ij} \right). \]  

(15)

In order to use the DEMATEL method for group decision making in a fuzzy environment, the analytical procedure of the proposed fuzzy DEMATEL method is summarized from Wu and Lee (2007) below.

Step 1: Identifying the decision goal and forming a committee.
Step 2: Developing evaluation factors and designing the fuzzy linguistic scale.
Step 3: Acquiring and aggregating the assessments of decision makers.
Step 4: Establishing and analyzing the structural model (Table I).

At the end of step 4, the causal diagram is constructed with the horizontal axis (D+R) named “Prominence” and the vertical axis (D−R) named “Relation.” The horizontal axis shows how much importance the factor has, while the vertical axis may divide factors into cause group and effect group. Usually, when the (D−R) axis is plus, the factor belongs to the cause group. Otherwise, the factor belongs to the effect group if the (D−R) axis is minus. Therefore, causal diagrams can visualize the complicated causal relationships of factors into a visible structural model, providing valuable insight for problem solving. Moreover, with the help of a causal diagram, proper decisions by recognizing the difference between cause and effect factors can be made.

5. Implementation of the study

As it was mentioned in the literature review part, personnel selection is a popular topic for researchers. Therefore, different kinds of criteria have been suggested in the literature so far. It is inevitable that the Industry 4.0 also affects the recruiting processes, especially for the departments which are related to operations management. In that sense, the need for including Industry 4.0 based personnel selection criteria arises. For this study, criteria have been selected according to the changes in job profiles due to Industry 4.0 from operations management perspective. The implementation of the proposed method had been conducted for a high-technology firm, which is planning to transform its processes according to new industrial revolution. According to the literature review which was conducted for this new research area, 11 criteria which were frequently used in the literature are revealed (Dombrowski and Wagner, 2014; Gehrke et al., 2015; Lorenz et al., 2015) and hired in the proposed method. Selected criteria are given below:

- combining know-how related to a specific job or process (C1);
- flexibility to adapt new roles and work environments (C2);

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Triangular fuzzy numbers</th>
</tr>
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<tbody>
<tr>
<td>Very high influence (VH)</td>
<td>(0.75, 1.0, 1.0)</td>
</tr>
<tr>
<td>High influence (H)</td>
<td>(0.5, 0.75, 1.0)</td>
</tr>
<tr>
<td>Low influence (L)</td>
<td>(0.25, 0.5, 0.75)</td>
</tr>
<tr>
<td>Very low influence (VL)</td>
<td>(0, 0.25, 0.5)</td>
</tr>
<tr>
<td>No influence (No)</td>
<td>(0, 0, 0.25)</td>
</tr>
</tbody>
</table>

Table I. The fuzzy linguistic scale
continual interdisciplinary learning and cooperation (C3); knowledge on IT and production technologies (C4); organizational and processual understanding (C5); ability to interact with modern interfaces (C6); trust in new technologies (C7); awareness of IT security and data protection (C8); ability of fault and error recovery (C9); ability of dealing with complexity and problem solving (C10); and thinking in overlapping process (C11).

Before moving to the implementation of the study, it is essential to explain the selected criteria above, briefly. Combining know-how related to a specific job or process (C1) refers to the integration of knowledge of the employee with the new technology or process. Since the physical part of the production is done by the fully automated smart machines in Industry 4.0, it is important for employees to combine their knowledge with their daily operations. Second criterion, flexibility to adapt new roles and work environments (C2) indicates the flexibility of employee to adapt high technological features of Industry 4.0 such as CPS or IoT. Due to the dynamic and multidimensional environment of Industry 4.0, it is also important for employees to have continual interdisciplinary learning and cooperation, which is third criterion (C3). Moreover, knowledge on IT and production technologies (C4) is selected as a criterion because of the technology-driven nature of production systems in Industry 4.0 and it is expected to be a desirable feature of workforce 4.0. Furthermore, due to decreasing demand for specific technical skills and increase in the need of broaden vision, organizational and processual understanding (C5) is taken as a criterion. Moreover, ability to interact with modern interfaces (C6) and trust in new technologies (C7) are also hired, since, advanced technologies are the core of the Industry 4.0. Data security is another significant topic in Industry 4.0 environment, since all production technologies are managed by internet. Therefore, awareness of IT security and data protection (C8) is another criterion. Moreover, to deal with complex problems, ability of fault and error recovery (C9) is chosen as a criterion. As it was mentioned before, Industry 4.0 has a dynamic and complex environment. Therefore, ability of dealing with complexity and problem solving (C10) is another aspect of workforce 4.0. Finally, since the processes are connected to each other in Industry 4.0, thinking in overlapping process (C11), in other words linking multidimensional processes, is important.

The features of Industry 4.0 are inherent in all of these criteria. According to the Final Report of the Industrie 4.0 Group (2013), which is a corner stone in the Fourth Industrial Revolution, socio-technical approach needs to be adopted in Industry 4.0 environment. Since many factors such as continuing professional development measures, technology and software architectures are developed in close conjunction in order to provide better relations between workers and the system, the need of interdependent criteria for personnel selection significantly arises. Moreover, Dombrowski and Wagner (2014) also suggest that the interdisciplinary collaboration is increasing due to changes in job profiles. Therefore, to deal with these interdependent criteria used in recruitment processes in Industry 4.0 environment, it is important to present their relationship with each other by using fuzzy DEMATEL method.

Implementation of the proposed method is conducted in a high-technology firm which has started making changes according to Industry 4.0. One of this transformation was introducing a new department, which is responsible of this transformation. Application of the fuzzy DEMATEL method was made by the participation of one executive manager and four experts who are currently playing an important role in this Industry 4.0 related
Participants have been asked to assess 11 criteria by using a linguistic scale. In order to solve the equations of fuzzy DEMATEL method, all the formulas and linguistic assessments of participants entered into Microsoft Excel. For example, the assessment of the executive manager is given in Table II.

After defuzzification by using CFCS method and aggregation of these assessments, the initial direct-relation matrix and normalized direct-relation matrix are produced by using Equations 2–15 and shown in Tables III and IV, respectively.

In the next step of the method, by using Equation 4, total-relation matrix is constructed and shown in Table V.
Finally, in order to establish and analyze the structural model, by using Equations 6 and 7, D+R and D−R values are calculated and shown in Table VI. The causal diagram is constructed by mapping the data set D+R and D−R and presented in Figure 5.

According to the results, cause group includes: C1, C4, C8 and C9, while the effect group was composed of C2, C3, C5, C6, C7, C10 and C11. Moreover, descending order of importance level of the criteria is found as C10, C11, C2, C3, C5, C7, C9, C1, C6, C8 and C4. Results of the fuzzy DEMATEL application are summarized in Table VII and discussions related to these results are made in the following section.

<table>
<thead>
<tr>
<th>T</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.601</td>
<td>1.937</td>
<td>1.894</td>
<td>0.81</td>
<td>1.931</td>
<td>1.768</td>
<td>1.77</td>
<td>1.367</td>
<td>1.689</td>
<td>2.17</td>
<td>1.899</td>
</tr>
<tr>
<td>C2</td>
<td>1.496</td>
<td>1.641</td>
<td>1.701</td>
<td>0.71</td>
<td>1.767</td>
<td>1.622</td>
<td>1.551</td>
<td>1.214</td>
<td>1.444</td>
<td>1.915</td>
<td>1.693</td>
</tr>
<tr>
<td>C3</td>
<td>1.773</td>
<td>2.015</td>
<td>1.838</td>
<td>0.84</td>
<td>2.043</td>
<td>1.858</td>
<td>1.768</td>
<td>1.421</td>
<td>1.674</td>
<td>2.209</td>
<td>2.010</td>
</tr>
<tr>
<td>C4</td>
<td>1.646</td>
<td>1.85</td>
<td>1.814</td>
<td>0.786</td>
<td>1.936</td>
<td>1.798</td>
<td>1.717</td>
<td>1.441</td>
<td>1.641</td>
<td>2.124</td>
<td>1.925</td>
</tr>
<tr>
<td>C5</td>
<td>1.547</td>
<td>1.842</td>
<td>1.769</td>
<td>0.772</td>
<td>1.734</td>
<td>1.685</td>
<td>1.600</td>
<td>1.269</td>
<td>1.514</td>
<td>2.052</td>
<td>1.812</td>
</tr>
<tr>
<td>C6</td>
<td>1.538</td>
<td>1.813</td>
<td>1.747</td>
<td>0.770</td>
<td>1.814</td>
<td>1.566</td>
<td>1.551</td>
<td>1.228</td>
<td>1.484</td>
<td>1.971</td>
<td>1.782</td>
</tr>
<tr>
<td>C7</td>
<td>1.680</td>
<td>1.914</td>
<td>1.835</td>
<td>0.820</td>
<td>1.876</td>
<td>1.781</td>
<td>1.613</td>
<td>1.39</td>
<td>1.574</td>
<td>2.099</td>
<td>1.897</td>
</tr>
<tr>
<td>C8</td>
<td>1.553</td>
<td>1.795</td>
<td>1.750</td>
<td>0.855</td>
<td>1.847</td>
<td>1.684</td>
<td>1.671</td>
<td>1.263</td>
<td>1.584</td>
<td>2.038</td>
<td>1.848</td>
</tr>
<tr>
<td>C9</td>
<td>1.744</td>
<td>1.95</td>
<td>1.873</td>
<td>0.876</td>
<td>2.018</td>
<td>1.800</td>
<td>1.761</td>
<td>1.452</td>
<td>1.613</td>
<td>2.236</td>
<td>2.015</td>
</tr>
<tr>
<td>C10</td>
<td>1.929</td>
<td>2.217</td>
<td>2.163</td>
<td>0.912</td>
<td>2.298</td>
<td>2.062</td>
<td>1.983</td>
<td>1.61</td>
<td>1.921</td>
<td>2.346</td>
<td>2.236</td>
</tr>
<tr>
<td>C11</td>
<td>1.740</td>
<td>1.985</td>
<td>1.900</td>
<td>0.86</td>
<td>2.028</td>
<td>1.87</td>
<td>1.821</td>
<td>1.475</td>
<td>1.757</td>
<td>2.254</td>
<td>1.917</td>
</tr>
</tbody>
</table>

Table V. Total-relation matrix

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
<th>C6</th>
<th>C7</th>
<th>C8</th>
<th>C9</th>
<th>C10</th>
<th>C11</th>
</tr>
</thead>
<tbody>
<tr>
<td>D+R</td>
<td>37.06</td>
<td>37.71</td>
<td>39.74</td>
<td>27.69</td>
<td>38.78</td>
<td>36.759</td>
<td>37.26</td>
<td>37.02</td>
<td>37.23</td>
<td>44.98</td>
</tr>
<tr>
<td>D−R</td>
<td>−4.207</td>
<td>−0.838</td>
<td>9.667</td>
<td>−3.658</td>
<td>−2.23</td>
<td>−0.345</td>
<td>2.758</td>
<td>1.438</td>
<td>−1.77</td>
<td>−1.43</td>
</tr>
</tbody>
</table>
6. Findings and discussions

Proposed method is useful to deal with vague and imprecise judgements since it allows to apply both linguistic variables and a fuzzy aggregation method. Cause group factors point the meaning of influencing factors, while effect group factors represent the meaning of influenced factors. This shows that cause group is difficult to change, whereas effect group can be easily moved. Results show that cause group includes, in descending order, knowledge on IT and production technologies (C4), awareness of IT security and data protection (C8), ability of fault and error recovery (C9), and combining know-how related to a specific job or process (C1); effect group includes flexibility to adapt new roles and work environments (C2), organizational and processual understanding (C5), ability to interact with modern interfaces (C6), ability of dealing with complexity and problem solving (C10), thinking in overlapping process (C11), continual interdisciplinary learning and cooperation (C3) and trust in new technologies (C7). Moreover, the importance level of selected criteria was found in descending order as ability of dealing with complexity and problem solving (C10), thinking in overlapping process (C11), continual interdisciplinary learning and cooperation (C3), organizational and processual understanding (C5), trust in new technologies (C7), ability of fault and error recovery (C9), combining know-how related to a specific job or process (C1), ability to interact with modern interfaces (C6), awareness of IT security and data protection (C8) and knowledge on IT and production technologies (C4).

Findings of the study reveal the need for analytical thinking and system approach because the top three most important criteria were ability of dealing with complexity and problem solving, thinking in overlapping process and flexibility.

On the other hand, “relation” results of the fuzzy DEMATEL applications show that technical abilities and applying theoretical knowledge to practice are the needs from employees in Industry 4.0 because the cause group criteria include knowledge on IT and production technologies, awareness of IT security and data protection, ability of fault and error recovery, and combining know-how related to a specific job or process. It can be said that these results are directly related with the transformation of traditional production systems to smart factories and the increase in the need of intelligence level from employees.

<table>
<thead>
<tr>
<th>Prominence (D+R)</th>
<th>Cause group</th>
<th>Relation (D−R)</th>
<th>Effect group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ability of dealing with complexity and problem solving (C10)</td>
<td>Knowledge on IT and production technologies (C4)</td>
<td>Flexibility to adapt new roles and work environments (C2)</td>
<td></td>
</tr>
<tr>
<td>2. Thinking in overlapping process (C11)</td>
<td>Awareness of IT security and data protection (C8)</td>
<td>Organizational and processual understanding (C5)</td>
<td></td>
</tr>
<tr>
<td>3. Flexibility to adapt new roles and work environments (C2)</td>
<td>Ability of fault and error recovery (C9)</td>
<td>Ability to interact with modern interfaces (C6)</td>
<td></td>
</tr>
<tr>
<td>4. Continual interdisciplinary learning and cooperation (C3)</td>
<td>Combining know-how related to a specific job or process (C1)</td>
<td>Ability of dealing with complexity and problem solving (C10)</td>
<td></td>
</tr>
<tr>
<td>5. Organizational and processual understanding (C5)</td>
<td></td>
<td>Thinking in overlapping process (C11)</td>
<td></td>
</tr>
<tr>
<td>6. Trust in new technologies (C7)</td>
<td></td>
<td>Continual interdisciplinary learning and cooperation (C3)</td>
<td></td>
</tr>
<tr>
<td>7. Ability of fault and error recovery (C9)</td>
<td></td>
<td>Trust in new technologies (C7)</td>
<td></td>
</tr>
<tr>
<td>8. Combining know-how related to a specific job or process (C1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Ability to interact with modern interfaces (C6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. Awareness of IT security and data protection (C8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. Knowledge on IT and production technologies (C4)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table VII. Summary of the fuzzy DEMATEL results
Moreover, since the effect group of the study includes flexibility to adapt new roles and work environments, organizational and processual understanding, ability to interact with modern interfaces, and ability of dealing with complexity and problem solving, the requirements of Industry 4.0 include flexible and interdisciplinary working structure. From this point of view, authors suggest that matrix organizations are suitable for Industry 4.0 by taking the team work and organizational understanding as priorities.

As it was mentioned in the Final Report of the Industrie 4.0 Group (2013), there are two main trends that affect job and skill profiles, in other words, expectations from employees. The first one is, despite to clear division of labor in Industry 3.0, new organizational environment requires features such as decision taking, coordination, control and support. Second, organizing and coordinating interactions between machines, control systems and production systems are another trends that affect workforce in Industry 4.0. These facts clearly show the advantages and suitability of the proposed selection criteria. These results were also consistent with the recent literature which states the increase in level of intelligence and complexity in Industry 4.0 environment (Qin et al., 2016).

In this study, due to the similar experience levels, participants were equally weighted. In further studies, when the participants are not equally weighted, it is suggested to implement a sensitivity analysis based on varying weights given to participants.

7. Conclusions

Industry 4.0, in other words Fourth Industrial Revolution, has started to transform manufacturing environment completely in developed countries and is expected to spread all around the world during the next decades. These transformation processes affect the expectations from employees as well. Especially, in the departments related to operations management, personnel selection criteria are expected to alter due to changing job profiles.

Literature on Industry 4.0 has been rapidly expanded and most of the studies are directly related to the manufacturing environment. Since Industry 4.0 is still an emergent area, most of the studies are based on creating conceptual models. Fundamental concepts of Industry 4.0 can be summarized as: smart factory, CPS, self-organization and IoT.

Even though research works about Industry 4.0 and its concepts exist commonly, there are only few studies which focus on changes in expectations from employees and its features. Only few of them focused on changes in job profiles in Industry 4.0 and there is nearly no evidence of presenting new personnel selection criteria in Industry 4.0 to support HR activities during the recruitment process in operations management related departments. Therefore, it is important to state the related criteria for firms and ways to evaluate them.

The aims of this study are as follows: first, to present a structural competency model for Workforce 4.0; second, to remark new criteria for personnel selection in Industry 4.0 environment and present an importance order and a causal relationship between these criteria by using Fuzzy DEMATEL, in a Turkish high-technology firm, which has started to modify its processes according to Industry 4.0 by introducing new departments; third, to contribute to the operations management literature by focusing on recruitment process in Industry 4.0 environment and supporting human resources activities with Industry 4.0 related criteria; and finally, point out new research field in Industry 4.0 environment.

Fuzzy DEMATEL method is based on evaluating complex relations by using matrix operations. The main advantage of the fuzzy DEMATEL is to present a detailed and clear cause and effect relationship between criteria. Fuzzy DEMATEL method is selected for this study due to its multidirectional results. Comparing to other MCDM methods, fuzzy DEMATEL gives the opportunity to examine both prominence of criteria and the causal relation between them. This gives a wider chance to analyze the criteria and make a more
reliable decision because relation between criteria affects the performance of selection process. Fuzzy logic also helps to represent and handle vagueness in this decision-making process. Since, this study does not aim to select personnel but to present interdependencies among criteria in order to support recruitment processes in Industry 4.0, fuzzy DEMATEL is one of the most suitable techniques for this study.

At the implementation of the study, five participants who are currently playing an important role in the selected firms Industry 4.0 related department were asked to assess criteria and results show that most important criteria are ability of dealing with complexity and problem solving, thinking in overlapping process, and flexibility to adapt new roles and work environments. Moreover, while cause group includes criteria such as combining know-how related to a specific job or process, knowledge on IT and production technologies, awareness of IT security and data protection, ability of fault and error recovery, effect group includes flexibility to adapt new roles and work environments, continual interdisciplinary learning and cooperation, organizational and processual understanding, and ability to interact with modern interfaces.

