An overview of intuitionistic linguistic fuzzy information aggregations and applications

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Received 4 June 2018
Accepted 4 June 2018

Abstract

Purpose – Intuitionistic linguistic fuzzy information (ILFI), characterized by linguistic terms and intuitionistic fuzzy sets (IFSs), can easily express the fuzzy information in the process of muticriteria decision making (MCDM) and muticriteria group decision making (MCGDM) problems. The purpose of this paper is to provide an overview of aggregation operators (AOs) and applications of ILFI.
Design/methodology/approach – First, some meaningful AOs for ILFI are summarized, and some extended MCDM approaches for intuitionistic uncertain linguistic variables (IULVs), such as extended TOPSIS, extended TODIM, extended VIKOR, are discussed. Then, the authors summarize and analyze the applications about the AOs of IULVs.
Findings – IULVs, characterized by linguistic terms and IFSs, can more detailed and comprehensively express the criteria values in the process of MCDM and MCGDM. Therefore, lots of researchers pay more and more attention to the MCDM or MCGDM methods with IULVs.
Originality/value – The authors summarize and analyze the applications about the AOs of IULVs Finally, the authors point out some possible directions for future research.

Keywords Applications, Intuitionistic linguistic fuzzy information, Intuitionistic uncertain linguistic variables, Aggregation operators

Paper type Research paper

1. Introduction

Due to the increasing complexity of decision-making problems, it is generally difficult to express criteria values of alternatives by exact numbers. Zadeh (1965) originally proposed the fuzzy set (FS) theory, which is an effective tool in dealing with fuzzy information. However, it is not suitable to handle the information with non-membership. As the generalization of FS, intuitionistic fuzzy set (IFS) introduced by Atanassov (1986, 1989, 1999) has a membership degree (MD), a non-membership degree (NMD) and a hesitancy degree (HD), which can further overcome the drawbacks of FS. Now, a large number of methods based on IFS have been utilized to a number of areas.

Up to date, many contributions have concentrated on the decision-making techniques based on IFSs, which are from three domains: the theory of foundations, for instance, operational rules (Chen and Han, 2018; Dymova and Sevastjanov, 2010, 2012, 2015, 2016), comparative approaches (Deepa and Kumar, 2018), distance and similarity measures (Atanassov, 1989), likelihood (Jiang and Hu, 2018), ranking function (Hao and Chen, 2018), consensus degree (Cheng, 2017), proximity measure (Ngan *et al.*, 2018) and so on; the extended muticriteria decision-making (MCDM) approaches for IFS, such as TOPSIS

Marine Economics and Management
Vol. 1 No. 1, 2018
pp. 55-78
Emerald Publishing Limited
2516-158X
DOI 10.1108/MAEM-06-2018-003

(Shen *et al.*, 2018), ELECTRE (Qu *et al.*, 2018), VIKOR (Sennaroglu and Celebi, 2018), TODIM (Atanoassov and Vassilev, 2018), entropy (Ansari and Mishra, 2018) and other methods, such as Choquet integral (CI) (Dymova and Sevastjanov, 2012), multi-objective linear programming (Singh and Yadav, 2018) or multi-objective nonlinear programming (NLP) (Jafarian *et al.*, 2018), Decision-Making Trial and Evaluation Laboratory (Bahar *et al.*, 2018), statistical convergent sequence spaces (Debnath *et al.*, 2018) and so on; and the MCDM techniques based on aggregation operators (AOs) of IFS, they have more superiority than the traditional MCDM techniques because of can acquire the comprehensive values of alternatives by aggregating all attribute values, and then rank the alternatives.

However, with the increasing of uncertainty and complexity, the IFS cannot depict the uncertain information comprehensively and accurately in the circumstance in which the MD and NMD with the form of IFS cannot be expressed as real values. For the sake of adequately expressing the fuzzy and uncertain information in real process of decision making, Zadeh (1975) proposed first the concept of linguistic variable (LV) and Herrera and Herrera-Viedma (2000) defined a discrete linguistic term set (LTS), that is, variables whose evaluation values are not real and exact numbers but linguistic terms, such as "very low," "low," "fair," "high," "very high," etc. Obviously, the decision maker can more easily to express his/her opinions and preferences by selecting the matching linguistic terms from the LTS. So based on the IFS and the LTS, a novel solution is that MD and NMD are denoted by LTS, which is called intuitionistic linguistic fuzzy set (ILFS), first introduced by Wang and Li (2010). As a generalization of IFS, LT and LTS, the ILFS can more adequately dispose the fuzzy and uncertain information than IFS, LT and LTS. Since appearance, IFLS has attracted more and more attention.

Based on the IFLS, different forms of IFLS are extended and some basic operational rules of IFLS are defined, such as intuitionistic uncertain linguistic set (IULS) (Liu and Jin, 2012; Liu, 2013a), interval-value intuitionistic uncertain linguistic set (IVIULS) (Wang, 2013; Meng and Chen, 2016), intuitionistic uncertain 2-tuple linguistic variable (IU2TLV) (Herrera and Martínez, 2000a, b, 2012). AOs of IFLS are a new branch of IFLS, which is a meaningful and significance research issue and has attracted more and more attention. For example, some basic intuitionistic linguistic (IL) fuzzy AOs, such as intuitionistic uncertain linguistic weighted geometric mean (IULWGM) operator (Liu and Jin, 2012), ordered intuitionistic uncertain linguistic weighted geometric mean (OIULWGM) operator (Liu and Jin, 2012), interval-value IULWGM (GIULWGM) operator (Liu, 2013b) and interval-value OIULWGM (GOIULWGM) operator (Liu, 2013b); the extended MCDM approaches for IUFS, such as the extended TOPSIS (ETOPSIS) approaches (Wei, 2014; Du and Zuo, 2011; Joshi *et al.*, 2018; Wei, 2011), the extended TODIM (ETODIM) approaches (Liu and Teng, 2015; Yu *et al.*, 2016; Wang and Liu, 2017), the extended VIKOR (EVIKOR) approach (Li *et al.*, 2017; Liu and Qin, 2017); some IL fuzzy AOs considering the interrelationships between criteria, such as IUL Bonferroni OWM (IULBOWM) operator (Liu, Chen and Chu, 2014), weighted IUL Bonferroni OWM (WIULBOWM) operator (Liu, Chen and Chu, 2014), IUL arithmetic Heronian mean (IULAHM) operator (Liu, Liu and Zhang, 2014), IUL geometric Heronian mean (IULGHM) operator (Liu, Liu and Zhang, 2014), weighted IUL arithmetic Heronian mean (WIULAHM) operator (Liu, Liu and Zhang, 2014), IUL geometric Heronian mean (WIULGHM) operator (Liu, Liu and Zhang, 2014), IUL Maclaurin symmetric mean (IULMSM) operator (Ju *et al.*, 2016), weighted ILMSM (WIULMSM) operator (Ju *et al.*, 2016); generalized intuitionistic linguistic fuzzy aggregation operators, such as generalized IL dependent ordered weighted mean (DOWM) (GILDOWM) operator (Liu, 2013a; Liu and Wang, 2014) and a generalized IL dependent hybrid weighted mean (DHWM) (GILDHWM) operator (Liu, 2013a; Liu and Wang, 2014); IL fuzzy AOs based on CI (Meng *et al.*, 2014); induced IL fuzzy AOs (Liu and Wang, 2014; Meng *et al.*, 2014; Xian and Xue, 2015; Yager and

Filev, 1999; Xian *et al.*, 2018; Xu, 2006; Xu and Xia, 2011; Meriglo *et al.*, 2012), such as, IFL induced ordered weighted mean (IFLIOWM) operator (Liu and Wang, 2014; Meng *et al.*, 2014), IFL induced ordered weighted geometric mean (IFLIOWGM) operator (Liu and Wang, 2014; Meng *et al.*, 2014).

To understand and learn these AOs and decision-making methods better and more conveniently, it is necessary to make an overview of interval-valued intuitionistic fuzzy information aggregation techniques and their applications. The rest of this paper is organized as follows: in Section 2, we review the basic concepts and operational rules of IFS, LTS, intuitionistic linguistic set (ILS), IULS and IVIULS. In Section 3, we review, summary analysis and discuss some kinds of AOs about ILS, IULS and IVIULS. At the same time, we divide the AOs into categories. In Section 4, we mainly review the applications in dealing with a variety of real and practice MCDM or muticriteria group decision-making (MCGDM) problems. In Section 5, we point out some possible development directions for future research. In Section 6, we discuss the conclusions.

2. Basic concepts and operations

2.1 The intuitionistic fuzzy set

Definition 1. (Xu, 2007) Let $E = \{\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\}$ be a nonempty set, an IFS R in E is given by $R = \{\langle \varepsilon, u_R(\varepsilon), v_R(\varepsilon)\rangle | \varepsilon \in E\}$, where u_R: $E \to [0, 1]$ and v_R: $E \to [0, 1]$, with the condition $0 \leqslant u_R(\varepsilon) + v_R(\varepsilon) \leqslant 1$, $\forall \varepsilon \in E$. The numbers $u_R(\varepsilon)$ and $v_R(\varepsilon)$ denote, respectively, the MD and NMD of the element ε to E.

In addition, $\pi(\varepsilon) = 1 - u_R(\varepsilon) - v_R(\varepsilon)$, $\forall \varepsilon \in E$, denotes the indeterminacy degree (ID) of the element ε to E. It is evident that $0 \leqslant \pi(\varepsilon) \leqslant 1$, $\forall \varepsilon \in E$.

For the given element ε, $\langle u_R(\varepsilon), v_R(\varepsilon)\rangle$ is called intuitionistic fuzzy number (IFN), and for convenience, we can utilize $\tilde{r} = (u_r, v_r)$ to denote an IFN, which meets the conditions, $u_R(\varepsilon)$, $v_R(\varepsilon) \in [0, 1]$ and $0 \leqslant u_R(\varepsilon) + v_R(\varepsilon) \leqslant 1$.