Due to the prominence results of fuzzy DEMATEL method in this study, it can be concluded that analytical thinking and system approach are the key topics for new supporting personnel selection criteria, which leads to need for the skills and qualifications in decision making and process management.

In addition to that results of the cause group criteria indicated the importance of technical abilities. It presents the need of technical abilities, such as coding, IT security and human-machine interfaces. On the other hand, effect group of the study takes the attention to the flexibility and interdisciplinary working structure that shows the suitability of matrix organization in the companies which follow the Industry 4.0 trends. Furthermore, according to the results of the study, team work is another arising key concept for organizations transforming to Industry 4.0. Therefore, recruitment processes need to be supported by Industry 4.0 related criteria, especially for operations management related departments.

Originality of this research results from three main points. The first one is the proposed competency structural model for Workforce 4.0 in terms of a road map. The second and the third ones are the suggested set of related criteria and the fuzzy MCDM-based methodology for companies, respectively. Therefore, the study would guide companies which alter their organizations according to Industry 4.0. However, the evaluation may change if the study is implemented in another company due to the unique needs and expectations. Each company will definitely have to take the unique needs and expectations of its own into account.

To sum up, this study is focused on a brand-new topic in Industry 4.0 environment and tried to contribute the operations management literature in Industry 4.0 from the point of personnel selection criteria. Main limitation of the research is that it is conducted for high-technology sector only. For future research works, conducting the research in various sectors and hiring different methods is suggested.

References


**Further reading**


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Hybrid revised weighted fuzzy c-means clustering with Nelder-Mead simplex algorithm for generalized multisource Weber problem

Tarik Kucukdeniz and Sakir Esnaf
Department of Industrial Engineering, Istanbul University, Istanbul, Turkey

Abstract

Purpose – The purpose of this paper is to propose hybrid revised weighted fuzzy c-means (RWFCM) clustering and Nelder-Mead (NM) simplex algorithm, called as RWFCM-NM, for generalized multisource Weber problem (MWP).

Design/methodology/approach – Although the RWFCM claims that there is no obligation to sequentially use different methods together, NM’s local search advantage is investigated and performance of the proposed hybrid algorithm for generalized MWP is tested on well-known research data sets.

Findings – Test results state the outstanding performance of new hybrid RWFCM and NM simplex algorithm in terms of cost minimization and CPU times.

Originality/value – Proposed approach achieves better results in continuous facility location problems.

Keywords Generalized multisource Weber problem, Nelder-Mead simplex algorithm, Revised weighted fuzzy c-means clustering

Paper type Research paper

1. Introduction

Continuous location-allocation problem, which is also referred as multisource Weber problem (MWP) is dealt with determining the location or coordinates of $c$ facilities in a plane to serve $n$ customers, having fixed locations, while allocating each customer to the facilities so that total costs are minimized. Weber problem in this category is the most studied problem in location theory and can be appeared in many real life problems. Nelder–Mead (NM) simplex algorithm based on derivative-free optimization is another well studied and most cited method. Like Weber problem, this algorithm attracts too many researchers and has a huge number of citations. Finally, fuzzy c-means (FCM) algorithm of Bezdek (1981) is one of the most popular fuzzy clustering algorithms and a pioneer in fuzzy cluster analysis. It can be easily said that reputation of FCM algorithm follows the Weber problem and NM simplex. Despite all these to the best of our knowledge this well-known triple has not been studied yet to date.

In this study, a hybrid algorithm which is the sequential use of the revised weighted fuzzy c-means (RWFCM) clustering, developed by Esnaf and Küçükdeniz (2013), and derivative-free NM simplex algorithm is proposed first time. This is the first and main contribution to the literature. The second main contribution is that the proposed hybrid algorithm is applied to the generalized MWP for the first time. According to the literature survey, there is no study to develop the hybrid of RWFCM clustering and NM algorithm for the generalized

Conflict of interest: the authors declare that they have no conflict of interest.

Compliance with ethical standards: this paper does not contain any studies with human participants or animals performed by any of the authors.
multisource Weber or continuous location-allocation problems. As a third contribution, this is the first study that applies fuzzy c-means and NM algorithms to these types of Weber problems. The forth contribution is lower transportation and computation costs provided by this hybrid method, which runs two algorithms consecutively.

The rest of the paper is organized as follows. The generalized MWP is described in the second section. In the third section, literature review is discussed. In the fourth section of this paper, the method, which consists of the RWFCM and NM algorithm, is explained. An illustrative example has been presented in the fifth section so that the model can be understood better. For a comparison, in the sixth section, the proposed hybrid method is applied to ten well-known data sets. The obtained results are compared with the known fuzzy clustering and particle swarm optimization (PSO) based continuous location-allocation methods in the literature. In the seventh and last section, conclusion is discussed.

2. Problem definition

In location-allocation problem, total transportation cost is the main criteria for location-allocation decisions. Location of the facilities and the allocation of the customers to these facilities have to be made with the objective of minimizing the total transportation cost resulting from the transportation among facilities and customers. Thus, the location-allocation model, which is adapted from Brimberg et al. (2008) and Gamal and Salhi (2003) can be formulated as follows:

\[
\text{Min} \sum_{i=1}^{c} \sum_{j=1}^{n} w_{ij} d(X_i, a_j),
\]

subject to:

\[
\sum_{i=1}^{c} w_{ij} = w_j, j = 1, 2, ..., n,
\]

\[
w_{ij} \geq 0 \text{ for all } i \text{ and } j,
\]

where \(w_{ij}\) the quantity assigned from facility \(i\) to fixed point \(j\) also denoting the allocation of customers to the open facilities; \(d(X_i, a_j)\): the Euclidean distance from the unknown location (coordinates) of facility \(i\), to the location of a customer \(j\); \((X_i, Y_i)\): the unknown location of facility \(i\); \((X_j, Y_j)\): the known location of a customer \(j\); \(w_j\): the demand or the weight of the customer \(j\).

The objective function is to minimize the sum of weighted distances from the demand points to the nearest facilities, in other words, total transportation cost. Constraint, which is given in formula (2), guarantee that total demand of each customer is satisfied. Finally, constraint (3) provides the non-negativity conditions. \((X_i, Y_i)\) and \(w_j\) represent the decision variables of the model.

If we solve the problem according to formula (1) it is called as classical MWP (Brimberg et al., 2014). The problem is referred to as the MWP when all quantities or weights are equal to unity and as the generalized MWP when they are unequal (Gamal and Salhi, 2003). It is both nondifferentiable and a nonconvex mathematical problem with a large number of local minima. Therefore, it is a global optimization problem (Xavier et al., 2014).

In this study, the second case, uncapacitated generalized MWP, is considered, which the weights are unequal and capacity of all facilities are infinite.
3. Literature review

The uncapacitated generalized MWP can be defined as any type of location problem which the final locations (facilities) are unknown and the data points (customers) have geographical location information like coordinates. Problems with this type usually occurs in supply chain management, emergency infrastructure planning and communication network planning applications. Its main limitation is that it does not take into account the demands/weights of the data points during the location calculation process.

The assignment of the data points to the found locations will determine the capacity of these locations after the problem solved. The main difficulty in solving MWP arises from the non-convexity of the objective function and the existence of multiple local minima. Exact solution methods have limited applicability due to the non-polynomial (NP) nature of the problem. Two facility cases, customers on a straight line with very small instances are to be solved by exact algorithms (Brimberg et al., 2008). However, in real cases, especially when struggling with large data, it is expensive to reach optimum solutions. In recent years, fuzzy and crisp clustering analysis based algorithms and methods are proposed by Levin and Ben-Israel (2004), Žalik (2006), Ayyoub et al. (2007), Esnaf and Küçükdeniz (2009), Iyigün and Ben-Israel (2010), Küçükdeniz et al. (2012), Gupta et al. (2017) and Gao et al. (2017) to solve the continuous location–allocation problems. Besides, in recent studies, Khoei et al. (2017) investigated how to limit the CO₂ emission in green supply chains, Akyüz (2018) analyzed several constraints like presence of barriers, and Guo et al. (2018) studied uncertain location of future demand points. All studies need decomposition of the multi-facility location problem (MFLP)s into several single-facility location problems. Esnaf and Küçükdeniz (2013) proposed a single phased, robust and simultaneous FCM algorithm instead of solving the MFLPs with a two-phased approach. This algorithm is fuzzy clustering part of the hybrid method proposed in this paper. The aim of the rest of the literature review part is to search for a clue about studies that involves with FCM and NM to solve MWP.s.

In the past decades, there has been an emerge of hybrid algorithms, combining global heuristic methods together with traditional local exploitation algorithms. One of the traditional local exploitation algorithms used with global heuristics is NM algorithm. NM is a simplex search algorithm which is a derivative-free search method originally developed by Nelder and Mead (1965) for multivariate unconstrained nonlinear optimization (Aksen, et al., 2009). This direct search algorithm is invited to solve different types of optimization functions in many studies. Selected of the papers are summarized below. Chelouah and Siarry (2003) proposed a hybrid method combining Genetic and NM algorithms for the global optimization of multiminima functions. Wei and Zhao (2005) combined NM simplex algorithm with genetic algorithms, called niche hybrid genetic algorithm, to solve continuous multimodal problems. Their niche hybrid algorithm helps to reduce premature convergence and empower weak exploitations features of genetic algorithms. Another study in 2005, Chelouah and Siarry developed another hybrid method combines tabu search and NM algorithms for the global optimization of multiminima functions. Fan et al. (2006) solved response surface optimization problems and nonlinear continuous optimization problems by integrated use of NM simplex algorithm with genetic algorithm and PSO separately. Fan and Zahara (2007) proposed the hybrid PSO–NM algorithm based on NM simplex search method and PSO for unconstrained optimization. Baulac et al. (2007) applied a multiple-criteria optimization method for complex road noise barriers using genetic algorithms and NM local search. Caponio et al. (2009) developed super-fit memetic differential evolution which combines PSO, NM algorithm and Rosenbrock algorithm within the differential evolution framework. Gao and Han (2012) investigated a new implementation of NM simplex algorithm for high dimensional problems. Hansen et al. (2016) adapted Zhao et al.’s (2009) modified Nelder–Mead (MNM) algorithm in order to handle the selection of a local search is left totally to the user which is the weakness of variable neighborhood search (VNS) method. Drazic et al. (2016) presented four versions of
standard NM algorithm called MNM, restarted Nelder–Mead (RNM), restarted modified Nelder–Mead (RMNM) and RMNM with expansions. The standard NM and RNM were the local search functions of VNS-based heuristics.

NM algorithm helps to improve the results of the conventional and fuzzy clustering algorithms. Hathaway and Bezdek (1995) used the fmins unconstrained function routine of MATLAB, which uses the NM simplex method for reformulation (R) of clustering criteria optimization including Hard C-Means-R and Fuzzy C-Means-R. (Hathaway and Bezdek, 1995). Kao et al. (2008) developed three hybrid-clustering methods called NM-PSO, K-Means-PSO and K-Means-NM-PSO for well-known data sets such as IRIS, Cancer, Wine, Glass and the others. Gallagher (2014) employed NM simplex search as a benchmark algorithm for clustering. Alci and Beyhan (2017) used the NM simplex algorithm to fine-tune the nonlinear parameters after calculated by FCM. Gallagher (2016) compared two covariance matrix adaptation-evolution strategy (CMA–ES) algorithm with NM, Random Search and K-means algorithms for well-known clustering data sets such as Iris, Ruspini and German towns (spath).

Although the limited number of papers published compared to other topics, another application area of the NM algorithm is facility location problems. Aras et al. (2008) proposed a new mixed-integer non-linear model to find the optimal number and locations of collection points as well as centralized return centers and the optimal premium offered by the company to product holders depending on the condition of their items. In their study NM simplex search is used to find best premium and related net profits for each location group. Yushimoto et al. (2010) presented a heuristic algorithm based on Voronoi diagrams for determining the strategic locations of distribution centers hold critical supplies to minimize the response time, which is critical in disasters and defined through deprivation cost function. In their heuristic algorithm, after constructed a Voronoi diagram for each randomly determined initial point, NM algorithm solved the constrained nonlinear optimization sub-problem. Akella et al. (2010) studied the problem of locating an additional base station or more to an existing cellular network to maximize the call completion probability. Function of optimal new cell tower location was formulated and solved using three algorithms such as total enumeration, simulated annealing and NM simplex search. Mimis (2012) developed a method for solving the problem of determining an optimal location of number of facilities that will be added to the existing network. This method combined Geographical Information Systems with continuous location model using Voronoi diagrams and a directed tabu search global optimization algorithm enriched by NM local search technique. Yaghini et al. (2013) proposed a heuristic algorithm that combines previously developed local branching and relaxation induced neighborhood search algorithms for capacitated p-median problem. The mixed-integer programming solver is utilized for the latter algorithm. Parameters of the proposed heuristic were fine-tuned by design of experiments and were optimized by NM simplex search. There is no study in the literature corresponding to hybrid use of RWFCM with NM for any application area and FCM or RWFCM with NM for facility location problems. In addition, FCM-NM and RWFCM-NM algorithms for the generalized MWP s are proposed the first time.

4. Hybrid RWFCM and NM method
In this section, the revised weighted model of the fuzzy c-means clustering algorithm and its hybrid usage with the NM method is introduced. Considering the weight as a parameter during the clustering steps of the FCM algorithm improves the clustering performance when there is a factor that affects the quality of the final solution, such as the demand of the customers in a facility location problem. After a solution is obtained from this RWFCM clustering algorithm, the solution can be further improved by feeding the final clusters as starting points for the NM simplex algorithm and employing the total transportation cost as the objective function.
4.1 RWFCM algorithm

The revised fuzzy c-means (RWFCM) is proposed by Esnaf and Küçükdeniz (2013). The mechanism of the algorithm is given below.

4.1.1 Use of weights in estimating cluster centers. The objective function for the RWFCM employed herein is the same as the followings; Bezdek (1981) Tsekouras (2005) and Tsekouras et al. (2005). It is given below:

\[ J_p(U, v) = \sum_{k=1}^{c} \sum_{i=1}^{c} w_k(u_{ik})^p \left\| a^k - v_i \right\|^2, \]  

where \( c \) is the number of the final clusters, \( U = \{u_{ik} \mid 1 \leq i \leq c, 1 \leq k \leq n \} \) is the partition matrix, \( V = \{v_i \mid 1 \leq i \leq c \} \) with \( v_i \in \mathbb{R}^m \) is the vector of the final cluster prototypes, \( a^k(1 \leq k \leq n) \) are the data to be clustered, \( p \in (1, \infty) \) is a factor to adjust the membership degree weighting effect and \( w_k \) is the weight of significance that is assigned to \( a^k \).

The optimization problem is to minimize \( J_p(U, V) \) under the following constraint:

\[ \sum_{i=1}^{c} u_{ik} = 1 \quad \forall k. \]  

The final prototypes and the respective membership functions that solve this constraint optimization problem are given by the following equations (Bezdek, 1981):

\[ v_i = \frac{\sum_{k=1}^{n} w_k(u_{ik})^p a_k}{\sum_{k=1}^{n} w_k(u_{ik})^p}, 1 \leq i \leq c, \]  

\[ u_{ik} = 1 / \left( \sum_{j=1}^{c} \left( \frac{\left\| a^k - v_j \right\|}{\left\| a^k - v_i \right\|} \right)^{2/(p-1)} \right)^{1/(p-1)}, 1 \leq i \leq c, 1 \leq k \leq n. \]  

Bezdek (1981), Tsekouras (2005), Tsekouras et al. (2005), Turmchokkasam and Mitaim (2006) and Hore et al. (2007) use computed weights. Eschrich et al. (2003) find the weights corresponding to the number of aggregated feature vectors. Unlike above studies, here, \( w_k \) is neither calculated nor aggregated and is not changed, in other words the weights are not artificially existed, during clustering iterations.

4.1.2 Steps of the RWFCM algorithm. Equations (6) and (7) develop an iterative optimization procedure, which is described by the following steps:

- Step 1: select the number of clusters \( c \), a value for the factor \( p \) and initial values for prototypes \( v_1, v_2, ..., v_c \).
- Step 2: employ Equation (7) to calculate the membership values \( u_{ik} (1 \leq i \leq c, 1 \leq k \leq n) \).
- Step 3: calculate the updated cluster center values \( v_1^{new}, v_2^{new}, ..., v_c^{new} \) using Equation (6).
- Step 4: if \( \max \{ \left\| v_i - v_i^{new} \right\| \}_i < \varepsilon \) then stop else go to Step 2.

It should also be noted that, in the case where all weights are equal, the weighted FCM algorithm is identical to the classical FCM method.
4.2 NM simplex algorithm

The simplex search method, first proposed by Spendley et al. (1962) and later refined by Nelder and Mead (1965), is a derivative-free search method that was particularly designed for traditional unconstrained minimization scenarios, such as the problems of nonlinear least squares, nonlinear simultaneous equations, and other types of function minimization (Fan et al., 2006).

The NM algorithm was proposed as a method for minimizing a $n$ dimensional real-valued function $f(x)$ for $x \in \mathbb{R}^n$ (Lagarias et al., 1998). A simplex is a geometric figure in $n$ dimensions that is the convex hull of $n+1$ vertices. We denote a simplex with vertices $x_1, x_2, \ldots, x_{n+1}$ by $\Delta$.

The NM method iteratively generates a sequence of simplices to approximate an optimal point of $f(x)$. At each iteration, the vertices $x_j$ of the simplex are ordered according to the objective function values:

$$f(x_1) \leq f(x_2) \leq \cdots \leq f(x_{n+1}).$$

(8)

Referring to $x_1$ as the best vertex and to $x_{n+1}$ as the worst vertex. If several vertices have the same objective values, consistent tie-breaking rules such as those given in Lagarias et al. (1998) are required for the method to be well defined (Gao and Han, 2012).

Four scalar parameters must be specified to define a complete NM algorithm: coefficients of reflection ($\alpha$), expansion ($\beta$), contraction ($\gamma$) and shrink ($\delta$). According to the original paper of Nelder and Mead (1965), these parameters should satisfy $\alpha > 0$, $\beta > 1$, $0 < \gamma < 1$, $0 < \delta < 1$. Standard values of these parameters are $\alpha = 1$, $\beta = 2$, $\gamma = (1)/(2)$, and $\delta = (1)/(2)$ (see Lagarias et al., 1998; Gao and Han, 2012; Nesamalar et al., 2016).