Let $\tilde{r} = (u_r, v_r)$ and $\tilde{t} = (u_t, v_t)$ be two IFNs, $\delta \geqslant 0$, then the operations of IFNs are defined as follows (Xu, 2007):

$$\tilde{r} \oplus \tilde{t} = (u_r + u_t - u_r u_t, v_r v_t), \tag{1}$$

$$\tilde{r} \otimes \tilde{t} = (u_r u_t, v_r + v_t - v_r v_t), \tag{2}$$

$$\delta \tilde{r} = \left(1 - (1 - u_r)^{\delta}, (v_r)^{\delta}\right), \tag{3}$$

$$\tilde{r}^{\delta} = \left((u_r)^{\delta}, 1 - (1 - v_r)^{\delta}\right). \tag{4}$$

2.2 The linguistic term set and intuitionistic linguistic set

Suppose $S = \{s_0, s_1, \ldots, s_m\}$ is a complete and finite ordered discrete LTS, where m is the even value. As a general rule, m is equal to 2, 4, 6, 8, etc., in real decision making. For example, when $m = 8$, the LTS S and their corresponding semantics can be given as follows:

$$S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\} = \{s_0(\text{extremly low}), s_1(\text{very low}), s_2(\text{low}), s_3(\text{slightly low}), s_4(\text{medium}), s_5(\text{slightly high}), s_6(\text{high}), s_7(\text{very high}), s_8(\text{extremly high})\}.$$

In general, for any LTS $S = \{s_0, s_1, \ldots, s_m\}$, it is compulsory that s_x and s_y must satisfy the following additional characteristics:

(1) the set is ordered: $s_x < s_y$, if $x < y$;

(2) maximum operator: max $(s_x, s_y) = s_x$, if $s_x \geqslant s_y$;

(3) minimum operator: min $(s_x, s_y) = s_x$, if $s_x \leqslant s_y$; and

(4) a negation operator: $neg(s_x) = s_y$, such that $y = t - x$.

For relieving the information loss in the decision making, Xu (Xu, 2006; Xu and Xia, 2011) extended discrete linguistic set $S = \{s_0, s_1, \ldots, s_m\}$ to continuous linguistic set $\widehat{S} = \{s_l | l \in [0, t]\}$. For any LV $s_x, s_y \in \widehat{S}$, the operations of LV can be defined as follows:

$$\delta s_x = s_{\delta \times x}, \quad \delta \geqslant 0, \tag{5}$$

$$s_x \oplus s_y = s_{x+y}, \tag{6}$$

$$s_x / s_y = s_{x/y}, \tag{7}$$

$$(s_x)^\delta = s_{x^\delta}, \quad \delta \geqslant 0, \tag{8}$$

$$\delta\left(s_x \oplus s_y\right) = \delta s_x \oplus \delta s_y, \quad \delta \geqslant 0, \tag{9}$$

$$(\delta_1 + \delta_2) s_x = \delta_1 s_x + \delta_2 s_x, \quad \delta_1, \delta_2 \geqslant 0. \tag{10}$$

Definition 2. (Xu and Yager, 2006) An ILS U in E is defined as $R = \{\langle \varepsilon[s_{\varphi(\varepsilon)}, (u_R(\varepsilon), v_R(\varepsilon))]\rangle | \varepsilon \in E\}$, where $s_{\varphi(\varepsilon)} \in \widehat{S}$, u_R: $E \to [0, 1]$ and v_R: $E \to [0, 1]$, with the condition $0 \leqslant u_R(\varepsilon) + v_R(\varepsilon) \leqslant 1$, $\forall \varepsilon \in E$. The numbers $u_R(\varepsilon)$ and $v_R(\varepsilon)$ denote, respectively, the MD and NMD of the element ε to linguistic index $s_{\varphi(\varepsilon)}$.

In addition, $\pi(\varepsilon) = 1 - u_R(\varepsilon) - v_R(\varepsilon)$, $\forall \varepsilon \in E$, denotes the ID of the element ε to E. It is evident that $0 \leqslant \pi(\varepsilon) \leqslant 1, \forall \varepsilon \in E$.

For the given element ε, $\langle s_{\varphi(\varepsilon)}, (u_R(\varepsilon), v_R(\varepsilon))\rangle$ is called intuitionistic linguistic fuzzy number (ILFN), and for convenience, we can utilize $\tilde{\varepsilon} = \langle s_{\varphi(\varepsilon)}, (u(\varepsilon), v(\varepsilon))\rangle$ to denote an ILFN, which meets the conditions, $u_R(\varepsilon), v_R(\varepsilon) \in [0, 1]$ and $0 \leqslant u_R(\varepsilon) + v_R(\varepsilon) \leqslant 1$.