The NM method which is the version of Lagarias et al. (1998) and Gao and Han (2012) is outlined below.

One iteration of the NM algorithm:

1. Sort. Evaluate $f$ at the $n+1$ vertices of $\Delta$ and sort the vertices so that (8) holds.
2. Reflection. Compute the reflection point $x_r$ from:

$$x_r = (1 + \alpha)x_{n+1} - \alpha x_n.$$  

(9)

Evaluate $f_r = f(x_r)$. If $f_1 \leq f_r < f_n$, replace $x_{n+1}$ with $x_r$.
3. Expansion. If $f_e < f_1$ then compute the expansion point $x_e$ from:

$$x_e = \beta x_r + (1 - \beta)x_1.$$  

(10)

and evaluate $f_e = f(x_e)$. If $f_e < f_n$, replace $x_{n+1}$ with $x_e$; otherwise replace $x_{n+1}$ with $x_r$.
4. Outside contraction. If $f_r < f_e < f_{n+1}$, compute the outside contraction point $x_{oc}$ from:

$$x_{oc} = \gamma x_r + (1 - \gamma)x_1.$$  

(11)

and evaluate $f_{oc} = f(x_{oc})$. If $f_{oc} < f_r$, replace $x_{n+1}$ with $x_{oc}$; otherwise go to Step 6.
5. Inside contraction. If $f_r > f_{n+1}$, compute the inside contraction point $x_{ic}$ from:

$$x_{ic} = (1 + \delta)x_r - \delta x_{n+1},$$  

(12)

and evaluate $f_{ic} = f(x_{ic})$. If $f_{ic} \leq f_{n+1}$, replace $x_{n+1}$ with $x_{ic}$; otherwise go to Step 6.
6. Shrink. For $2 \leq i < n + 1$, define:

$$x_i = \delta x_i + (1 - \delta)x_1.$$  

(13)
The stopping criterion is a measure of how far the simplex was moved from one iteration \( k \) to the following one \( (k+1) \). The algorithm stops when: \( \frac{1}{2} \sum_{i=1}^{n} \| x_i^k - x_i^{k+1} \|^2 < \varepsilon \), where \( x_i^{k+1} \) is the vertex replacing \( x_i^k \) at the iteration \( (k+1) \) and \( \varepsilon \) is a given “small” positive real number.

4.3 The Hybrid RWFCM-NM algorithm based method for generalized MWPs

The RWFCM algorithm determines the locations and numbers of warehouses, distribution centers or facilities that will meet the demands of demand points by trying to minimize the total transportation cost. How the RWFCM algorithm is applied to the generalized MWP is explained in detail.

Demand points with coordinates \( X, Y \) are assumed to be data points set \( a_k \). The demand quantities of demand points are the weights in the RWFCM, and they are symbolized with \( w_k \). As explained above, transportation costs are the product of distances from facility locations to demand points and the demand quantities to be transported. The \( X, Y \) coordinates of demand points, quantities of demand and the cost parameters are known beforehand. It is assumed that capacity of each facility is unlimited.

The cluster centers found by the RWFCM algorithm give the first facility locations for the generalized MWP. Facility-demand point assignment is thus determined by membership values of the clustering algorithm. After RWFCM, NM simplex algorithm is used to determine final facility locations. The implementation of the RWFC-NM hybrid method is itemized as follows:

1. Define the data set of location of \( n \) customers: \( a^k = (x_k, y_k) \) where customer \( k \) in a plane, \( k = 1, 2, \ldots, n \);
2. Find the cluster centers \( v_i \) for \( c \) facilities by applying the RWFCM algorithm to \( a^k \);
3. Assign each customer to a cluster (facility) by using the membership values \( u_{ik} \);
4. Within each cluster calculate the optimized facility locations with NM algorithm and finalize \( v_i, a^k, w_k \); and
5. \( X_i \rightarrow v_i, a_i \rightarrow a^k, w_j \rightarrow w_k \), according to the final assignments by NM algorithm determine \( u_{ij} \) and calculate the total transportation cost function which is given in the first section with formula (1).

5. Illustrative example

Proposed hybrid method is applied to a small data set to further explain the process of the solution to the generalized MWP. Location and demand data belonging to eight randomly generated customers can be seen in Table I. The first step is to apply RWFCM clustering algorithm to this data and to get the cluster centers that will be the locations of facilities by using formulas (4)–(7) of the RWFCM algorithm. Parameters for the RWFCM algorithm are membership degree weighting effect, \( p \), and termination tolerance of the clusters, \( \varepsilon \) are taken

<table>
<thead>
<tr>
<th>Customers ( j )</th>
<th>( X )</th>
<th>( Y )</th>
<th>Demand ( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<td>19</td>
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<td>1</td>
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<td>4</td>
</tr>
<tr>
<td>7</td>
<td>32</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>44</td>
<td>21</td>
<td>1</td>
</tr>
</tbody>
</table>

Table I. Location and demand data of the customers for the illustrative example
as 1.7 and $10^6$, respectively. Iterative optimization procedure of the algorithm consisting of four steps is carried out with formulas (6) and (7). The number of clusters, therefore number of facilities, must be given to the clustering algorithm as an input. In this small illustrative example, eight customers will be served by two facilities.

The cluster centers calculated by the RWFCM algorithm after eight iterations can be seen in Table II.

If formula (1) is applied to this solution, total transportation cost of the RWFCM method can be calculated, which is 108.70 in this case. To further improve the total cost of the solution, the cluster centers found by the RWFCM algorithm is fed into the NM simplex algorithm as the initial points. The NM simplex algorithm is executed to minimize the total transportation cost, defined in formula (1), by searching better values for the location of the facilities, which are the decision variables. The NM simplex algorithm accomplishes this with six functions that sequentially use the formulas (8) through (13) proper to the iterative procedure. Employed parameters in this example are $\alpha = 1, \beta = 2, \gamma = (1)/(2)$ and $\delta = (1)/(2)$. The final facility locations calculated by the NM Simplex algorithm after 78 iterations can be seen in Table III. The final cluster centers for the RWFCM and RWFCM and NM hybrid solutions can be seen in Figure 1.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$X_i$</th>
<th>$Y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster no. 1</td>
<td>35.45</td>
<td>18.82</td>
</tr>
<tr>
<td>Cluster no. 2</td>
<td>21.47</td>
<td>17.91</td>
</tr>
</tbody>
</table>

Table II. Cluster centers obtained by the RWFCM algorithm on the illustrative example data set

<table>
<thead>
<tr>
<th>$c$</th>
<th>$X_i$</th>
<th>$Y_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Facility no. 1</td>
<td>32.00</td>
<td>19.99</td>
</tr>
<tr>
<td>Facility no. 2</td>
<td>20.00</td>
<td>19.99</td>
</tr>
</tbody>
</table>

Table III. Cluster centers obtained by the proposed RWFCM and NM hybrid algorithm on the illustrative example data set

![Figure 1](image-url)

Notes: (a) RWFCM; (b) RWFCM and NM
The total transportation cost of the NM hybrid solution is 94.86, which is lower than the RWFCM solution. The same calculation procedure applied to the NM-based benchmark methods, which are given in Section 6.2. Fuzzy c-means and Center of Gravity hybrid method uses Center of Gravity formulas after fuzzy c-means algorithm. The rest of the methods calculates the total transportation cost according to the first step explained above.

6. Experimental study
The RWFCM-NM hybrid approach has been applied to the selected test problems. Various benchmark algorithms were also coded and compared using the same configurations given in 5.2.

6.1 Solving different generalized MWPs by the RWFCM-NM
In total, 26 different configurations generated from ten benchmarking problems solved by Osman and Christofides (1994), Bongartz et al. (1994), Lorena and Pereira (2002) and Taillard (2003) are used to prove this claim. The details of the original data sets are given in Esnaf and Küçükerdem (2013).

Data sets in configurations include customer number, X and Y coordinates and demands of each customer. These data sets are originally created to solve the capacitated p-median facility location problem, in which the desired number of facilities can be placed on the given demand point locations only. However, the RWFCM-NM solves the uncapacitated generalized MWP in continuous search space. Thus, a facility location may or may not be on any of the given demand points. This fundamental difference makes it impossible to compare the results of the RWFCM-NM with the original solutions on these data sets.

For each data set, the customer location data with demand information is fed into RWFCM algorithm. The RWFCM solves the clustering problem by considering the demands as weights and gives cluster centers (facility locations) for a given number of clusters. Then, NM simplex algorithm is run to minimize the total transportation cost in each cluster by taking the location of the facilities as decision variables. The found cluster centers of the facilities from the RWFCM algorithm is given as the initial points to the NM algorithm. The solution of the NM algorithm gives the final cluster centers (facility locations). The total transportation cost is calculated by multiplying demand for each customer by its distance to the center of the cluster (facility location) that the customer belongs to.

6.2 Comparing the performance of the RWFCM-NM with benchmark methods
The aim of this section is to test the RWFCM-NM’s performance against the similar clustering based multi-facility location methods. Thus, it is tried to be shown that the RWFCM-NM solves the uncapacitated generalized MWP with a lower cost than the benchmark methods. To test the RWFCM-NM’s actual performance, FCM clustering, PSO, and the RWFCM based multi-facility location methods are used. The results of the RWFCM-NM method are compared with five benchmarking methods on 26 different configurations of ten large and well-known problem sets. Brimberg et al. (2008) state that exact methods are restricted to a small problem size due to the NP-hard nature of the problem and they do not guarantee the optimal solutions. For this reason, calculation of the deviation from the optimal values is not possible for these problems. Original weighted c-means algorithms of Bezdek (1981), Tsekouras et al. (2005), Tsekouras (2005) or Eschrich et al. (2003) cannot be applied to the continuous location-allocation problems due to their computed or arranged weights. Weights, which are assumed customer demands here, cannot be computed or aggregated during each iteration, unlike above-mentioned algorithms.
The computer that was used during test runs has the following configuration; Intel Core i7 processor at 2.4 GHz with 8 GB RAM. Codes of the FCM, the COG, the PSO, the RWFCM and the RWFCM-NM algorithms were developed and executed by MATLAB R2015b.

Selected benchmarking methods are explained in the following.

6.2.1 FCM algorithm based method. Demand points are clustered according to their geographical location with FCM algorithm. Resulting cluster centers are the final location of the facilities.

6.2.2 FCM based center of gravity (FCM and COG) method (Esnaf and Küçükdeniz, 2009). Within each cluster of former FCM results, a center of gravity point is calculated according to the demands at each cluster. These COG points are the ultimate locations of the facilities.

6.2.3 FCM based NM (FCM and NM) method. Within each cluster of FCM results, NM fine-tuned the cluster centers according to the demands at each cluster. NM simplex algorithm is directly applied to minimize the total transportation cost for each problem by changing the cluster centers which are the ultimate locations of the facilities.

6.2.4 Revised weighted FCM algorithm based method (Esnaf and Küçükdeniz, 2013). Details and steps of the RWFCM method proposed by Esnaf and Küçükdeniz (2013) is given in subtitle 4.1.

For all FCM-based methods including the proposed RWFCM ε, termination tolerance of the clusters, is taken as 10⁻⁶. The membership degree weighting effect, p, is 1.7.

6.2.5 PSO method. PSO method is applied to the facility location problem. In this algorithm, each particle represents a possible solution to the problem. Thus, the dimensions of the particles, as in the study of Merwe and Engelbrecht (2003), are the geographical coordinates of the facilities. Contrary to the study of Merwe and Engelbrecht (2003), objective function of the PSO is the total transportation cost of the represented facilities. Clerc and Kennedy (2002) analyzed the optimal values of PSO coefficients. The parameters used for the PSO algorithm in this paper were taken from their work and are as follows: the size of the population is 20 particles; the social and cognitive parameters were taken as c₁ = 2.05 and c₂ = 2.05; and inertia weight, w, was taken as 0.99. For each benchmark problem, PSO algorithm is conducted for 30 replications. Maximum iteration counts for each PSO run is 1000. Mean and minimum objective function values of 30 replications are noted.

In all algorithms, for the objective function value comparison, each customer is only served by a single facility and will be assigned to its nearest facility. Transportation costs of all methods for each data set are given in Table IV, and transportation cost differences (as percent) of the RWFCM-NM hybrid method with respect to other benchmarking methods are presented in Table V. The bold values in Table IV represents the lowest (the best) solution for the data set at that row. PSO algorithms has two values at each line for the mean of 30 runs and the best of those 30 runs, for each data set. The number of facilities/clusters is given in parentheses in Table V after the name of each data set.

The percentages of the transport cost differences for each data set of the generalized multi-source Weber problem of FCM, FCM and COG, FCM and NM, PSO and RWFCM by the RWFCM-NM hybrid method are computed with the following formula:

\[ \Delta = \left( \frac{H - M}{H} \right) \times 100, \]  

where \( H \) represents the objective function value, i.e. transportation cost, generated by benchmark methods for each data set, \( M \) is the transportation cost generated by the RWFCM method for the corresponding data set.
CPU-time performances of the all methods are also given in Table VI. Figure 2 is illustrated to be an example of the cluster map solutions of the proposed and benchmark algorithms for the ten-facility problem of Bongartz et al. (1994). As the cluster maps in the Figure 2 shows, each algorithm has different final cluster centers and facility-customer assignments.
7. Conclusions
This paper investigated employing weights in a fuzzy clustering algorithm for generalized MWP by proposing a new hybrid RWFCM and NM simplex search algorithm. This hybridization is proposed first with this paper. Another novelty in this study is application of this hybrid algorithm to generalized MWPs. Furthermore, not only RWFCM algorithm, FCM algorithm hybridized with NM simplex for the same type of Weber problems. Finally, transportation and computation costs are improved by this novel hybrid method. The RWFCM-NM clusters the data while considering the demand without loss of sensitivity. The proposed RWFCM-NM algorithm based method is benchmarked with FCM, FCM-COG, FCM and NM, PSO and RWFCM methods for generalized MWP. Ten different data sets with 26 different settings from Osman and Christofides (1994), Bongartz et al. (1994), Lorena and Pereira (2002) and Taillard (2003) were handled. Facility location models with different cluster numbers on different settings were inspected. Except only one case, the RWFCM-NM algorithm based method outperformed FCM, FCM-COG, FCM and NM, PSO and RWFCM methods developed for continuous location–allocation problems which are also called planar multi-facility location problems in related papers.

When the percentage differences examined, it was found that the differences between the RWFCM-NM and the benchmark methods were remarkable for the data sets of Bongartz et al. (1994), Bongartz et al. (1994) and Christofides (1994), Bongartz et al. (1994), Lorena and Pereira (2002) and Taillard (2003). The RWFCM has been given better results in all instances. For Bongartz et al’s (1994) data with 20 clusters, the RWFCM-NM improved the transportation cost by 60.02 percent compared to the FCM algorithm based method. This percentage value is the largest improvement rate on the whole table.

<table>
<thead>
<tr>
<th>Continuous location–allocation data</th>
<th>FCM (%)</th>
<th>FCM and COG (%)</th>
<th>FCM and NM (%)</th>
<th>PSO (Mean)</th>
<th>RWFCM (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bongartz et al. (1994) (10)</td>
<td>46.76</td>
<td>22.57</td>
<td>11.51</td>
<td>14.72</td>
<td>4.85</td>
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<td>29.77</td>
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<td>SJC818(20)</td>
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<td>14.04</td>
<td>4.20</td>
<td>17.53</td>
<td>1.53</td>
</tr>
<tr>
<td>SJC818(40)</td>
<td>19.93</td>
<td>9.10</td>
<td>3.01</td>
<td>26.21</td>
<td>4.02</td>
</tr>
<tr>
<td>SJC818(60)</td>
<td>30.58</td>
<td>10.32</td>
<td>6.42</td>
<td>36.72</td>
<td>3.97</td>
</tr>
<tr>
<td>CPmedcap1(25)</td>
<td>3.37</td>
<td>1.96</td>
<td>1.36</td>
<td>6.74</td>
<td>1.51</td>
</tr>
<tr>
<td>CPmedcap1(40)</td>
<td>3.88</td>
<td>2.12</td>
<td>1.60</td>
<td>13.67</td>
<td>1.84</td>
</tr>
<tr>
<td>CPmedcap2(25)</td>
<td>2.37</td>
<td>1.41</td>
<td>0.13</td>
<td>8.07</td>
<td>0.89</td>
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<tr>
<td>CPmedcap2(40)</td>
<td>2.67</td>
<td>1.72</td>
<td>0.52</td>
<td>10.97</td>
<td>1.32</td>
</tr>
<tr>
<td>Taillard (2003) (50)</td>
<td>31.01</td>
<td>11.67</td>
<td>5.79</td>
<td>31.62</td>
<td>2.90</td>
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<tr>
<td>Taillard (2003) (100)</td>
<td>36.35</td>
<td>12.74</td>
<td>9.71</td>
<td>43.87</td>
<td>3.85</td>
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<tr>
<td>Taillard (2003) (200)</td>
<td>42.43</td>
<td>12.65</td>
<td>13.28</td>
<td>52.33</td>
<td>3.25</td>
</tr>
<tr>
<td>Average percentage of differences</td>
<td>25.84</td>
<td>11.91</td>
<td>5.95</td>
<td>19.62 (min) 23.37 (mean)</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table V. Percentages of transportation costs differences of the RWFCM-NM method from the FCM, the FCM&COG, the FCM and NM, the PSO and the RWFCM methods.
On the other hand, for SJC708 with 20 clusters, result of the RWFCM-NM was slightly worse which was 0.63 percent greater than the FCM and NM method. According to the average differences for all data sets, the RWFCM-NM has given 25.84, 11.91, 5.95 and 3.10 percent better results than the methods using FCM, FCM and COG, FCM and NM and RWFCM, respectively. When the RWFCM-NM is compared with the PSO algorithm, RWFCM-NM was found to be 19.62 percent better on minimum score and 23.37 percent better on average score than PSO algorithm, respectively.

Average CPU time of the RWFCM-NM algorithm was 5.68 and 69.71 percent times faster than the FCM and NM and PSO (Mean) algorithms, respectively. However, FCM, FCM and COG and the RWFCM algorithms were faster than RWFCM-NM algorithm. Thus, it is concluded that using constant weights in fuzzy clustering hybridized with NM is a preferable approach and the proposed RWFCM-NM algorithm, which has not been observed in the literature yet, is a new and strong algorithm for uncapacitated planar multi-facility location problems. Also, FCM-NM was proposed and discussed for continuous location–allocation problems first in this study.

Facility location problem is one of the most important areas of decision in supply chain management context. The location of the facility directly affects the logistics costs. By finding a good location for its facilities and/or distribution centers, a company may gain competitive advantage over its competitors. Same problem also applies to the hub location, public facility location, wireless ad hoc network location and similar problems.

Future research can be focused on a different hybrid type of the proposed algorithm, which is strengthened by a third method and capacitated generalized MWPs. In real life applications, capacities of the facilities may be regarded as fixed priori, due to the
investment budgets or technological constraints. When solving the location–allocation problem with the capacity constrained facilities, covering area of service will be defined for each facility, which affects the final locations. In addition, customers can be assigned to more than one facility by their membership degrees computed. This requires a redefinition of the MFLP problem according to the multi-assignment structure. Public services and emergency infrastructure related location planning problems are also important areas of application. Stochastic nature of the demand and customer location like in mobile

Figure 2. Cluster map solutions for the problem of Bongartz et al. (1994) with 10 clusters
telecommunication applications can be further investigated. Information of geographical location becomes more important in these problems and therefore inspecting the location problem as a Weber problem will improve the quality of the solutions.

References


Further reading

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Hybrid flow shop with multiprocessor task scheduling based on earliness and tardiness penalties

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Abstract
Purpose – Hybrid flow shop with multiprocessor task (HFSMT) has received considerable attention in recent years. The purpose of this paper is to consider an HFSMT scheduling under the environment of a common time window. The window size and location are considered to be given parameters. The research deals with the criterion of total penalty cost minimization incurred by earliness and tardiness of jobs. In this research, a new memetic algorithm in which a global search algorithm is accompanied with the local search mechanism is developed to solve the HFSMT with jobs having a common time window. The operating parameters of memetic algorithm have an important role on the quality of solution. In this paper, a full factorial experimental design is used to determining the best parameters of memetic algorithm for each problem type. Memetic algorithm is tested using HFSMT problems.

Design/methodology/approach – First, hybrid flow shop scheduling system and hybrid flow shop scheduling with multiprocessor task are defined. The applications of the hybrid flow shop system are explained. Also the background of hybrid flow shop with multiprocessor is given in the introduction. The features of the proposed memetic algorithm are described in Section 2. The experiment results are presented in Section 3.

Findings – Computational experiments show that the proposed new memetic algorithm is an effective and efficient approach for solving the HFSMT under the environment of a common time window.

Originality/value – There is only one study about HFSMT scheduling with time window. This is the first study which added the windows to the jobs in HFSMT problems.

Keywords Memetic algorithm, Hybrid flow shop, Multiprocessor task, Time window

Paper type Research paper

1. Introduction
Low-cost manufacturing is vital for the manufacturing companies to be one step ahead of the others in terms of competition. In this scope, determining good and optimal (if possible) job schedules while minimizing the production lead time needs to be focused on as it can drastically reduce the machine idle time and job waiting time and, therefore, minimize the extra costs.

The hybrid flow shop (HFS) manufacturing system, which can often be found in the production of integrated circuit packaging and printed circuit board fabrication (Linn and Zhang, 1999), berth allocation of container terminal, real-time machine-vision systems, workforce management (Chou, 2013), is evolved from combining the properties of parallel machine and flow shop manufacturing system to meet the requirements of changing manufacturing environment. Coupling hybrid flow shop with multiprocessor task (HFSMT) brought out a new challenging research topic that drew attention among the researchers recently.

HFSMT contains a set of n jobs (j ∈ {1, 2, …, n}) to be processed on k-stage flow shop. There are mi identical parallel processors at stage i, and the expression sizeij indicates the number of machines the jobs requires at each stage. Each task of job j needs to be processed simultaneously on sizeij out of mi machines for pij units of time. All processors and jobs are available since time zero. The main objective, as in this study, is to find a schedule S* that minimizes the maximum completion time of all jobs (e.g. minimizing the makespan).
The problem can be denoted as $F_k(P_{m1}, ..., P_{mk}) \mid size_j \mid C_{max}$ using the popular three-field notation for scheduling problems. The problem can be decomposed into the following two sequential decision problems: determining a permutation of jobs (e.g., a solution containing a schedule) at stage 1, and a decoding method to sequence jobs in subsequent stages. HFS scheduling with multiprocessor task problems is shown as NP-Hard (non-deterministic polynomial time hard) by Linn and Zhang (1999).

Although HFS and HFSMT have been attracting considerable attention among the researchers recently, to the best of our knowledge, HFSMT accompanied with a common due window has not been studied yet. Time windows are practically important in real world. Companies need to be efficient in production, in order to deliver the products right on time, no sooner or later than the given due dates. Delivering right on time would also increase customer satisfaction and consumer perception for the companies.

The rest of the paper is organized as follows: Section 2 presents a review of the current literature, Section 3 describes the problem formulation, Section 4 includes the features of the proposed memetic algorithm, Section 5 includes the test results and Section 6 concludes the study and provides future directions.

2. Literature review

In the literature, there are a considerable number of studies dealing with HFSMT. Öğuz et al. (2004) proposed the first benchmark problem set and lower bound for HFSMT. They proposed a genetic algorithm (GA) to test the performance of heuristic in terms of percentage deviation from the lower bound or optimal solution, if available. Serifoğlu and Ulusoy (2004) provided different benchmark problem set for HFSMT and proposed a GA for minimizing the completion time of all the tasks in the last stage for the HFSMT problem. Their GA included roulette wheel selection, one-point and uniform order-based crossover types and the job exchange-replace mutation operator. The initial population in their proposed GAs is fed with three chromosomes: one found by the shortest processing time sequence and one found by the longest processing time of completion times at first stage and also one found by the shortest total processing time method. They found the optimal solution for small-scale problems of HFSMT by running total enumeration method, i.e., testing each possible solution. Their GA results are compared with improved lower bound; solutions obtained by SPT, LPT and STPT heuristics, optimal solutions for small problems and estimated optimal values for larger problems. They integrated a two-opt local search heuristic into their GA and stated that adding a list scheduling (LS) decoding method did not lead to a statistically significant improvement in results obtained. Also, they stated that the lower bound equation developed for HFSMT problem is inadequate. A GA by Oğuz and Ercan (2005) studied on HFSMT and proposed a GA with preliminary test to determine the best combination of the control parameters. They tested four versions of their algorithm and concluded that the algorithm approaches to lower bound better (e.g., average percentage deviation) when the new crossover operator (NXO) is used along with insertion mutation operator.

Almost all of the authors studying HFSMT proposed metaheuristics, such as a particle swarm optimization by Tseng et al. (Tseng and Liao, 2008), a heuristic coupled with two local search methods by Ying et al. (Ying and Lin, 2009), a memetic algorithm that combines GA and constraint programming by Jouglet et al. (2009), a parallel greedy approach by Kahraman et al. (2010), a simulated annealing including three different decoding methods by Wang et al. (2011), PSO algorithm by Chou (2013), a shuffled frog-leaping algorithm for HFSMT by Xu et al. (2013), a new discrepancy search called climbing depth-bounded adjacent discrepancy search by Lahimer et al. (2013), a new hybrid metaheuristic called “twin particle swarm optimization” algorithm by Yu (2014) and an improved cuckoo search metaheuristic algorithm by Marichelvam et al. (2014), were presented for the HFSMT problem. Engin et al. (2011) proposed a GA for HFSMT and they stated that their results
suggested that the computational performance of GA depends on the factors of initial population, reproduction, crossover and mutation operators and coefficients. They managed to see a reduction in makespan compared to Kahraman et al. (2010), Oguz and Ercan (2005) and Oguz et al. (2004) for benchmark problem set provided by Oguz (2006).

As to some studies dealing with the minimization of earliness and tardiness, Cheng et al. (Yeung et al., 2004) dealt with two-stage flow shop scheduling problem focused on minimizing the earliness and tardiness of the jobs by introducing a common due window, for which they proposed a branch and bound algorithm and a heuristic to solve. Liou et al. (Liou and Hsieh, 2015) extended the multi-stage flow shop scheduling problem by adding sequence dependent setup and transportation times. For the simpler machine scheduling environments, such as single machine, minimization of earliness, tardiness or both have been studied extensively in the literature (Ying, 2008; Hino et al., 2005; Weng and Fujimura, 2008; Talebi et al., 2009; Feldmann and Biskup, 2003; Ronconi and Kawamura, 2010). Hino et al. (2005) dealt with minimizing earliness and tardiness penalties in a single machine problem with a common due date. They proposed a tabu search-based heuristics and GA and the function, \( \sum_{i=1}^{n} (\alpha_j E_j + \beta_j T_j) \) is used as objective function, where \( \alpha_j \) and \( \beta_j \) represent job earliness and tardiness penalties, respectively. Baker et al. (Baker, 2014) dealt with the stochastic version of single machine earliness and tardiness minimization problem and proposed branch and bound approach, while Weng and Fujimura (2008), Talebi et al. (2009) and Feldmann and Biskup (2003) proposed different solution approach for single machine scheduling with respect to earliness and tardiness minimization. This problem is known as NP-hard in the literature. For more complex manufacturing environment, such as parallel machine (Sun and Wang, 2003; Sivrikaya-Şerifoğlu and Ulusoy, 1999; Ventura and Kim, 2003; Toksar and Güner, 2009; Jeong and Kim, 2008; Radhakrishnan and Ventura, 2000), job shop (Vepsalainen and Morton, 1987), multi machine (Zhu and Heady, 2000), flow shop (Bulbul et al., 2004) have been dealt with respect to earliness and tardiness minimization. However, to the best of our knowledge, there has been no research on HFSMT involving earliness and tardiness minimization objective. This literature review encouraged us to fill this gap by studying the earliness and minimization in HFSMT problem.

3. Problem formulation

The following mathematical model is based on the formulation presented in the original work of Lin et al. (2013). We reformulated the mathematical model to address the problem of Fk(Pm1, . . . , Pmk), \( \sum_{i=1}^{n} (\alpha_j E_j + \beta_j T_j) \). The objective is to minimize the sum of earliness and tardiness penalties. The following assumptions for HFSMT problem are also valid for HFSMT with common time windows:

1. all processors and all jobs are available on time \( t = 0 \);
2. processors used at each stage cannot process tasks corresponding to any other stages;
3. each processor can process not more than one job at a time; and
4. preemption of jobs is not allowed.

Table I contains the parameters of the model.

Table II contains the decision variables of the model.

Let job 0 be the dummy initial job. The mathematical model is as follows:

\[
\text{Min } Z(\pi) = \sum_{i=1}^{n} (\alpha_j E_j + \beta_j T_j),
\]  

(1)
subject to:

\[ \sum_{j=1}^{n} X_{ijt} \text{size}_{ij} \leq m_i, \quad i = 1, \ldots, k, \quad t = 1, \ldots, T, \] (2)

\[ C_{i-1,j} \leq C_{i,j} - p_{ij}, \quad i = 2, \ldots, k, \quad j = 1, \ldots, n, \] (3)

\[ C_{ij} - S_{ij} + 1 = p_{ij}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \] (4)

\[ \sum_{t=1}^{T} X_{ijt} = p_{ij}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \] (5)

\[ S_{ij} \leq t + T (1 - X_{ijt}), \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \quad t = 1, \ldots, T, \] (6)

\[ t X_{ijt} \leq S_{ij} + p_{ij} - 1, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \quad t = 1, \ldots, T, \] (7)

\[ C_{\text{max}} \geq C_{kj}, \quad j = 1, \ldots, n, \] (8)

\[ E_j \geq e - c_{ij}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \] (9)

\[ T_j \geq c_{ij} - d, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \] (10)

\[ X_{ijt} \in \{0, 1\}, \] (11)

\[ S_{ij} \in \{1, \ldots, T\}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \quad t = 1, \ldots, T, \] (12)

\[ C_{10} = 0, \quad i = 1, \ldots, k, \] (13)

<table>
<thead>
<tr>
<th>Table I.</th>
<th>Parameters of the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>Number of jobs</td>
</tr>
<tr>
<td>( k )</td>
<td>Number of stages</td>
</tr>
<tr>
<td>( m_i )</td>
<td>Number of identical parallel machines at stage ( i )</td>
</tr>
<tr>
<td>( p_{ij} )</td>
<td>Processing time of job ( j ) at stage ( i )</td>
</tr>
<tr>
<td>( \text{size}_{ij} )</td>
<td>Number of parallel machines required to process job ( j ) at stage ( i )</td>
</tr>
<tr>
<td>( T )</td>
<td>Planning horizon for which the schedule is to be developed</td>
</tr>
<tr>
<td>( e ) and ( d )</td>
<td>The earliest time and the latest time that the jobs can be completed without incurring any penalties, respectively</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II.</th>
<th>Decision variables of the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{ijt} )</td>
<td>A binary time index, which is equal to 1 if job ( j ) is processed at stage ( i ) in time period ( t ) and equal to 0, otherwise</td>
</tr>
<tr>
<td>( S_{ij} )</td>
<td>Starting processing time of job ( j ) at stage ( i )</td>
</tr>
<tr>
<td>( C_{ij} )</td>
<td>Completion time of job ( j ) at stage ( i )</td>
</tr>
</tbody>
</table>
\[ C_{ij} \in \{1, \ldots, T\}, \quad i = 1, \ldots, k, \quad j = 1, \ldots, n, \quad t = 1, \ldots, T, \quad \text{(14)} \]

\[ T_j, E_j \geq 0, \quad j = 1, 2, \ldots, n, \quad \text{(15)} \]

where \( E_j = \max(0, e - c_{ij}) \) and \( T_j = \max(0, c_{ij} - d) \) for a job \( j \in N \), \( k \) is the last stage, \( e \) and \( d \) is the earliest and latest due date that the jobs can be completed without incurring any penalties and \( \alpha_j \) and \( \beta_j \) are the penalties per unit time of earliness and tardiness which are given in advance, respectively. Equation (2) determines the maximum number of machines at each stage in each time period, Equation (3) restrains a task of job to start before the completion of its preceding task. Equation (4) denotes the starting time calculated by processing time requirement, Equation (5) determines the time occupation of each job at each stage. The required number of processors occupation of each job at each stage from starting time until their finishing time is denoted by Equations (6) and (7), respectively. Equation (8) calculates the makespan. Equations (9) and (10) define the decision variables related to earliness and tardiness, respectively. Finally, Equations (11)–(15) set the restrictions on decision variables. To the best of our knowledge, no efficient exact methods are available in the literature to solve \( F_k(P_{m1}, \ldots, P_{mk}) | size_{ij} | C_{max} \) and its version in which earliness and tardiness penalties are to be minimized.

4. Developed memetic algorithm

Memetic algorithms are basically the combination of a global search algorithm (exploration) and local search approach (exploitation), for intense search around defined neighborhood. Memetic algorithm that has been developed to solve the problem \( F_k(P_{m1}, \ldots, P_{mk}) | size_{ij} | \sum_{t=1}^{n}(\alpha_iE_j + \beta_jT_j) \) is based GA that causes the population to evolve through four genetic operators, i.e., selection, crossover, mutation and replacement, and the local improvement method to further improve the offspring generated by crossover operator is given in detail in this section.

4.1 Initial population generation

Population size parameter determines how many individuals are to be included in the population. The initial population consists of “population size” randomly generated individuals, where the “population size” is an input parameter. And the population size is kept constant through the generations.

4.2 Encoding scheme and decoding method

What kind of encoding scheme researchers need to consider are generally driven by the problem type and, therefore, its nature. By definition, HFSMT allows the solver to determine only the sequence of jobs, i.e., the permutation of \( n \) jobs, at each stage. Each individual will carry the information of the sequence of jobs.

As to decoding method, studies in the literature dealing with HFSMT differ from each other in either adopting the permutation at first stage for the remaining stages, or decoding the each individual right after a stage to a full schedule by using different algorithms, such as LS (Oğuz and Ercan, 2005), which resequences the jobs for the stage in non-decreasing order of their completion times at previous stage, or permutation scheduling and first-fit algorithm used in Wang et al. (2011). In this study, the LS method is adopted, the description of which can be seen in Oğuz and Ercan (2005). An example to shed light on how LS decoding method works is given in detail in this section.

Example: consider an example with eight jobs, two stages and four processors at both stages. The processing time and the number of machines required for each jobs are given in Table III.
For a given permutation \((\pi_1 = 4, 1, 8, 2, 7, 3, 6, 5)\) at the stage 1, it is decoded with LS method for the stage 2.

Memetic algorithm operates through the individuals represented by the permutation of jobs at stage 1. LS decodes each individuals into schedules for the remaining stages \((i > 1)\), as it can be seen in Figure 1. In this example, a semi-active schedule is constructed as follows: jobs from the permutation \((\pi_1 = 4, 1, 8, 2, 7, 3, 6, 5)\) are iteratively assigned to the processors available starting from the time 0. We need to pay attention to not assigning a job before its preceding jobs in the permutation, still the jobs can start at the same time if there are enough processors. For example, the job 7 might have started at the same time with the job 8 at stage 1, as there are enough processors for it to be processed. However, this would yield the same results with the permutation of \((\pi\# = 4, 1, 8, 7, 2, 3, 6, 5)\), for instance. If this was not the case; it would be very likely to obtain the same solution with different permutations, and this would significantly hamper the performance of memetic algorithm as operators such as crossover, mutation or local search cannot make much difference.

4.3 Fitness functions
Individuals compete each other with their fitness values calculated, i.e., makespan, which is the completion time of the last job at the processors at the last stage, and defined by \(C_{\text{max}}\). The \(C_{\text{max}}\) values are used to locate the common due window. After using makespan as a fitness function, as the time window will be added to the problem later on, instead of makespan, the total penalty cost function is used for the extended problem with time window (Equation (1)).