Let $\tilde{\varepsilon}_1 = \langle s_{\varphi(\varepsilon_1)}, (u(\varepsilon_1), v(\varepsilon_2))\rangle$ and $\tilde{\varepsilon}_2 = \langle s_{\varphi(\varepsilon_2)}, (u(\varepsilon_2), v(\varepsilon_2))\rangle$ be two ILFNs, then the operations of ILFN can be defined as follows (Xu and Yager, 2006):

$$\tilde{\varepsilon}_1 \oplus \tilde{\varepsilon}_2 = \left\langle s_{\varphi(\varepsilon_1) + \varphi(\varepsilon_2)}, (1 - (1 - u(\varepsilon_1))(1 - u(\varepsilon_2)), v(\varepsilon_1) v(\varepsilon_2))\right\rangle, \tag{11}$$

$$\tilde{\varepsilon}_1 \otimes \tilde{\varepsilon}_2 = \left\langle s_{\varphi(\varepsilon_1) \times \varphi(\varepsilon_2)}, (u(\varepsilon_1) u(\varepsilon_2), v(\varepsilon_1) + v(\varepsilon_2) - v(\varepsilon_1) v(\varepsilon_2))\right\rangle, \tag{12}$$

$$\delta \tilde{\varepsilon}_1 = \left\langle s_{\delta \times \varphi(\varepsilon_1)}, \left(1 - (1 - u(\varepsilon_1))^\delta\right), (v(\varepsilon_1))^\delta \right\rangle, \tag{13}$$

$$\tilde{\varepsilon}_1^\delta = \left\langle s_{(\varphi(\varepsilon_1))^\delta}, \left((u(\varepsilon_1))^\delta, 1 - (1 - v(\varepsilon_1))^\delta\right) \right\rangle. \tag{14}$$

2.3 The uncertain linguistic variable and intuitionistic uncertain linguistic set

Definition 3. (Xu, 2004) Suppose $\widehat{s}=[s_j,s_k]$, $s_j,s_k\in\widehat{S}$ and $j\leqslant k$, s_j is the lower limit of $\widehat{s}$ and s_k is the upper limit of $\widehat{s}$, then $\widehat{s}$ can be called an ULV.

Let $\widehat{s}_1=[s_{j_1},s_{k_1}]$ and $\widehat{s}_2=[s_{j_2},s_{k_2}]$ are ULVs, then the operations of ULV are defined as follows:

$$\widehat{s}_1\oplus\widehat{s}_2=[s_{j_1},s_{k_1}]+[s_{j_2},s_{k_2}]=[s_{j_1+j_2},s_{k_1+k_2}], \tag{15}$$

$$\widehat{s}_1\otimes\widehat{s}_2=[s_{j_1},s_{k_1}]\times[s_{j_2},s_{k_2}]=[s_{j_1\times j_2},s_{k_1\times k_2}], \tag{16}$$

$$\delta\widehat{s}_1=\delta[s_{j_1},s_{k_1}]=[s_{\delta j_1},s_{\delta k_1}],\quad \delta\geqslant 0, \tag{17}$$

$$\widehat{s}_1^{\ \delta}=[s_{j_1},s_{k_1}]^{\delta}=\left[s_{j_1^{\ \delta}},s_{k_1^{\ \delta}}\right],\quad \delta\geqslant 0. \tag{18}$$

It is easy to know that the operation rules (15)–(18) have some limitations which the ULVs obtained by calculating are lower than the maximum number s_t are not assured. For example, $S=\{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$, $\widehat{s}_1=[s_5,s_6]$ and $\widehat{s}_1+\widehat{s}_2=[s_{12},s_{14}]$, then $\widehat{s}_1+\widehat{s}_2=[s_{12},s_{14}]$. It is obviously that the upper and lower limits are all greater than s_6 which is the largest number of S. For the sake of overcoming the above limitation, some literatures give some new modified operational laws for ULVs.

Let $\widehat{s}_1=[s_{j_1},s_{k_1}]$ and $\widehat{s}_2=[s_{j_2},s_{k_2}]$ are ULVs, then the operations of ULV are defined as follows:

$$\widehat{s}_1\oplus\widehat{s}_2=[s_{j_1},s_{k_1}]+[s_{j_2},s_{k_2}]=\left[s_{j_1+j_2-(j_1j_2/t)},s_{k_1+k_2-(k_1k_2/t)}\right], \tag{19}$$

$$\widehat{s}_1\otimes\widehat{s}_2=[s_{j_1},s_{k_1}]\times[s_{j_2},s_{k_2}]=[s_{(j_1\times j_2)/t},s_{(k_1\times k_2)/t}], \tag{20}$$

$$\delta\widehat{s}_1=\delta[s_{j_1},s_{k_1}]=\left[s_{t\left(1-(1-(j_1/t))^{\delta}\right)},s_{t\left(1-(1-(j_2/t))^{\delta}\right)}\right],\quad \delta\geqslant 0, \tag{21}$$

$$\widehat{s}_1^{\delta}=[s_{j_1},s_{k_1}]^{\delta}=\left[s_{t(j_1/t)^{\delta}},s_{t(j_2/t)^{\delta}}\right],\quad \delta\geqslant 0. \tag{22}$$

Definition 4. (Liu and Jin, 2012) Let $R=\{\langle\varepsilon,[[s_{\varphi(\varepsilon)},\ s_{\vartheta(\varepsilon)}],\ (u_R(\varepsilon),\ v_R(\varepsilon))]\rangle|\varepsilon\in E\}$ be IULS, $[[s_{\varphi(\varepsilon)},\ s_{\vartheta(\varepsilon)}],\ (u_R(\varepsilon),\ v_R(\varepsilon))]$ is called an intuitionistic uncertain linguistic variable (IULV). where $s_{\varphi(\varepsilon)},s_{\vartheta(\varepsilon)}\in\widehat{S}$, u_R: E→[0, 1] and v_R: E→[0, 1], with the condition $0\leqslant u_R(\varepsilon)+v_R(\varepsilon)\leqslant 1$, $\forall\varepsilon\in E$. The numbers $u_R(\varepsilon)$ and $v_R(\varepsilon)$ denote, respectively, the MD and NMD of the element ε to linguistic index $[s_{\varphi(\varepsilon)},\ s_{\vartheta(\varepsilon)}]$.

In addition, $\pi(\varepsilon)=1-u_R(\varepsilon)-v_R(\varepsilon)$, $\forall\varepsilon\in E$, denotes the ID of the element ε to E. It is evident that $0\leqslant\pi(\varepsilon)\leqslant 1,\forall\varepsilon\in E$.

Let $\tilde{\varepsilon}_1=\langle[s_{\varphi(\varepsilon_1)},s_{\vartheta(\varepsilon_1)}],(u(\varepsilon_1),v(\varepsilon_1))$ and $\tilde{\varepsilon}_2=\langle[s_{\varphi(\varepsilon_2)},s_{\vartheta(\varepsilon_2)}],(u(\varepsilon_2),v(\varepsilon_2))$ be two IULVs, $s_{\varphi(\varepsilon_1)},s_{\vartheta(\varepsilon_1)},s_{\varphi(\varepsilon_2)},s_{\vartheta(\varepsilon_2)}\in\widehat{S}$, $\delta\geqslant 0$, then the operations of IULV can be defined as follows (Liu and Jin, 2012):

$$\tilde{\varepsilon}_1\oplus\tilde{\varepsilon}_2=\left\langle\left[s_{\varphi(\varepsilon_1)+\varphi(\varepsilon_2)},s_{\vartheta(\varepsilon_1)+\vartheta(\varepsilon_2)}\right],(1-(1-u(\varepsilon_1))(1-u(\varepsilon_2)),v(\varepsilon_1)v(\varepsilon_2))\right\rangle, \tag{23}$$

$$\tilde{\varepsilon}_1 \otimes \tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_1)\times\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_1)\times\vartheta(\varepsilon_2)}\right], (u(\varepsilon_1)u(\varepsilon_2), v(\varepsilon_1)+v(\varepsilon_2)-v(\varepsilon_1)v(\varepsilon_2))\right\rangle, \tag{24}$$

$$\delta\tilde{\varepsilon}_1 = \left\langle \left[s_{\delta\times\varphi(\varepsilon_1)}, s_{\delta\times\vartheta(\varepsilon_1)}\right], \left(1-(1-u(\varepsilon_1))^{\delta}\right), (v(\varepsilon_1))^{\delta}\right\rangle, \tag{25}$$

$$\tilde{\varepsilon}_1{}^{\delta} = \left\langle \left[s_{(\varphi(\varepsilon_1))^{\delta}}, s_{(\vartheta(\varepsilon_1))^{\delta}}\right], \left((u(\varepsilon_1))^{\delta}, 1-(1-v(\varepsilon_1))^{\delta}\right)\right\rangle. \tag{26}$$

From Liu and Jin (2012), Liu (2013a), Xu (2004) and Wang and Wang (2015), we can find that there are some shortcomings in the process of calculation by taking some examples, which the IULVs obtained by calculating are lower than the maximum number s_t are not assured. For supplying this gap, some modified operational laws of IULV are presented in some literatures.

Let $\tilde{\varepsilon}_1 = \left\langle \left[s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)}\right], (u(\varepsilon_1), v(\varepsilon_1))\right\rangle$ and $\tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)}\right], (u(\varepsilon_2), v(\varepsilon_2))\right\rangle$ be two IULVs, $s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)}, s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)} \in \widehat{S}$, $\delta \geqslant 0$, then the modified operations of IULV can be defined as follows:

$$\tilde{\varepsilon}_1 \oplus \tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_1)+\varphi(\varepsilon_2)-\left((\varphi(\varepsilon_1)\varphi(\varepsilon_2))/t\right)}, s_{\vartheta(\varepsilon_1)+\vartheta(\varepsilon_2)-\left((\vartheta(\varepsilon_1)+\vartheta(\varepsilon_2))/t\right)}\right], (1-(1-u(\varepsilon_1))(1-u(\varepsilon_2)), v(\varepsilon_1)v(\varepsilon_2))\right\rangle, \tag{27}$$

$$\tilde{\varepsilon}_1 \otimes \tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_1)\times\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_1)\times\vartheta(\varepsilon_2)}\right], (u(\varepsilon_1)u(\varepsilon_2), v(\varepsilon_1)+v(\varepsilon_2)-v(\varepsilon_1)v(\varepsilon_2))\right\rangle, \tag{28}$$

$$\delta\tilde{\varepsilon}_1 = \left\langle \left[s_{t\left(1-\left(1-\left((\varphi(\varepsilon_1))/t\right)\right)^{\delta}\right)}, s_{t\left(1-\left(1-\left((\vartheta(\varepsilon_1))/t\right)\right)^{\delta}\right)}\right], \left(1-(1-u(\varepsilon_1))^{\delta}\right), (v(\varepsilon_1))^{\delta}\right\rangle, \tag{29}$$

$$\tilde{\varepsilon}_1{}^{\delta} = \left\langle \left[s_{t\left((\varphi(\varepsilon_1))/t\right)^{\delta}}, s_{t\left((\vartheta(\varepsilon_1))/t\right)^{\delta}}\right], \left((u(\varepsilon_1))^{\delta}, 1-(1-v(\varepsilon_1))^{\delta}\right)\right\rangle. \tag{30}$$

Besides, Liu and Shi (2015) defined the operations of IULVs based on the Einstein t-norm (TN) and t-conorm (TC), which can be used to demonstrate the corresponding intersections and unions of IULVs.