4.4 Genetic operators: selection, crossover and mutation operators
As the selection method, the tournament selection method is chosen, as a study in the literature showed that tournament selection method outperforms other selection methods (Noraini and Geraghty, 2011). NXO is presented by Öğuz et al. (Öğuz and Ercan, 2005),

<table>
<thead>
<tr>
<th>Jobs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_j)</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(size_j)</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(p_{j'})</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(size_{j'})</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Table III. Necessary data for the example

![Figure 1](image.png)

The schedule of the permutation \((\pi_1 = 4, 1, 8, 2, 7, 3, 6, 5)\) after being decoded by the LS algorithm
especially for the HFSMT considering its nature. The insight into the algorithm logic and some examples to the implementation of NXO can be found in Öğuz and Ercan (2005). For mutation operator, inversion mutation is chosen.

4.5 Introducing a common time window
In order to study on the due dates, there are two main approaches. In the first scenario, due dates are given from the customers and asked to meet as much as possible to avoid the penalty cost by scheduling the jobs accordingly. In the other scenario, due dates (whether a common due date valid for all the jobs, or due date set for each job) are determined so that the incurred penalty cost is minimized (Baker, 2014).

As the study by Yeung et al. (2004) sets the due window at the center of $C_{\text{max}}$ found by Johnson’s rule, where the both ends of time window $e$ and $d$ equal to $(0.4 \times C_{\text{max}})$ and $(0.6 \times C_{\text{max}})$, respectively, we adopted the same approach in our research; however, we used $C_{\text{max}}$ values found previously in our study (Engin, 2016) to determine the earliest and latest times of common due window. If a job is completed before the time $e$, earliness penalty will incur, and if a job is completed after the time $d$, tardiness penalty will incur, and if a job is completed between the common due window, no penalty will incur.

5. Computational experiments
In this study, the algorithms were tested on benchmark problem instances that can be found in Öğuz (2006) which consists of 240 problems divided into two types; Type Q and Type P with different combinations of $n$ and $k$ ($n=10, 20, 50, 100$; $k=2, 5, 8$), where each combination contains ten instances. Whereas the number of processors at each stage in Type Q is uniformly distributed ($U(1, 5)$); it is fixed as five in Type P problems. Problem names are decoded as “Problem type/Number of jobs/Number of stages/Problem index.” For example, the problem Q20S5T1 is the first problem in its category (Index = 1) and belongs to Type Q problem which is harder to solve than Type P relatively, consisting of 20 jobs and 5 stages.

After dealing with HFSMT and obtaining $C_{\text{max}}$ (not necessarily optimal values) for the problems listed in Table V, we introduced a common time window to jobs and tried to minimize the total earliness and tardiness occurred. The computer code was written using C# language and the experiments were run on an Intel Core 2 Duo with a 1.7 GHz processor and 2.0 GB of RAM memory.

The parameter set used in this study is given in Table IV. This parameter set is formed by Full Factorial Design implemented in the study (Engin, 2016). The penalties per unit time of earliness $\alpha_j$ and tardiness $\beta_j$ which are taken 1 for all jobs for the simplicity, since the focus in this study is on developing earliness and tardiness minimization for the first time and to see if the local search approach makes any significant difference.

To assess if the local search in MA statistically improved the result, Paired $t$-test is conducted. Same benchmark instances provided by Öğuz et al. (2004) are solved with MA and MA missing local search approach (referred as GA).

Table V contains the average total cost functions of five independent runs. $p$-Value is calculated as 0.00. Since this is less than 0.05 (95% confidence level), it suggests that there is strong evidence that the mean of total costs obtained from MA and GA is not the same.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.9</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.1</td>
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Table V. Comparison of MA with GA in terms of earliness and tardiness (total penalty costs)
which leads to the notion that local search significantly improves the solution for the problems listed in Table V.

Finally, Table VI illustrates that MA outperforms GA in solving HFSMT with jobs having a common due window using the same parameter set. Local search is the key to make this noticeable difference in total cost function.

6. Conclusions
In this study, our motivation was to introduce a common time window to HFSMT for the first time in the literature. A memetic algorithm, which is a population-based metaheuristic coupled with neighborhood search (local search), is developed in order to provide good solutions to the problem in hand. As the probability of finding quality solutions with MA relies upon the parameter choices, such as crossover rate, mutation rate, population size, iteration size and parent selection rate, in this case, the Full Factorial design is implemented to decide the best parameter set for each problem type, i.e. solving an instance out of each problem type with each possible parameter combination. Finally, as the algorithm is given its final scheme, the benchmark problem set provided by Özgöz et al. (2004) was solved with

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<th>MA</th>
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Table V. Comparison of GA and MA

Table VI. Which leads to the notion that local search significantly improves the solution for the problems listed in Table V. Finally, Table VI illustrates that MA outperforms GA in solving HFSMT with jobs having a common due window using the same parameter set. Local search is the key to make this noticeable difference in total cost function.

6. Conclusions
In this study, our motivation was to introduce a common time window to HFSMT for the first time in the literature. A memetic algorithm, which is a population-based metaheuristic coupled with neighborhood search (local search), is developed in order to provide good solutions to the problem in hand. As the probability of finding quality solutions with MA relies upon the parameter choices, such as crossover rate, mutation rate, population size, iteration size and parent selection rate, in this case, the Full Factorial design is implemented to decide the best parameter set for each problem type, i.e. solving an instance out of each problem type with each possible parameter combination. Finally, as the algorithm is given its final scheme, the benchmark problem set provided by Özgöz et al. (2004) was solved with
the developed MA. The efficiency of local search method is assessed by paired t-test using the comparison between MA and GA results. The findings indicated that the local search improved the results significantly. Local search may increase the computational effort; however, it was negligible in this study.

Time windows are practically important in real world, as jobs can be completed either too early for them to be delivered to the customers as jobs finished and waiting in the inventory may incur inventory cost; or delivered late which would cause customer dissatisfaction or penalty because of late delivery, as stated in the introduction. Therefore, realizing that due window is an important part of real-world situations; it is decided to develop HFS scheduling problem with multiprocessor task with the jobs having a common due window. By minimizing the penalties (i.e. another source of high cost) incurred by earliness and tardiness, companies can save considerable amount of capital to invest more, increase their efficiency, and therefore competitiveness. In this study, best $C_{max}$ values found for each problem are used to locate the common time windows for each job. As a future direction, due dates (whether a common due date valid for all the jobs, or due date set for each job) may be determined so that the incurred penalty cost is minimized, instead of using a common time window. Also, other non-population-based algorithms may be developed to compare the results with population-based algorithms.

References
Engin, B.E. (2016), A Memetic Algorithm for Hybrid Flow-Shop Scheduling with Multiprocessor Tasks And Due Windows, Selcuk University, Konya.


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Demand prediction in health sector using fuzzy grey forecasting

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Ferhan Çebi
Department of Management, Istanbul Technical University, Istanbul, Turkey

Abstract

Purpose – The purpose of this paper is to apply GM (1, 1) and TFGM (1, 1) models on the healthcare sector, which is a new area, and to show TFGM (1, 1) forecasting accuracy on this sector.

Design/methodology/approach – GM (1, 1) and TFGM (1, 1) models are presented. A hospital’s nine months (monthly) demand data is used for forecasting. Models are applied to the data, and the results are evaluated with MAPE, MSE and MAD metrics. The results for GM (1, 1) and TFGM (1, 1) are compared to show the accuracy of forecasting models. The grey models are also compared with Holt–Winters method, which is a traditional forecasting approach and performs well.

Findings – The results of this study indicate that TFGM (1, 1) has better forecasting performance than GM (1, 1) and Holt–Winters. GM (1, 1) has 8.01 per cent and TFGM (1, 1) 7.64 per cent MAPE, which means excellent forecasting power. So, TFGM (1, 1) is also an applicable forecasting method for the healthcare sector.

Research limitations/implications – Future studies may focus on developed grey models for health sector demand. To perform better results, parameter optimisation may be integrated to GM (1, 1) and TFGM (1, 1). The demand may be predicted not only for the total demand on hospital, but also for the demand of hospital departments.

Originality/value – This study contributes to relevant literature by proposing fuzzy grey forecasting, which is used to predict the health demand. Therefore, the new application area as the health sector is handled with the grey model.

Keywords Demand forecasting, Fuzzy grey forecasting, Healthcare demand, GM (1, 1)

Paper type Research paper

1. Introduction

Forecasts are made to guide decisions in a variety of fields (Diebold, 2007). Forecasting the demand in healthcare is very important because the usage and the allocation of hospitals’ resources effectively depend on predicting the demand correctly. Forecasting accuracy effects all the planning of the system.

There has been applications of numerous forecasting methods in healthcare sector literature mostly times series through methods such as exponential smoothing, Holt–Winters, etc. Forecasting is also important in other sectors such as finance, energy, meteorology, etc. (Lin et al., 2004). In the process of time, papers about forecasting techniques are on the development and increase.

The systems with incomplete information are referred to as grey systems where grey means poor, incomplete, etc. Incompleteness in information is the fundamental meaning of being grey. The meaning of grey can be expanded or stretched. Table I, shows the details of grey, white and black (Liu et al., 2012).

The aim of grey systems and applications is making a bridge between social and natural science. A grey system can be applied to agriculture, ecology, economy, meteorology, medicine, history, geography, industry, military affairs, sports, traffic, management, biological protection, judicial system, etc. (Julong, 1989). The grey system theory, probability and statistics and fuzzy logic are the methods dealing with uncertain problems. Lin et al. (2004) explain the differences between these methods clearly.
Grey forecasting is the sub-concept of grey theory and it includes: series forecasting, calamities (seasonal) forecasting, topological forecasting, systematic forecasting. All of them are based on GM (1, 1) (Julong, 1989).

This paper focuses on predicting the demand for the healthcare sector with GM (1, 1) under fuzzy environment. Dissimilarly from the forecasting demand on healthcare literature, this study applies GM (1, 1) and TFGM (1, 1) methods to forecast the demand a hospital. Grey models are used in the healthcare sector which is a new area.

In this paper, triangular fuzzy grey model (TFGM) is proposed to predict the monthly demand of a hospital. GM (1, 1) and triangular fuzzy grey model (1, 1) are applied to the same data, and the accuracy of forecasting methods is compared. The comparison is made by some evaluation metrics, namely, mean absolute percentage error (MAPE), mean absolute deviation (MAD) and mean squared error, which are the most common metrics to evaluate forecasting performance. So, we show the applicability of GM (1, 1) and TFGM (1, 1) to the demand on the healthcare sector.

The rest of the paper proceeds as follows: Section 2 describes the literature about grey system theory, GM (1, 1), and some forecasting methods, which are used in the healthcare sector. Sections 3 and 4 present GM (1, 1) and TFGM (1, 1), respectively. In Section 5, Holt–Winters, GM (1, 1) and TFGM (1, 1) are applied to hospital data. This section is the first application of TFGM (1, 1) on the healthcare sector. Conclusion and future works take part in Section 6.

2. Literature review

The grey system theory was first introduced in 1982 by Julong Deng. Julong (1989) proposed a research about the grey system theory and its essential contents and topics such as: grey relational space, grey forecasting, grey decision making, etc. In the coming years, the grey theory was used in optimisation problems (Zheng and Lewis, 1993). Grey theory-based approaches were applied in many fields, for example, earthquake, agriculture, rock mechanics (An et al., 2012; Xiuwen, 1994).

When we look at the 20 years from the first introduction of the grey system theory, there are many papers about the fields like science, electric power, information technology, economics, economic development, transportation, geology, medicine, ecology, education and finance (Lin et al., 2004). Grey forecasting techniques are applied frequently for products which have a short life cycle (Hsu, 2003). Grey forecasting, GM (1, 1), is also applied to forecast opto-electronics output value and TFT-LCD panels in Taiwan (Lin and Yang, 2003; Chang, 2005).

The grey forecasting which is applied to the energy sector frequently (Zhou et al., 2006; Huang et al., 2007; Nai-ming et al., 2015; Hamzacebi and Es, 2014) is applied to Turkey’s electric demand prediction under different scenarios, and Akay and Atak conclude that grey forecasting provides accurate forecasting results in short-term data (2007).

<table>
<thead>
<tr>
<th>Situation</th>
<th>Concept</th>
<th>Information</th>
<th>Appearance</th>
<th>Processes</th>
<th>Properties</th>
<th>Methods</th>
<th>Attitude</th>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>White</td>
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<td>Clear</td>
<td>Old</td>
<td>Order</td>
<td>Confirmation</td>
<td>Rigorous</td>
<td>Unique solution</td>
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<tr>
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<td>Incomplete</td>
<td>Blurred</td>
<td>Changing</td>
<td>Multivariate</td>
<td>Change for better</td>
<td>Tolerant</td>
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<tr>
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<td>Black</td>
<td>Unknown</td>
<td>Dark</td>
<td>New</td>
<td>Chaotic</td>
<td>Negation</td>
<td>Letting go</td>
<td>No solution</td>
</tr>
</tbody>
</table>

Table I. Extensions of grey concept
When we did research about grey models in the healthcare sector, only a paper about health management was found (Chen et al., 2008). In literature research, we did not discover any paper about demand forecasting in the healthcare sector using the grey theory.

The fuzzy set theory was first introduced by Zadeh (1965). It is widely applied in many fields such as decision making (Kahraman et al., 2003), optimisation (Bolat et al., 2014) and forecasting (Geng et al., 2015), etc.

Tsaur (2010) constructed a crisp-input and fuzzy-output fuzzy grey model GM (1, 1). A dynamic grey fuzzy Markov prediction model is proposed to forecast the biofuel production in China for 2010–2013. It has strong applicability (Geng et al., 2015). In 2016, the fuzzy grey model is proposed to forecast the consumer price index and power load (Zeng et al., 2016).

The following literature review is about the healthcare sector. As we mentioned before, grey model methods were not used in the research about the forecasting demand in the healthcare sector. Traditional forecasting methods are used to compare the proposed methods performance. Box–Jenkins univariate and Tiao–Box multiple time series approaches are evaluated against Holt–Winters exponential smoothing model using the MSE, MAD and MAPE (Lin, 1989). In 1998, autoregressive integrated moving average (ARIMA) models are used in a study for forecasting monthly patient volume at a primary healthcare clinic. ARIMA (4, 2, 0) gives the best result of all ARIMA models in the study (Abdel-Aal and Mangoud, 1998). In a recent study of Holt–Winters, ARIMA methods are used to forecast emergency department demand (Aboagye-Sarfo et al., 2015). In a research paper, the linear regression model was used to forecast the demand of emergency department visits accordingly website visits. MAPE was used to evaluate the forecasting accuracy. Emergency department visits was the dependent variable, and website visits was the independent variable. They checked the correlation between these variables at first. MAPE were between 5.2 and 13.1 per cent (Ekström et al., 2015). Bu using three different methods, a Swiss academic hospital’s bed needs were predicted. In this case, three scenarios were used (Seematter-Bagnoud et al., 2015).

Moving average, exponential smoothing, Holt–Winters and linear regression forecasting methods are used to forecast consumption of serum set of a hospital. The forecasting result of Holt–Winters additive method has the lowest error rate (Yigit, 2016). In a paper, six different approaches proposed to forecast healthcare sector time series data (healthcare sector index). Monthly data of January 2010–December 2016 period were used for forecasting. The models were built by using Holt–Winters and ARIMA with different size of training data set. RMSE was used for comparing the six models. While the ARIMA-based approach with a forecast horizon of 12 months performed best with lowest RMSE value, the Holt–Winters method with a forecast horizon of 12 months performed worst with the highest RMSE value (Sen and Chaudhuri, 2017). Bon and Ng (2017) aimed to optimise the overall inventory demand by ten forecasting techniques which are single moving average, single exponential smoothing, double moving average, double exponential smoothing, regression, Holt–Winters additive, seasonal additive, Holt–Winters multiplicative, seasonal multiplicative and ARIMA. They used historical demand data of Panadol 650 mg for 68 months from January 2009 to August 2014 in University Health Centre. Regression analysis had the best RMSE result. The aim of the paper is to predict an hospital daily outpatient visit. They used seasonal ARIMA (SARIMA) and seasonal exponential smoothing (SES) models to build a combinatorial model based on SARIMA and SES. MAPE was used to evaluate forecasting accuracy. Combinatorial model had better forecasting performance than SARIMA and SES models (Luo et al., 2017). To forecast the pre-hospital emergency medical services (EMS) demand for diabetic emergencies, Ambulance Victoria (AV) electronic database between 2009 and 2015 was used. Different models were built by using SARIMA. MAPE was used to evaluate the models and with an MAPE 4.2 per cent. SARIMA (0, 1, 0, 12) model had the best forecasting accuracy. They conclude that the pre-hospital EMS demand for diabetic emergencies is increasing (Villani et al., 2017).
In a research to forecast short-term load, the Holt–Winters method, which is regarded as a well-performing forecasting method, is used to combine with wavelet transform and weighted nearest neighbour models (Sudheer and Suseelatha, 2015).

3. Grey forecasting
Grey forecasting is the sub-topic of the grey system. In this section, GM (1, 1) is presented for predicting the demand.

The following equations are the steps of GM (1, 1). (Akay and Atak, 2007; Liu and Lin, 2006; Lee and Tong, 2011; Liu et al., 2016).

$t$ is the independent variable, $a$ is the coefficient and $b$ is the grey control coefficient.