2.4 Interval-value intuitionistic uncertain linguistic set (IVIULS)

Definition 5. (Wang, 2013) Let $R = \{\langle \varepsilon, [[s_{\varphi(\varepsilon)}, s_{\vartheta(\varepsilon)}], ([u_{lR}(\varepsilon), u_{uR}(\varepsilon)], [v_{lR}(\varepsilon), v_{uR}(\varepsilon)])]\rangle | \varepsilon \in E\}$ be IVIULS, $[[s_{\varphi(\varepsilon)}, s_{\vartheta(\varepsilon)}], ([u_{lR}(\varepsilon), u_{uR}(\varepsilon)], [v_{lR}(\varepsilon), v_{uR}(\varepsilon)])]$ is called an IVIULV. Here $s_{\varphi(\varepsilon)}, s_{\vartheta(\varepsilon)} \in \widehat{S}$, $[u_{lR}(\varepsilon), u_{uR}(\varepsilon)] \in [0, 1]$ and $[v_{lR}(\varepsilon), v_{uR}(\varepsilon)] \in [0, 1]$, with the condition $0 \leqslant u_{uR}(\varepsilon)+v_{uR}(\varepsilon) \leqslant 1$, $\forall \varepsilon \in E$. The interval values $[u_{lR}(\varepsilon), u_{uR}(\varepsilon)]$ and $[v_{lR}(\varepsilon), v_{uR}(\varepsilon)]$ denote, respectively, the MD and NMD of the element ε to linguistic index $[s_{\varphi(\varepsilon)}, s_{\vartheta(\varepsilon)}]$.

It is obviously that if $u_{lR}(\varepsilon) = u_{uR}(\varepsilon)$ and $v_{lR}(\varepsilon) = v_{uR}(\varepsilon)$ for each $\varepsilon \in E$, then IVIULS reduces to be the IULS. Furthermore, if $s_{\varphi(\varepsilon)} = s_{\vartheta(\varepsilon)}$, then it reduces to be the ILS.

Let $\tilde{\varepsilon}_1 = \langle [s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)}], ([u_{lR}(\varepsilon_1), u_{uR}(\varepsilon_1)], [v_{lR}(\varepsilon_1), v_{uR}(\varepsilon_1)])\rangle$ and $\tilde{\varepsilon}_2 = \langle [s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)}], ([u_{lR}(\varepsilon_2), u_{uR}(\varepsilon_2)], [v_{lR}(\varepsilon_2), v_{uR}(\varepsilon_2)])\rangle$ be two IVIULVs, $s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)}, s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)} \in \widehat{S}$, $\delta \geqslant 0$, then the operations of IVIULV can be defined as follows (Wang, 2013):

$$\tilde{\varepsilon}_1 \oplus \tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_1)+\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_1)+\vartheta(\varepsilon_2)}\right], ([1-(1-u_l(\varepsilon_1))(1-u_l(\varepsilon_2)), 1-(1-u_u(\varepsilon_1))(1-u_u(\varepsilon_2))], [v_l(\varepsilon_1)v_l(\varepsilon_2), v_u(\varepsilon_1)v_u(\varepsilon_2)])\right\rangle, \tag{31}$$

$$\tilde{\varepsilon}_1 \otimes \tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_1)\times\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_1)\times\vartheta(\varepsilon_2)} \right], ([u_l(\varepsilon_1)u_l(\varepsilon_2), u_u(\varepsilon_1)u_u(\varepsilon_2)], [v_l(\varepsilon_1)+v_l(\varepsilon_2)-v_l(\varepsilon_1)v_l(\varepsilon_2), v_u(\varepsilon_1)+v_u(\varepsilon_2)-v_u(\varepsilon_1)v_u(\varepsilon_2)]) \right\rangle, \tag{32}$$

$$\delta\tilde{\varepsilon}_1 = \left\langle \left[s_{\delta\times\varphi(\varepsilon_1)}, s_{\delta\times\vartheta(\varepsilon_1)} \right], \left(\left[1-(1-u_l(\varepsilon_1))^{\delta}, 1-(1-u_u(\varepsilon_1))^{\delta}\right], \left[(v_l(\varepsilon_1))^{\delta}, (v_u(\varepsilon_1))^{\delta}\right] \right) \right\rangle, \tag{33}$$

$$\tilde{\varepsilon}_1{}^{\delta} = \left\langle \left[s_{(\varphi(\varepsilon_1))^{\delta}}, s_{(\vartheta(\varepsilon_1))^{\delta}} \right], \left(\left[(u_l(\varepsilon_1))^{\delta}, (u_u(\varepsilon_1))^{\delta}\right], \left[1-(1-v_l(\varepsilon_1))^{\delta}, 1-(1-v_u(\varepsilon_1))^{\delta}\right] \right) \right\rangle. \tag{34}$$