The following equation is the white differential equation of grey model (1, 1):

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b.$$  \hspace{1cm} (1)

The original time sequence is:

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)).$$ \hspace{1cm} (2)

The following equation is the first-order accumulated generated operation (1-AGO) of Equation (2):

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)), \hspace{1cm} (3)$$

$$X^{(1)} = \left( \sum_{t=1}^{1} x^{(0)}(t), \sum_{t=1}^{2} x^{(0)}(t), \ldots, \sum_{t=1}^{n} x^{(0)}(t) \right).$$

By using the least mean square method, $a$ is estimated:

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} = \left( B^T \times B \right)^{-1} \times B^T \times Y_N.$$ \hspace{1cm} (4)

1-AGO (first-order accumulated generating operation) matrix $B$:

$$B = \begin{bmatrix}
-\frac{1}{2} [x^{(1)}(1) + x^{(1)}(2)] & 1 \\
-\frac{1}{2} [x^{(1)}(2) + x^{(1)}(3)] & 1 \\
\vdots & \vdots \\
-\frac{1}{2} [x^{(1)}(n-1) + x^{(1)}(n)] & 1
\end{bmatrix},$$ \hspace{1cm} (5)

$$Y = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T.$$ \hspace{1cm} (6)
By using estimated coefficients $a$ and $b$, the grey forecasting equation can be estimated as in the following equation which gives the cumulated value at time $t + 1$:

$$x_1(t + 1) = \left( x_0(1) - \frac{b}{a} \right) e^{-at} + \frac{b}{a}.$$  

(7)

The estimated value in equation (7) gives the first-order accumulated operation. So by performing inverse accumulated generating operation), it gives the following equation:

$$x_0(t + 1) = x_1(t + 1) - x_1(t).$$  

(8)

$$\hat{x}(0) = \left( \hat{x}(0)(1), \hat{x}(0)(2), \ldots, \hat{x}(0)(n+1) \right).$$  

(9)

$\hat{x}(0)(n+1)$ is the first predicted grey value of $x(n+1)$.

4. Triangular fuzzy grey model (1, 1)

The following equations are the steps of TFGM (1, 1) (Zeng et al., 2016):

$$\tilde{x} = \{\tilde{x}(1), \tilde{x}(2), \ldots, \tilde{x}(n)\}, \quad \tilde{x}(i) = [x_L(i), x_M(i), x_R(i)].$$  

(10)

Definition 1:

$$\hat{x}(1)(i) = \sum_{k=1}^{i} \tilde{x}(k) = \left[ \sum_{k=1}^{i} x_L(k), \sum_{k=1}^{i} x_M(k), \sum_{k=1}^{i} x_R(k) \right]$$

$$= \left[ x_L^{(1)}(i), x_M^{(1)}(i), x_R^{(1)}(i) \right], \quad i = 1, 2, \ldots, n.$$  

(11)

Definition 2:

$$\tilde{z}^{(1)} = \{\tilde{z}^{(1)}(2), \tilde{z}^{(1)}(3), \ldots, \tilde{z}^{(1)}(n)\}.$$  

$$\tilde{z}^{(1)}(i) = 0.5(\tilde{z}^{(1)}(i-1) + \hat{x}(1)(i))$$

$$= \left[ 0.5 \left( \sum_{k=1}^{i-1} x_L(k) + \sum_{k=1}^{i} x_L(k) \right), \ 0.5 \left( \sum_{k=1}^{i-1} x_M(k), \sum_{k=1}^{i} x_M(k) \right), \sum_{k=1}^{i} x_R(k) \right]$$

$$= \left[ \tilde{z}_L^{(1)}(i), \tilde{z}_M^{(1)}(i), \tilde{z}_R^{(1)}(i) \right], \quad i = 2, 3, \ldots, n.$$  

(12)

Definition 3: TFGM (1, 1) grey differential equation is:

$$\tilde{x}(i) + a \times \tilde{z}^{(1)}(i) = \tilde{b}, \quad i = 2, 3, \ldots, n.$$  

(13)
\[
\begin{align*}
[ x_L^{(1)}(i), x_M^{(1)}(i), x_R^{(1)}(i) ] + a \times [ x_L^{(1)}(i), x_M^{(1)}(i), x_R^{(1)}(i) ] &= [ b_L, b_M, b_R ], \\
x_L(2) + a_L x_L^{(1)}(2) &= b_L, \quad x_L(3) + a_L x_L^{(1)}(3) = b_L, \quad x_L(n) + a_L x_L^{(1)}(n) = b_L, \\
x_M(2) + a_M x_M^{(1)}(2) &= b_M, \quad x_M(3) + a_M x_M^{(1)}(3) = b_M, \quad x_M(n) + a_M x_M^{(1)}(n) = b_M, \\
x_R(2) + a_R x_R^{(1)}(2) &= b_R, \quad x_R(3) + a_R x_R^{(1)}(3) = b_R, \quad x_R(n) + a_R x_R^{(1)}(n) = b_R,
\end{align*}
\]

Theorem 1:
\[
\tilde{x}(i) = \frac{2(2-a)^{i-2} (b-a \times \tilde{x}(1))}{(2+a)^{i-1}} \quad i = 2, 3, \ldots
\]
5. Methodology

In this section, TFGM (1, 1) is applied to forecast the monthly demand of a hospital (İrmak et al., 2012). To evaluate the performance of the model, we compare it with the traditional GM (1, 1) and Holt–Winters method, which is widespread in forecasting studies and performs well. Accuracy of the prediction is evaluated by MAD, mean square error (MSE) and the general accepted metric, MAPE (Tang and Yin, 2012; De Gooijer and Hyndman, 2006; Chen et al., 2012; Liu et al., 2016):

\[
\hat{x}(i) = [\hat{x}_L(i), \hat{x}_M(i), \hat{x}_R(i)],
\]

\[
\hat{x}_L(i) = \frac{2(2-a)^{-2}(b_L-ax_L(1))}{(2+a)^{-1}}, \\
\hat{x}_M(i) = \frac{2(2-a)^{-2}(b_M-ax_M(1))}{(2+a)^{-1}}, \\
\hat{x}_R(i) = \frac{2(2-a)^{-2}(b_R-ax_R(1))}{(2+a)^{-1}}.
\]

The criterion of MAPE is shown in Table II (Liu et al., 2016).

First, four months are used as training data and the following five months are predicted. GM (1, 1) is applied. The following calculation shows the fifth predicted value.

First, Equation (2) is applied to the training data \(X^{(0)}\), which shows the raw data:

\[
X^{(0)} = (64,515,58,330,66,385,64,946).
\]

Then, Equation (3) is used to obtain 1-AGO vector:

\[
X^{(1)} = (64,515,122,845,189,230,254,176).
\]
B (Equation 5) and Y (Equation 6) matrices are calculated in order to be used in the least square method on Equation (4). Hence, \( a \) and \( b \) coefficients are determined by Equation (4):

\[
B = \begin{bmatrix}
-93,680 \\
-156,037.5 \\
-221,703
\end{bmatrix}, \quad Y = \begin{bmatrix}
58,330 \\
66,385 \\
64,946
\end{bmatrix},
\]

\[
\hat{a} = \begin{bmatrix}
a \\
b
\end{bmatrix} = \begin{bmatrix}
-0.051028 \\
55,201.765
\end{bmatrix}.
\]

By Equation (7), the estimated value of AGO vector is obtained:

\[
x^{(1)}(5) = \left(x^{(0)}(1) - \frac{55,201.765}{-0.051028}\right) \times e^{0.051028 \times 4} + \frac{55,201.765}{-0.051028} = 324,080.
\]

By subtracting the raw value at time 4 from the estimated AGO value at time 5, the estimated IAGO value at time 5 is attained (Equation 8):

\[
\hat{x}^{(0)}(5) = 324,080 - 254,176 = 69,904.
\]

The predicted values are shown in Table III, and the accuracy of the prediction is evaluated by MAD, MSE, MAPE, as shown in Table IV.

Second, Holt–Winters method is applied. Predicted values and evaluation metrics’ results are shown in Table V.

And third, TFGM (1, 1), TFGM (1, 1) is applied to data.

\( x_M \) is the number of the patients that take an appointment from the system. In real world, some patients do not show up or some of them come for examination and doctors decide that there is no need for examination. These patients occur in the lower bound of the triangular fuzzy number. Some patients come without any appointment or unexpected patients show up. Those kinds of patients occur in the upper bound of the

---

### Table III

<table>
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<th>Month</th>
<th>Demand</th>
<th>Predicted demand</th>
<th>PE (%)</th>
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<tbody>
<tr>
<td>January 2009</td>
<td>64,515</td>
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<td></td>
</tr>
<tr>
<td>February 2009</td>
<td>58,330</td>
<td></td>
<td></td>
</tr>
<tr>
<td>March 2009</td>
<td>66,385</td>
<td></td>
<td></td>
</tr>
<tr>
<td>April 2009</td>
<td>64,946</td>
<td></td>
<td></td>
</tr>
<tr>
<td>May 2009</td>
<td>62,632</td>
<td>69,904</td>
<td>11.61</td>
</tr>
<tr>
<td>June 2009</td>
<td>67,427</td>
<td>65,893</td>
<td>2.28</td>
</tr>
<tr>
<td>July 2009</td>
<td>64,100</td>
<td>68,327</td>
<td>6.59</td>
</tr>
<tr>
<td>August 2009</td>
<td>59,534</td>
<td>65,929</td>
<td>12.42</td>
</tr>
<tr>
<td>September 2009</td>
<td>59,310</td>
<td>63,547</td>
<td>7.14</td>
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### Table IV

<table>
<thead>
<tr>
<th>Metrics</th>
<th>MAPE</th>
<th>MAD</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM (1, 1)</td>
<td>8.01%</td>
<td>4,933.04</td>
<td>29,148,087.33</td>
</tr>
</tbody>
</table>

---

JEIM 31,6
triangular fuzzy number. \( x_M \) is the number of the patients who take an appointment from the system (Table VI).

The fifth value of the data is predicted as follows:

A, B, C, \( Y_L \), \( Y_M \) and \( Y_R \) are obtained from Equation (16):

\[
A = \begin{bmatrix} -84.665 & 1 \\ -141.522 & 1 \\ -201.685 & 1 \end{bmatrix}, \quad Y_L = \begin{bmatrix} 53.330 \\ 60.385 \\ 59.940 \end{bmatrix},
\]

\[
B = \begin{bmatrix} -93.680 & 1 \\ -156.037 & 1 \\ -221.703 & 1 \end{bmatrix}, \quad Y_M = \begin{bmatrix} 58.330 \\ 66.385 \\ 64.946 \end{bmatrix},
\]

\[
C = \begin{bmatrix} -98.180 & 1 \\ -165.045 & 1 \\ -237.280 & 1 \end{bmatrix}, \quad Y_R = \begin{bmatrix} 61.400 \\ 72.330 \\ 72.140 \end{bmatrix}.
\]

Via Equation (15), \( a_L \), \( b_L \), \( a_M \), \( b_M \), \( a_R \), \( b_R \) are obtained:

\[
\begin{align*}
(a_L) &= (-0.0559) \begin{bmatrix} 49.916 & 8965 \\ 55.201 & 765 \\ 55.919 & 836 \end{bmatrix},
(b_L) &= \begin{bmatrix} 49.916 & 8965 \\ 55.201 & 765 \\ 55.919 & 836 \end{bmatrix},
(a_R) &= (-0.076144) \begin{bmatrix} 49.916 & 8965 \\ 55.201 & 765 \\ 55.919 & 836 \end{bmatrix},
(b_R) &= \begin{bmatrix} 49.916 & 8965 \\ 55.201 & 765 \\ 55.919 & 836 \end{bmatrix}.
\end{align*}
\]

Equations (17) and (18) are applied and triangular fuzzy values are obtained. Table VII shows the predicted triangular fuzzy values.

<table>
<thead>
<tr>
<th>Month</th>
<th>Predicted demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 2009</td>
<td>62,632</td>
</tr>
<tr>
<td>June 2009</td>
<td>67,427</td>
</tr>
<tr>
<td>July 2009</td>
<td>64,100</td>
</tr>
<tr>
<td>August 2009</td>
<td>59,534</td>
</tr>
<tr>
<td>September 2009</td>
<td>59,310</td>
</tr>
</tbody>
</table>

Table V. Forecasting results and evaluation metrics for Holt and Winter’s model

<table>
<thead>
<tr>
<th>Month</th>
<th>( x_L )</th>
<th>( x_M )</th>
<th>( x_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 2009</td>
<td>58,000</td>
<td>64,515</td>
<td>67,480</td>
</tr>
<tr>
<td>February 2009</td>
<td>53,330</td>
<td>58,330</td>
<td>61,400</td>
</tr>
<tr>
<td>March 2009</td>
<td>60,385</td>
<td>66,385</td>
<td>72,330</td>
</tr>
<tr>
<td>April 2009</td>
<td>59,940</td>
<td>64,946</td>
<td>72,140</td>
</tr>
<tr>
<td>May 2009</td>
<td>59,632</td>
<td>62,632</td>
<td>68,435</td>
</tr>
<tr>
<td>June 2009</td>
<td>60,680</td>
<td>67,427</td>
<td>72,230</td>
</tr>
<tr>
<td>July 2009</td>
<td>59,100</td>
<td>64,100</td>
<td>70,510</td>
</tr>
<tr>
<td>August 2009</td>
<td>55,530</td>
<td>59,534</td>
<td>65,470</td>
</tr>
<tr>
<td>September 2009</td>
<td>53,200</td>
<td>59,310</td>
<td>66,244</td>
</tr>
</tbody>
</table>

Table VI. Monthly demand of an hospital
Table VIII shows the percentage errors of the bounds; from Equation (17), crisp error rates are established. The results of Holt–Winters, GM (1, 1) and TFGM (1, 1) are shown in Table IX. For the Holt–Winters model, MAPE, MAD and MSE evaluation metrics results are 11.81 per cent, 7,297.12 and 59,072,686.63, respectively. For GM (1, 1), MAPE, MAD and MSE evaluation metrics results are 8.01 per cent, 4,933.04 and 29,148,087.33 respectively. For TFGM (1, 1), they are 7.64 per cent, 4,754.71 and 29,064,897.93, respectively. So, grey models error rates show that the forecasting accuracy is acceptable (MAPE, 8.01 per cent < 10 per cent and 7.64 per cent < 10 per cent), and TFGM (1, 1) forecasting performance is better than the Holt–Winters method and the GM (1, 1) in this case.

6. Conclusion and future works

Recent studies address that grey forecasting methods on small data generally perform better than big data. Moreover, grey forecasting is one of the novel grey application areas in the healthcare sector.

In this paper, small data are used for forecasting the demand on health sector with GM (1, 1) and TFGM (1, 1). The aim of the paper is to indicate the accuracy of forecasting performance with grey model-integrated triangular fuzzy number. Grey forecasting (TFGM (1, 1)) is the first grey application in the healthcare sector. To compare grey methods with a traditional forecasting technique, the Holt–Winters method, which performs well, is applied.

The results show that both GM (1, 1) and TFGM (1, 1) methods give good results. Besides, TFGM (1, 1) performs better than GM (1, 1) and Holt–Winters.

Future studies may focus on developed grey models for health sector demand. For better results, parameter optimisation may be integrated to GM (1, 1) and TFGM (1, 1). The demand may be predicted not only for the total demand on hospitals, but also for the demand of hospital departments. Big data may be analysed for the forecasting accuracy of GM (1, 1) and TFGM (1, 1) for this sector.

<table>
<thead>
<tr>
<th>May 2009</th>
<th>(65,490, 69,904, 59,869)</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2009</td>
<td>(60,543, 65,893, 64,279)</td>
</tr>
<tr>
<td>July 2009</td>
<td>(63,996, 68,327, 64,948)</td>
</tr>
<tr>
<td>August 2009</td>
<td>(61,864, 66,929, 65,927)</td>
</tr>
<tr>
<td>September 2009</td>
<td>(58,722, 63,547, 67,750)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table VII. Predicted triangular fuzzy values</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 2009</td>
</tr>
<tr>
<td>June 2009</td>
</tr>
<tr>
<td>July 2009</td>
</tr>
<tr>
<td>August 2009</td>
</tr>
<tr>
<td>September 2009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table VIII. Percentage error of the predicted values</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 2009</td>
</tr>
<tr>
<td>June 2009</td>
</tr>
<tr>
<td>July 2009</td>
</tr>
<tr>
<td>August 2009</td>
</tr>
<tr>
<td>September 2009</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table IX. Evaluation metrics of Holt–Winter’s, GM (1, 1) and TFGM (1, 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPE (%)</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Holt–Winter’s</td>
</tr>
<tr>
<td>GM (1, 1)</td>
</tr>
<tr>
<td>TFGM (1, 1)</td>
</tr>
</tbody>
</table>
References


Diebold, F.X. (2007), Elements of Forecasting, Thomson South-Western, Mason, OH.


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Stochastic AHP and fuzzy VIKOR approach for warehouse location selection problem

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Department of Industrial Engineering, Ataturk University, Erzurum, Turkey

Abstract
Purpose – The purpose of this paper is to develop a stochastic multi-criteria decision-making approach to solve the warehouse location problem in the stochastic environment which contains uncertain condition.
Design/methodology/approach – In developed approach, the weight of criteria was calculated by using the stochastic analytic hierarchy process (SAHP) method. Alternative ranking was made and evaluated by fuzzy VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje).
Findings – This study dealt with warehouse location selection problem of a supermarket that has sellers in many regions in Turkey and selected proper warehouse.
Originality/value – This study combined SAHP and fuzzy VIKOR methods as a solution approach for warehouse location selection problems.
Keywords Fuzzy VIKOR, The warehouse location selection, Stochastic AHP
Paper type Research paper

1. Introduction
Rapidly changing and evolving conditions have necessitated businesses to make the right decisions for their companies. In such an environment, companies must make a valid and healthy decision in order to provide a competitive advantage. In order to carry out the decision-making process effectively, MCDM techniques are used which help companies choose the best alternative among more than one criterion. MCDM is a frequently used methodology for solving evaluation and ranking problems involving many conflicting criteria.

In real-life problems, there may be cases in which the criteria values of the alternatives and the criterial weights are not precisely known. Therefore, in this study, the authors proposed a stochastic analytic hierarchy process (SAHP) that can handle uncertain information and identify weights of criteria in the MCDM problem (Cobuloglu and Buyuktahtakin, 2015). Two methods to rank alternatives – SAHP and fuzzy VIKOR (ViseKriterijumska Optimizacija I Kompromisno Resenje) – were hybridized in this study.

The remainder of this paper is organized as follows: Section 2 reviews the literature on the warehouse location selection problem, SAHP and fuzzy VIKOR. Section 3 develops SAHP method and presents fuzzy VIKOR. Section 4 illustrates the case study of a supermarket and Section 5 concludes the paper.
2. Literature review

2.1 Warehouse location selection problem

Considering the literature of warehouse location selection problem in recent years, it is seen that different methods are used. Through studies using Heuristic and programming methods, the paper studied a successful application of multi-criteria Choquet integral to a real warehouse location selection problem of a big Turkish logistic firm (Demirel et al., 2010). Tabu Search algorithm was used for warehouse layout problem. The proposed system proved that is suitable to solve the problem in negligible time (Nehzati et al., 2011).