Theorem 2. (Wang, 2013) Let $\tilde{\varepsilon}_1 = \left\langle \left[s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)} \right], (u(\varepsilon_1), v(\varepsilon_2)) \right\rangle$ and $\tilde{\varepsilon}_2 = \left\langle \left[s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)} \right], (u(\varepsilon_2), v(\varepsilon_2)) \right\rangle$ be two IULVs, $s_{\varphi(\varepsilon_1)}, s_{\vartheta(\varepsilon_1)}, s_{\varphi(\varepsilon_2)}, s_{\vartheta(\varepsilon_2)} \in \widehat{S}$, then the modified operations of IULV have some properties as follows:

$$\tilde{\varepsilon}_1 \oplus \tilde{\varepsilon}_2 = \tilde{\varepsilon}_2 \oplus \tilde{\varepsilon}_1, \tag{35}$$

$$\tilde{\varepsilon}_1 \otimes \tilde{\varepsilon}_2 = \tilde{\varepsilon}_2 \otimes \tilde{\varepsilon}_1, \tag{36}$$

$$\delta(\tilde{\varepsilon}_1 \oplus \tilde{\varepsilon}_2) = \delta\tilde{\varepsilon}_1 + \delta\tilde{\varepsilon}_2, \quad \delta \geqslant 0, \tag{37}$$

$$\delta_1\tilde{\varepsilon}_1 \oplus \delta_2\tilde{\varepsilon}_1 = (\delta_1+\delta_2)\tilde{\varepsilon}_1, \quad \delta_1, \delta_2 \geqslant 0, \tag{38}$$

$$\tilde{\varepsilon}_1{}^{\delta} \otimes \tilde{\varepsilon}_2{}^{\delta} = (\tilde{\varepsilon}_2 \otimes \tilde{\varepsilon}_1)^{\delta}, \quad \delta \geqslant 0, \tag{39}$$

$$\tilde{\varepsilon}_1{}^{\delta_1} \otimes \tilde{\varepsilon}_1{}^{\delta_2} = \tilde{\varepsilon}_1{}^{\delta_1+\delta_2}, \quad \delta_1, \delta_2 \geqslant 0. \tag{40}$$

We know if $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are two IVIULVs, then have the same above properties as the IULVs.

Furthermore, two symmetrical IVL hybrid aggregation operators are introduced by Meng and Chen (2016).

2.5 Intuitionistic uncertain 2-tuple linguistic variable (IU2TLV)

Definition 6. (Herrera and Martínez, 2000a, b, 2012) Let $S = \{s_0, s_1, \ldots, s_m\}$ be an ordered linguistic label set. The symbolic translation between the 2-tuple linguistic representation and numerical values can be defined as follows:

$$\nabla: [0, t] \rightarrow S \times [-0.5, 0.5), \tag{41}$$

where $\nabla(\eta) = (s_i, \kappa)$ with $i = Round(\eta)$ and $\kappa = \eta - i$, $\nabla^{-1}(s_i, \kappa) = i + \kappa = \eta$.

Definition 7. (Beg and Rashid, 2016; Nie *et al.*, 2017; Liu and Chen, 2018) An IU2TLV in R is defined as $R = \{\langle \varepsilon, [(s_{\varphi(\varepsilon)}, \chi_{\vartheta(\varepsilon)}), (u_R(\varepsilon), v_R(\varepsilon))] \rangle | \varepsilon \in E\}$, where $(s_{\varphi(\varepsilon)}, \chi_{\vartheta(\varepsilon)}) \in S$, u_R: E→[0, 1] and v_R: E→[0, 1], with the condition $0 \leqslant u_R(\varepsilon) + v_R(\varepsilon) \leqslant 1$, $\forall \varepsilon \in E$. The numbers $u_R(\varepsilon)$ and $v_R(\varepsilon)$ denote, respectively, MD and NMD of the element ε to linguistic index $(s_{\varphi(\varepsilon)}, \chi_{\vartheta(\varepsilon)})$. $(s_{\varphi(\varepsilon)}, \chi_{\vartheta(\varepsilon)})$, $(u_R(\varepsilon), v_R(\varepsilon))$ is called an IU2TLV.

Suppose $\tilde{\varepsilon}_1 = \left\langle \left(s_{\varphi(\varepsilon_1)}, \chi_{\vartheta(\varepsilon_1)} \right), (u_R(\varepsilon_1), v_R(\varepsilon_1)) \right\rangle$ and $\tilde{\varepsilon}_2 = \left\langle \left(s_{\varphi(\varepsilon_2)}, \chi_{\vartheta(\varepsilon_2)} \right), (u_R(\varepsilon_2), v_R(\varepsilon_2)) \right\rangle$ are any two IU2TLVs, then the operational rules of IU2TLV are defined as follows (Beg and Rashid, 2016):