An automobile spare part warehouse location problem dealt with particle swarm optimization was improved for the problem (Yaobao et al., 2013). A model was presented that looks for the optimal allocation of goods in order to maximize the storage space available within the restrictions of the warehouse. Computational tests performed on a set of randomly generated and real warehouse instances showed the effectiveness of the proposed methods (Quintanilla et al., 2014). Rath and Gutjahr (2014) developed a “math-heuristic” for a three-objective warehouse location–routing problem in disaster relief. A continuous approximation model was proposed for location warehouse. Proposed methodology was applied to the real case of a company in Santiago, Chile (Pulido et al., 2015).

Multi-criteria decision-making techniques such as AHP, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), ELECTRE (Elimination and Choice Translating Reality) and Grey Theory methodologies used in previous studies were used for a warehouse location selection problem. According to results obtained, the best alternative was chosen with TOPSIS and ELECTRE calculation (Ozcan et al., 2011). Silva et al. (2015) proposed a multi-criteria decision model to support the process of assigning storage locations to products in a warehouse. The alternatives were ranked using the Simple Multi-Attribute Rating Technique Extended to Ranking and lexicographic methods (Silva et al., 2015). Temur (2016) presented a new supportive decision tool named as CBDO which tackles high uncertainty. The proposed model was applied in a warehouse location selection problem (Temur, 2016). Fuzzy Step-wise Weight Assessment Ratio Analysis and fuzzy MOORA were used for selection problem in a plastic industry (Mavi et al., 2017). Dey et al. (2017) proposed a new multi-criteria group decision-making approach. A real case example on warehouse location selection was illustrated to demonstrate the validity and effectiveness of the proposed approach. Sensitivity analysis and other investigations found the proposed approach as a rational effective and precise decision-making aid to the supply chain managers (Dey et al., 2017). A multi-criteria decision model was proposed to perform the products classification and to solve the storage location assignment problem (SLAP) in a multi-layer warehouse. It could conclude that the ELECTRE TRI allows minimizing the computational effort in SLAP (Fantana and Nepomuceno, 2017).

2.2 Stochastic AHP

Warehouse location selection model was developed to minimize logistics costs. They have studied three different methods which are AHP, VIKOR and MOORA to solve the problem of warehouse location selection (Aktepe and Ersoz, 2014). Jalao et al. (2014) proposed SAHP and non-linear programming to calculate crisp criteria or alternative weights. Soner et al. (2017) proposed integration of AHP and VIKOR under interval type 2 fuzzy environment for hatch cover selection problem in the maritime transportation. The results showed that the proposed approach can be applied to similar decision-making problems in the maritime transportation industry (Soner et al., 2017). Van Bavel and Ghanmi (2017) presented an SAHP method for performing a high-dimensional sensitivity analysis of bid evaluation plans for military systems in-service support contracts.
2.3 Fuzzy VIKOR
Mandal et al. (2015) developed a methodology for human error identification and prioritization. They used HTA and SHERPA for human error analysis and used fuzzy VIKOR method for risk prioritization purpose. Singh et al. (2016) proposed an integrated AHP–VIKOR method to select sustainability strategy. The proposed method provides more flexibility to decision makers to deal with uncertainties compared to the other MCDM method based on fuzzy type-1 or crisp values (Singh et al., 2016). Awasthi and Kannan (2016) presented an integrated approach for evaluating and selecting best green supplier development program(s) for buyer organizations using nominal group technique and VIKOR under fuzzy environment. Ren et al. (2017) developed a new distance measure for dual hesitant fuzzy elements. They used a dual hesitant fuzzy VIKOR method for solving the multi-criteria group decision-making problems.

3. Methodology
3.1 Stochastic AHP
In this study, an SAHP model was developed by utilizing the works of Jalao et al. and Cobuloglu and Buyuktahtakin. Saaty’s (1977) pairwise comparison scale is shown in Table I.

The stages of the proposed method for the selection of the appropriate warehouse location are described in detail below (Cobuloglu and Buyuktahtakin, 2015).

Step 1: an AHP problem with \( n \) criteria and \( m \) alternatives has been structured.

Step 2: compare elements using pairwise comparisons. Experts are required to compare every element pairwise in their corresponding section. A section consists of elements that are placed at the same level under a specific element of the hierarchy where experts can provide crisp values, most-likely value (somewhat imprecise) with upper and lower bounds or a range (totally imprecise nine with lower and upper bounds for comparison of two elements. As indicated previously, Table I is employed for pairwise comparison since it is a widely used scale.

For example, assume that an expert indicates that the cost of land is “extreme importance” compared to the cost of labor in warehouse selection. Then, a numerical score, \( a_{lm} = 9 \) is assigned. On the other hand, the expert can also provide an imprecise linguistic pairwise comparison while comparing the criteria of proximity to competitors. In this case, he may decide that the cost of land is very strong importance, extreme importance or an intermediate value between the two compared to the proximity to competitors. Then, the value \( a_{lc} = [7, 8, 9] \) is assigned. Finally, if the expert provides a range because the cost of land has very strong importance or between strong importance and very strong importance compared proximity to suppliers, then we assign the value \( a_{lc} = [6, 7] \).

Step 3: convert imprecise preferences of experts into stochastic pairwise comparisons. Readers are referred to the study of Jalao et al. (2014) for a discussion of the \( \beta \) distribution and stochastic pairwise comparison.

To obtain crisp values of a stochastic pairwise comparison given alternatives \( i \) and \( j \), \( a_{ij} \), conversion is done according to the probability density function \( f_{ij}(a_{ij} \mid 0_{ij}) \) with parameters \( 0_{ij} \). Based on the previous example, \( a_{lm} \sim f_{lm} = 9 \) and \( a_{lc} = [7, 8, 9] \) is modeled

<table>
<thead>
<tr>
<th>Intensity of importance</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
</tr>
</tbody>
</table>

Note: Even scale of 2, 4, 6 and 8 are used to compromise slight differences between two classifications.
as a triangular distribution with a lower limit (LL), most likely (ML) and upper limit (UL) as $a_{ij} \sim f_a(LL, ML, UL) = T(\cdot, 8, 9)$. Finally $a_{ij} = [6, 7]$ is modeled as a uniform distribution as $a_{at} \sim f_b(LL, UL) = U(a, 3)$. Because of the difficulty of a priority weight calculation for these varying distributions, the stochastic pairwise comparisons are transformed into $\beta$-distributed pairwise comparisons, $\tilde{a}_{ij}$. The term $\tilde{a}_{ij}$ follows the $\beta$-distribution $\beta(\tilde{a}_{ij} | a, \beta, LL, UL)$, where $LL \leq \tilde{a}_{ij} \leq UL$ and $a, \beta \geq 1$.

In order to explicitly model all $a_{ij}$ as $\beta$-random variables, $\tilde{a}_{ij}$ the shape ($\alpha, \beta$) and location ($LL_{ij}, UL_{ij}$) parameters are estimated based on the method of moments (MOM). The sample mean and variance are obtained by taking the first and second moments:

$$E[\tilde{a}_{ij}] = LL + \frac{x}{x + \beta}(UL - LL).$$

(1)

$$Var(\tilde{a}_{ij}) = \frac{\alpha \beta}{(x + \beta)^2(x + \beta + 1)}(UL - LL).$$

(2)

Equating Equations (1) and (2) to the sample mean ($\tilde{a}_{ij}$) and sample variance ($S_{ij}^2$), respectively, we obtain the estimates of the shape parameters, $\hat{\alpha}_{ij}$, $\hat{\beta}_{ij}$ using the following equations:

$$\hat{\alpha}_{ij} = \left(\frac{\bar{a}_{ij} - LL}{UL - LL}\right) \left(\frac{(\bar{a}_{ij} - LL/UL - LL)(1 - (\bar{a}_{ij} - LL/UL - LL))}{s_{ij}^2/(UL - LL)} - 1\right).$$

(3)

$$\hat{\beta}_{ij} = \left(1 - \frac{\bar{a}_{ij} - LL}{UL - LL}\right) \left(\frac{(\bar{a}_{ij} - LL/UL - LL)(1 - (\bar{a}_{ij} - LL/UL - LL))}{s_{ij}^2/(UL - LL)} - 1\right).$$

(4)

Applying the MOM for converting stochastic pairwise comparisons to $\beta$-distributed pairwise comparisons, outputs are summarized below:

$$\tilde{a}_{ij} = a_{ij} \quad \text{if } a_{ij} \text{ is crisp},$$

(5)

$$\tilde{a}_{ij} \sim B(\tilde{a}_{ij} = 1, \hat{\beta}_{ij} = 1, LL_{ij}, UL_{ij}) \quad \text{if } a_{ij} \sim U(LL_{ij}, UL_{ij}),$$

(6)

$$\tilde{a}_{ij} \sim B(\tilde{a}_{ij}, \hat{\beta}_{ij}, LL_{ij}, UL_{ij}) \quad \text{if } a_{ij} \sim T(LL_{ij}, ML_{ij}, UL_{ij})$$

(7)

where $\tilde{a}_{ij}$ and $\hat{\beta}_{ij}$ in Equation (7) are obtained from Equations (3) and (4), respectively:

$$\pi_{ij} = (LL_{ij} + ML_{ij} + UL_{ij})/3,$$

and

$$S_{ij}^2 = \frac{(LL_{ij}^2 + ML_{ij}^2 + UL_{ij}^2 - LL_{ij}ML_{ij} - LL_{ij}UL_{ij} - ML_{ij}UL_{ij})/18}{(LL_{ij}^2 + ML_{ij}^2 + UL_{ij}^2 - LL_{ij}ML_{ij} - LL_{ij}UL_{ij} - ML_{ij}UL_{ij})/18}$$

(8)

(obtained from standard mean and variance formulation used by Jalao et al. (2014)).

Step 4: convert $\beta$-distributed pairwise comparisons to crisp values. The median value of the $\beta$-distribution is used for the crisp values of each $\tilde{a}_{ij}$. The median of $\beta$ distribution, $m(\tilde{a}_{ij}, \hat{\beta}_{ij})$, is obtained by employing the closed-form:

$$m(\tilde{a}_{ij}, \hat{\beta}_{ij}) \approx \frac{\tilde{a}_{ij} - 1/3}{\tilde{a}_{ij} + \hat{\beta}_{ij} - 2/3}.$$  

(9)
The median is lower-bounded by the mode and upper-bounded by the mean as shown in Equation (9):

\[
\frac{\hat{\mu}_{ij} - 1}{\hat{\mu}_{ij} + \hat{\beta}_{ij} - 2} \leq m(\hat{\mu}_{ij}, \hat{\beta}_{ij}) \leq \frac{\hat{\mu}_{ij}}{\hat{\beta}_{ij}}.
\]  

(10)

where in the case of \( \hat{\beta}_{ij} \leq \hat{\mu}_{ij} \) the order of the inequalities in Equation (10) is reversed. Then the numeric value of the median for comparison \( a_{ij} \) considering \( LL_{ij} \) and \( UL_{ij} \) parameters is obtained by using the formula defined in the following equation:

\[
a_{ij} = LL_{ij} + m(\hat{\mu}_{ij}, \hat{\beta}_{ij}) \times (UL_{ij} - LL_{ij}).
\]  

(11)

### 3.2 Fuzzy VIKOR technique

Fuzzy VIKOR technique is applying fuzzy logic to VIKOR technique. The method offers rational and systematic process for the best and compromise solution by handling linguistic expressions. In this process, implemented steps are as follows (Akyuz, 2012).

Stage 1: first, \( n \) decision makers, \( m \) alternatives and \( k \) criteria are determined to solve the problems.

Stage 2: linguistic variables and fuzzy numbers are defined concerning these variables. Linguistic variables are used to determine the weight of criteria and evaluate the alternatives. However, SAHP is used when determining criteria weight in the study.

Stage 3: evaluation of decision makers are combined and integrated fuzzy weight of each criterion is calculated with the aid of the following equation:

\[
\tilde{w}_j = \frac{1}{n} \left[ \sum_{e=1}^{n} \tilde{w}_j^e \right], j = 1, 2, \ldots, k.
\]  

(12)

Importance weight of \( i \)th alternative according to \( j \)th criteria is calculated with the aid of the following equation:

\[
\tilde{x}_{ij} = \frac{1}{n} \left[ \sum_{e=1}^{n} \tilde{x}_{ij}^e \right], i = 1, 2, \ldots, m.
\]  

(13)

Stage 4: fuzzy decision matrix is created as follows:

\[
\tilde{D} = \begin{bmatrix}
    \tilde{x}_{11} & \cdots & \tilde{x}_{1k} \\
    \vdots & \ddots & \vdots \\
    \tilde{x}_{m1} & \cdots & \tilde{x}_{mk}
\end{bmatrix}, i = 1, 2, \ldots, m; j = 1, 2, \ldots, k,
\]  

(14)

\( \tilde{x}_{ij}^e, \tilde{x}_{ij}^{me} \) and \( \tilde{x}_{ij}^n \) values in the fuzzy decision matrix represent small, medium and large value in triangular fuzzy numbers for an \( i \)th alternative in terms of \( j \)th criteria, respectively.

Decision makers have benefited from the scale in Table II in the creation of the fuzzy decision matrix (Vatansever and Ulukoy, 2013):

\[
\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_k], j = 1, 2, \ldots, k,
\]  

(15)

where \( \tilde{x}_{ij} \) is the degree of \( A_i \) alternative according to \( C_j \) criteria and \( \tilde{w}_j \) is importance weight of \( j \)th criteria.
Stage 5: fuzzy best ($\tilde{f}_j^*$) and worst ($\tilde{f}_j^-$) values are determined as follows:

\[
(\tilde{f}_j^*) = \max_i \tilde{x}_{ij}, \quad (\tilde{f}_j^-) = \min_i \tilde{x}_{ij}.
\]  

Stage 6: $\tilde{S}_i$ and $\tilde{R}_i$ values are calculated as follows:

\[
\tilde{S}_i = \sum_{j=1}^{k} \tilde{w}_j \left( \tilde{f}_j^* - \tilde{x}_{ij} / \left(\tilde{f}_j^* - \tilde{f}_j^-\right) \right),
\]  

\[
\tilde{R}_i = \max_j \left[ \tilde{w}_j \left( \tilde{f}_j^* - \tilde{x}_{ij} / \left(\tilde{f}_j^* - \tilde{f}_j^-\right) \right) \right],
\]  

where $\tilde{S}_i$ is a total of criteria value distance to fuzzy best value. $\tilde{R}_i$ is the maximum distance of alternative $A_i$ to the fuzzy worst value according to $j$th criteria. In other words, $\tilde{S}_i$ and $\tilde{R}_i$ values represent moderate and the worst scores of $A_i$ alternative.

Stage 7: $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$, $\tilde{R}^-$ and $\tilde{Q}_i$ values are calculated as follows:

\[
\tilde{S}^* = \min_i \tilde{S}_i, \quad \tilde{S}^- = \max_i \tilde{S}_i,
\]  

\[
\tilde{R}^* = \min_i \tilde{R}_i, \quad \tilde{R}^- = \max_i \tilde{R}_i,
\]  

\[
\tilde{Q}_i = v \left( \tilde{S}_i - \tilde{S}^- \right) / \left( \tilde{S}^- - \tilde{S}^* \right) + \left( 1 - v \right) \left( \tilde{R}_i - \tilde{R}^* \right) / \left( \tilde{R}^- - \tilde{R}^* \right),
\]  

where $\tilde{S}^*$ represents the maximum benefit of the group and $\tilde{R}^*$ represents minimum regret of opposite view. $\tilde{Q}_i$ index is calculated together with the assessment group of benefits and minimum regret. $v$ value represents the weight of strategy which ensures maximum group benefit. Compromise can be provided with "majority vote" ($v > 0.5$), "compromise" ($v = 0.5$) or "rejection" ($v < 0.5$).

Stage 8: $\tilde{Q}_i$ index is obtained by defuzzification using Equation (22). There are different defuzzification methods in the literature. BNP (Best Non-Fuzzy Performance Value) proposed by Hsieh et al. (2004) is used for defuzzification in this study. In the equation, $u_i$ is the upper value of a triangular fuzzy number, $m_i$ is the median value of a triangular fuzzy number and $l_i$ is the lower value of a triangular fuzzy number:

\[
\text{BNP} = [(u_i - l_i) + (m_i - l_i)]/3 + l_i, \forall i.
\]  

\[
\tilde{Q}_i \quad \text{indexes are arranged in increasing order. The alternative which has the lowest } \tilde{Q}_i \text{ value is the best alternative.}
\]

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Triangular fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very weak/low</td>
<td>(0, 0, 1)</td>
</tr>
<tr>
<td>Weak</td>
<td>(0, 1, 3)</td>
</tr>
<tr>
<td>Moderate weak</td>
<td>(1, 3, 5)</td>
</tr>
<tr>
<td>Moderate</td>
<td>(3, 5, 7)</td>
</tr>
<tr>
<td>Moderate well</td>
<td>(5, 7, 9)</td>
</tr>
<tr>
<td>Well</td>
<td>(7, 9, 10)</td>
</tr>
<tr>
<td>Very well</td>
<td>(9, 10, 10)</td>
</tr>
</tbody>
</table>

Table II. Linguistic variables used for alternative assessment
Stage 9: in this stage, the compromise solution is determined. If following two conditions are satisfied, obtained solution by using $Q_i$ index is compromise solution ($a'$):

1. Condition 1: acceptable advantage: with condition 1, it is established that there is a clear difference between the best and closest options:

$$Q(a'^*) - Q(a') \geq DQ.$$  

(23)

In the equation, $a$ is an alternative in the second when ordered $Q_i$ values:

$$DQ = \frac{1}{m-1} (\text{eğer } m \leq 4 \text{ ise } DQ = 0.25).$$  

(24)

2. Condition 2: Acceptable stability: An alternative must also be the best alternative when ordered based on S and/or R values. If $Q(a^{(m)}) - Q(a') < DQ$, if condition 1 is not satisfied, $a^{(m)}$ and $a'$ are the same compromise solutions. Because of similar compromise solution ($a'$, $a'^*$, ..., $a^{(m)}$), $a'$ does not have a comparative advantage. If condition 2 is not satisfied, decision making is not stable although has a comparative advantage. Therefore, compromise solution of $a'$ and $a'^*$ is similar.

Stage 10: finally, the best alternative is selected. Alternative which has minimum $Q$ value is the best solution.

4. Application and results

This section demonstrates how to implement SAHP–fuzzy VIKOR methods. The name and any other information pertaining to the supermarket are not used in the study due to the privacy policy of the supermarket owner.

In the first stage of the application, a decision team was created which consisted of the warehouse manager, purchasing manager and warehouse officer. With the help of the team, criteria and alternatives to be used in the solution of the warehouse location selection problem were determined. A total of 17 criteria were determined by the decision-making team: transportation diversity, proximity to suppliers, development rate, storage capacity, proximity to customers, proximity to competitors, labor costs, transportation costs, earthquake resistance, cost of land, climatic conditions, holding costs, communication systems, customer service levels, delivery time, proximity to producers and electricity, water and telephone infrastructure. Four identified choices of location were Trabzon, Gaziantep, Kars and Diyarbakir.

The decision team has evaluated the criteria stochastically so that it is more suited to real life. The stochastic assessment provides simplicity in cases where we cannot express deterministically. Linguistic variables are utilized when evaluating alternatives in Table II.

4.1 SAHP–fuzzy VIKOR calculation

In order to overcome the problems that may arise from ambiguity of the examined warehouse location problem, the value of the SAHP was used when assessing the criteria, and linguistic expressions were used when assessing the alternatives in the data collection phase.

In this study, SAHP calculations for weighting were performed as recommended by Jalao et al. (2014). All calculations and operations were performed as described Section 2. In Table I, previously mentioned Saaty’s pairwise comparison scale was used when assessing the criteria in SAHP. Each decision by the decision team has been made during the evaluation process by using this scale. Table III shows stochastic pairwise comparison matrix which is obtained through the assessment results produced by warehouse officer.

The values in the matrix were converted to $\beta$ distribution by applying equalities (5)–(7) to stochastic pairwise comparison matrix. Table IV shows stochastic pairwise comparison matrix of the warehouse officer following the conversion to the $\beta$ distribution. Shown in the
form of the $\beta$ distribution, values in the matrix were converted to net value by using equalities (9) and (10). Table V shows the form of $\beta$ distribution pairwise comparison matrix of warehouse officer converted to net value.

Pairwise comparison matrix of decision team is combined with a geometric mean method. Table VI shows combined pairwise comparison matrix. Combined pairwise comparison matrix in Table VII normalized pairwise comparison matrix.

Weight value of each criteria was calculated by applying calculation in AHP method to normalized pairwise comparison matrix. Calculated weight values of the criteria were as follows.

Cost of land 0.15, delivery time 0.14, storage capacity 0.12, holding costs 0.12, transportation costs 0.11, climatic conditions 0.07, proximity to producers 0.05, proximity to

<table>
<thead>
<tr>
<th>Labor cost</th>
<th>Transport costs</th>
<th>Holding costs</th>
<th>Climatic conditions</th>
<th>Cost of land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor cost</td>
<td>1</td>
<td>1/9</td>
<td>1/9</td>
<td>1/9</td>
</tr>
<tr>
<td>Transport costs</td>
<td>9</td>
<td>1</td>
<td>B(1.96, 0.37, 1/8, 1/6)</td>
<td>2</td>
</tr>
<tr>
<td>Holding costs</td>
<td>9</td>
<td>B(2.49, 2.49, 6, 8)</td>
<td>1</td>
<td>B(1, 1, 1/2, 1)</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>9</td>
<td>1/2</td>
<td>B(1, 1, 1/2)</td>
<td>1</td>
</tr>
<tr>
<td>Cost of land</td>
<td>9</td>
<td>B(1, 1, 1/9, 1/8)</td>
<td>B(1, 1, 1/2)</td>
<td>8</td>
</tr>
</tbody>
</table>

Table III.
Stochastic pairwise comparison matrix of warehouse officer

<table>
<thead>
<tr>
<th>Labor cost</th>
<th>Transport costs</th>
<th>Holding costs</th>
<th>Climatic conditions</th>
<th>Cost of land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor cost</td>
<td>1</td>
<td>1/9</td>
<td>1/9</td>
<td>1/9</td>
</tr>
<tr>
<td>Transport costs</td>
<td>9</td>
<td>1</td>
<td>B(1.96, 0.37, 1/8, 1/6)</td>
<td>2</td>
</tr>
<tr>
<td>Holding costs</td>
<td>9</td>
<td>B(2.49, 2.49, 6, 8)</td>
<td>1</td>
<td>B(1, 1, 1/2, 1)</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>9</td>
<td>1/2</td>
<td>B(1, 1, 1/2)</td>
<td>1</td>
</tr>
<tr>
<td>Cost of land</td>
<td>9</td>
<td>B(1, 1, 1/9, 1/8)</td>
<td>B(1, 1, 1/2)</td>
<td>8</td>
</tr>
</tbody>
</table>

Table IV.
The form of pairwise comparison matrix converted to $\beta$ distribution of warehouse officer

<table>
<thead>
<tr>
<th>Labor cost</th>
<th>Transport costs</th>
<th>Holding costs</th>
<th>Climatic conditions</th>
<th>Cost of land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor cost</td>
<td>1</td>
<td>1/9</td>
<td>1/9</td>
<td>1/9</td>
</tr>
<tr>
<td>Transport costs</td>
<td>9</td>
<td>1</td>
<td>0.16</td>
<td>2</td>
</tr>
<tr>
<td>Holding costs</td>
<td>9</td>
<td>7</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>9</td>
<td>1/2</td>
<td>1.50</td>
<td>1</td>
</tr>
<tr>
<td>Cost of land</td>
<td>9</td>
<td>0.12</td>
<td>1.50</td>
<td>8</td>
</tr>
</tbody>
</table>

Table V.
The form of $\beta$ distribution pairwise comparison converted to net value of warehouse officer

<table>
<thead>
<tr>
<th>Labor cost</th>
<th>Transport costs</th>
<th>Holding costs</th>
<th>Climatic conditions</th>
<th>Cost of land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor cost</td>
<td>1.00</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Transport costs</td>
<td>8.14</td>
<td>0.12</td>
<td>0.15</td>
<td>0.57</td>
</tr>
<tr>
<td>Holding costs</td>
<td>8.31</td>
<td>8.83</td>
<td>6.80</td>
<td>0.91</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>8.14</td>
<td>1.74</td>
<td>1.14</td>
<td>1.00</td>
</tr>
<tr>
<td>Cost of land</td>
<td>8.83</td>
<td>0.50</td>
<td>2.73</td>
<td>8.16</td>
</tr>
</tbody>
</table>

Table VI.
Combined pairwise comparison matrix

Warehouse location selection problem
suppliers 0.04, customer service levels 0.04, labor costs 0.04, transportation diversity 0.03, proximity to customers 0.03, proximity to competitors 0.02, communication systems 0.01, development rate 0.01, earthquake resistance 0.01 and electricity, water and telephone infrastructure 0.01. This value is used in weight found by calculating the fuzzy MOORA.

The decision team evaluated alternatives and formed fuzzy decision matrix by using linguistic variables, shown in Table II. Fuzzy decision matrix of the decision team was combined, and combined fuzzy decision matrix is shown in Table VIII.

VIKOR calculations in the application were used as proposed by Akyuz (2012). In the warehouse location selection problem-solving approach in the study, as mentioned earlier, the solution was made by considering 17 criteria and 4 alternatives.

Fuzzy best \( \tilde{f}^* \) and fuzzy worst \( \tilde{f}^\text{C0} \) values are calculated by the aid of equality (16), by examining combined fuzzy decision matrix in Table IX.

Subsequently, value of the alternative distance to the best value \( \tilde{s}_i \) and distance to the worst value \( \tilde{r}_i \) were calculated by the aid of equalities (17) and (18). \( \tilde{s}_i \) and \( \tilde{r}_i \) are shown in

<table>
<thead>
<tr>
<th>Criteria</th>
<th>( \tilde{f}^* )</th>
<th>( \tilde{f}^\text{C0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor cost</td>
<td>( (3, 7.7, 10) )</td>
<td>( (3, 5, 7) )</td>
</tr>
<tr>
<td>Transport costs</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Holding costs</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 7) )</td>
</tr>
<tr>
<td>Proximity to customers</td>
<td>( (7, 9.3, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Proximity to suppliers</td>
<td>( (7, 9.3, 10) )</td>
<td>( (0, 0, 1) )</td>
</tr>
<tr>
<td>Proximity to opponents</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 7) )</td>
</tr>
<tr>
<td>Proximity to producers</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 7) )</td>
</tr>
<tr>
<td>Delivery time</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
<tr>
<td>Communication systems</td>
<td>( (7, 9.6, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
<tr>
<td>Development rate</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Capacity of storage</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Customer service levels</td>
<td>( (7, 9.3, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Transportation diversity</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 0, 3) )</td>
</tr>
<tr>
<td>Earthquake resistance</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
<tr>
<td>Electricity, water and telephone infrastructure</td>
<td>( (7, 9.6, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
<tr>
<td>Climatic conditions</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
<tr>
<td>Cost of land</td>
<td>( (9, 10, 10) )</td>
<td>( (0, 1, 3) )</td>
</tr>
</tbody>
</table>

Table IX. Fuzzy best \( \tilde{f}^* \) and fuzzy worst \( \tilde{f}^\text{C0} \) values
Table X. $\tilde{S}_i^+, \tilde{S}_i^-, \tilde{R}_i^+$ and $\tilde{R}_i^-$ values calculated by the aid of equalities (19) and (20) are shown in Table XI.

In stage 7, $\tilde{Q}_i$ values are calculated by placing $\tilde{S}_i^+, \tilde{S}_i^- \tilde{R}_i^+$ and $\tilde{R}_i^-$ values in equality (21). $v$ value is taken as 0.5 to reflect the consensus. Equality (22) was used to de-fuzzify the obtained fuzzy numbers and $Q_i$, $S_i$ and $R_i$ index values are calculated. Alternatives are ranked according to these index values. $Q_i$, $S_i$ and $R_i$ index values and sequence of alternatives are shown in Table XII.

Stage of identifying a conciliatory solution: it is investigated if both conditions are satisfied in the ordering performed according to $q$ index.

Condition 1: acceptable advantage: according to equalities (23) and (24), because of $Q(a'') - Q(a') = 0.449 - 0 \geq 0.25$, $Q(a'') - Q(a') = 0.614 - 0 \geq 0.25$ and $Q(a''') - Q(a') = 1 - 0 \geq 0.25$ condition 1 is satisfied.

Condition 2: acceptable stability: the best alternative $a'$ which was used rank according to $Q_i$ index, in addition, if best alternative $a'$ which were used rank according to $S_i$ and/or $R_i$ index, this conciliatory solution stable decide process. Looking at Table XIII, Gaziantep has been the best alternative all rank according to $Q_i$, $S_i$ and $R_i$ index. Hence, condition 2 is satisfied.

The equation $v$ value is taken as 0.5 to reflect the consensus when $\tilde{Q}_i$ value is calculated.

The result of VIKOR method showed that Gaziantep is the best alternative. Gaziantep is used with other alternatives as follows: Trabzon, Diyarbakır and Kars.

4.1.1 Sensitivity analysis. In the study, a sensitivity analysis was conducted to check the results. In the analysis, results are recalculated for different $V$ values. Calculated results for $V = 0$ are shown in Table XIV and calculated results for $V = 1$ are shown in Table XV.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$\tilde{S}_i$</th>
<th>$\tilde{R}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>(0.705, 0.389, 0.00)</td>
<td>(0.15, 0.095, 0.00)</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>(0.054, 0.007, 0.00)</td>
<td>(0.04, 0.005, 0.00)</td>
</tr>
<tr>
<td>Kars</td>
<td>(1, 0.98, 0.912)</td>
<td>(0.15, 0.15, 0.15)</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>(0.545, 0.382, 0.203)</td>
<td>(0.12, 0.107, 0.12)</td>
</tr>
<tr>
<td></td>
<td>$\tilde{S}_i$ and $\tilde{R}_i$ values</td>
<td></td>
</tr>
</tbody>
</table>

| $\tilde{S}_i^+$ | (0.054, 0.007, 0.00) |
| $\tilde{S}_i^-$ | (1, 0.98, 0.912) |
| $\tilde{R}_i^+$ | (0.04, 0.005, 0.00) |
| $\tilde{R}_i^-$ | (0.15, 0.15, 0.15) |
|              | $\tilde{S}_i^+, \tilde{S}_i^-, \tilde{R}_i^+$ and $\tilde{R}_i^-$ |

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$\tilde{Q}_i$</th>
<th>$Q_i$</th>
<th>Ranking</th>
<th>$S_i$</th>
<th>Ranking</th>
<th>$R_i$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>(0.84, 0.506, 0)</td>
<td>0.449</td>
<td>2</td>
<td>0.365</td>
<td>2</td>
<td>0.0817</td>
<td>2</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>(0, 0, 0)</td>
<td>0</td>
<td>1</td>
<td>0.020</td>
<td>1</td>
<td>0.015</td>
<td>1</td>
</tr>
<tr>
<td>Kars</td>
<td>(1, 1, 1)</td>
<td>1</td>
<td>4</td>
<td>0.964</td>
<td>4</td>
<td>0.15</td>
<td>4</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>(0.786, 0.545, 0.511)</td>
<td>0.614</td>
<td>3</td>
<td>0.377</td>
<td>3</td>
<td>0.116</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$Q_i$, $S_i$ and $R_i$ index values and sequence of alternatives</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $Q_i$, $S_i$, $R_i$ | Gaziantep > Trabzon > Diyarbakır > Kars |
| Acceptable stability on decision making |

Table XIII.
4.2 Comparative analysis over other MCDM techniques

TOPSIS method which was proposed by Hwang and Yoon in 1981 is one of the most widely used multi-criteria decision analysis methods. The TOPSIS method measures the proximity and distance to the ideal as the VIKOR method. Therefore, in order to test the accuracy of the results, results obtained by the VIKOR method and the results obtained from the TOPSIS method were compared. TOPSIS calculations in the application were used as proposed by Walczak and Rutkowska (2017). Table XVI shows calculation of the relative proximity based on the similarity to the best alternative ($S_i$) by using TOPSIS method.

As shown in Table XVII, since the results obtained from the two methods are the same, it can be said that the proposed method is an effective method. At the same time, it can be applied to real cases.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$\tilde{Q}_i$</th>
<th>$Q_i$</th>
<th>Ranking</th>
<th>$S_i$</th>
<th>Ranking</th>
<th>$R_i$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>(1, 0.621, 0)</td>
<td>0.540</td>
<td>2</td>
<td>0.365</td>
<td>2</td>
<td>0.0817</td>
<td>2</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>(0, 0, 0)</td>
<td>0</td>
<td>1</td>
<td>0.020</td>
<td>1</td>
<td>0.015</td>
<td>1</td>
</tr>
<tr>
<td>Kars</td>
<td>(1, 1, 1)</td>
<td>1</td>
<td>4</td>
<td>0.964</td>
<td>4</td>
<td>0.15</td>
<td>4</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>(0.727, 0.703, 0.8)</td>
<td>0.743</td>
<td>3</td>
<td>0.377</td>
<td>3</td>
<td>0.116</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$\tilde{Q}_i$</th>
<th>$Q_i$</th>
<th>Ranking</th>
<th>$S_i$</th>
<th>Ranking</th>
<th>$R_i$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>(0.688, 0.393, 0)</td>
<td>0.360</td>
<td>2</td>
<td>0.365</td>
<td>2</td>
<td>0.0817</td>
<td>2</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>(0, 0, 0)</td>
<td>0</td>
<td>1</td>
<td>0.020</td>
<td>1</td>
<td>0.015</td>
<td>1</td>
</tr>
<tr>
<td>Kars</td>
<td>(1, 1, 1)</td>
<td>1</td>
<td>4</td>
<td>0.964</td>
<td>4</td>
<td>0.15</td>
<td>4</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>(0.519, 0.385, 0.22)</td>
<td>0.375</td>
<td>3</td>
<td>0.377</td>
<td>3</td>
<td>0.116</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>$S_i$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>0.6283</td>
<td>2</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>0.8005</td>
<td>1</td>
</tr>
<tr>
<td>Kars</td>
<td>0.2383</td>
<td>4</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>0.5121</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Stochastic AHP–fuzzy TOPSIS ($S_i$)</th>
<th>Stochastic AHP–fuzzy TOPSIS ranking</th>
<th>Stochastic AHP–fuzzy VIKOR ($Q_i$)</th>
<th>Stochastic AHP–fuzzy VIKOR ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trabzon</td>
<td>0.6283</td>
<td>2</td>
<td>0.449</td>
<td>2</td>
</tr>
<tr>
<td>Gaziantep</td>
<td>0.8005</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Kars</td>
<td>0.2383</td>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Diyarbakır</td>
<td>0.5121</td>
<td>3</td>
<td>0.614</td>
<td>3</td>
</tr>
</tbody>
</table>

Table XVI.
$\tilde{Q}_i$, $Q_i$ values and sequence of alternatives ($v = 0$)

Table XV.
$\tilde{Q}_i$, $Q_i$ and $R_i$ index values and sequence of alternatives ($v = 1$)

Table XVII.
Comparative analysis results
5. Conclusion
Warehouse location selection problem involves many different and vague criteria. Various methods have been developed to find solutions to such problems. In this study, a model was developed that has not been previously applied to storage location problems.

Proposed application solved effectively supermarket-owned warehouse location problems. The criteria are determined according to the consensus decision team which consists of the warehouse manager, purchasing manager and warehouse officer. While SAHP is used for weighting of determining criteria, fuzzy VIKOR technique from fuzzy multi-criteria decision-making techniques is used for ranking alternatives.

The model has been established for alternative warehouse places as an approach to solving real-life problems in the study. The results of the model showed that the warehouse should be established to Gaziantep because Gaziantep is the best alternative.

The proposed approach has been applied to the problem of deciding on specific warehouse location selection problems in this study, but it can be applied to different problems in different areas of decision making in future studies. Integration of SAHP and fuzzy VIKOR can be utilized for any other decision-making problems.

References


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