$$\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2 = \left\langle \left[\nabla\left(\underline{\eta}_{\varphi(\varepsilon_1)} + \underline{\eta}_{\varphi(\varepsilon_2)} \right), \nabla\left(\overline{\eta}_{\varphi(\varepsilon_1)} + \overline{\eta}_{\varphi(\varepsilon_2)} \right) \right], (u(\varepsilon_1) \vee u(\varepsilon_2), v(\varepsilon_1) \vee v(\varepsilon_2)) \right\rangle, \tag{42}$$

$$\tilde{\varepsilon}_1 \times \tilde{\varepsilon}_2 = \left\langle \left[\nabla\left(\underline{\eta}_{\varphi(\varepsilon_1)} \underline{\eta}_{\varphi(\varepsilon_2)} \right), \nabla\left(\overline{\eta}_{\varphi(\varepsilon_1)} \overline{\eta}_{\varphi(\varepsilon_2)} \right) \right], (u(\varepsilon_1) \wedge u(\varepsilon_2), v(\varepsilon_1) \wedge v(\varepsilon_2)) \right\rangle, \tag{43}$$

$$\delta\tilde{\varepsilon}_1 = \left\langle \left[\nabla\left(\delta\underline{\eta}_{\varphi(\varepsilon_1)} \right), \nabla\left(\delta\overline{\eta}_{\varphi(\varepsilon_1)} \right) \right], (u(\varepsilon_1), v(\varepsilon_1)) \right\rangle, \tag{44}$$

$$\tilde{\varepsilon}_1{}^{\delta} = \left\langle \left[\nabla\left(\left(\underline{\eta}_{\varphi(\varepsilon_1)} \right)^{\delta} \right), \nabla\left(\left(\overline{\eta}_{\varphi(\varepsilon_1)} \right)^{\delta} \right) \right], (u(\varepsilon_1), v(\varepsilon_1)) \right\rangle. \tag{45}$$

3. Intuitionistic linguistic fuzzy aggregation (ILFG) operators

3.1 Some basic intuitionistic linguistic fuzzy AOs

Based on the operational rules presented in Section 2, Liu and Jin (2012) developed IULWGM operator, OIULWGM operator. Liu (2013b) developed interval-value intuitionistic uncertain linguistic weighted geometric mean operator and interval-value intuitionistic uncertain linguistic weighted geometric mean operator.

Definition 8. (Liu and Jin, 2012) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by weighted geometric mean (WGM) operator is an IULV, and:

$$\mathrm{IULWGM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\prod_{i=1}^{n} (\varphi(\varepsilon_i))^{\mathrm{w}_i}}, s_{\prod_{i=1}^{n} (\vartheta(\varepsilon_i))^{\mathrm{w}_i}} \right], \left(\prod_{i=1}^{n} u(\varepsilon_i)^{\mathrm{w}_i}, 1 - \prod_{i=1}^{n} (1 - v(\varepsilon_i))^{\mathrm{w}_i} \right) \right\rangle, \tag{46}$$

where the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$.

Definition 9. (Liu and Jin, 2012) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by ordered weighted geometric mean (OWGM) operator is an IULV, and:

$$\mathrm{OIULWGM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\prod_{i=1}^{n} (\varphi(\varepsilon_{\theta_i}))^{\mathrm{w}_i}}, s_{\prod_{i=1}^{n} (\vartheta(\varepsilon_{\theta_i}))^{\mathrm{w}_i}} \right], \left(\prod_{i=1}^{n} u(\varepsilon_{\theta_i})^{\mathrm{w}_i}, 1 - \prod_{i=1}^{n} (1 - v(\varepsilon_{\theta_i}))^{\mathrm{w}_i} \right) \right\rangle, \tag{47}$$

where the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$. $(\theta_1, \theta_2, \ldots, \theta_n)$ is any permutation of $(1, 2, \ldots, n)$, such that $\tilde{\varepsilon}_{\theta_{i-1}} \geqslant \tilde{\varepsilon}_{\theta_i}$ for all $(i = 1, 2, \ldots, n)$.

It is easy to prove that the above operators have the properties of commutativity, idempotency, boundedness and monotonicity.

Definition 10. (Liu, 2013b) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of the IVIULVs, the aggregation value by WGM operator is still an IULV, and:

$$\mathrm{IVIULWGM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\prod_{i=1}^{n}(\varphi(\varepsilon_i))^{w_i}}, s_{\prod_{i=1}^{n}(\vartheta(\varepsilon_i))^{w_i}} \right], \left(\left[\prod_{i=1}^{n} u_{lR}(\varepsilon_i)^{w_i}, \prod_{i=1}^{n} u_{uR}(\varepsilon_i)^{w_i} \right], \right.\right.$$

$$\left.\left. \left[1-\prod_{i=1}^{n}(1-v_{lR}(\varepsilon_i))^{w_i}, 1-\prod_{i=1}^{n}(1-v_{uR}(\varepsilon_i))^{w_i} \right] \right) \right\rangle, \qquad (48)$$

where the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 11. (Liu, 2013b) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of IVIULVs. The value aggregated by OWGM operator is still an IULV, and:

$$\mathrm{IVIULWGM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\prod_{i=1}^{n}(\varphi(\varepsilon_{\theta_i}))^{w_i}}, s_{\prod_{i=1}^{n}(\vartheta(\varepsilon_{\theta_i}))^{w_i}} \right], \left(\left[\prod_{i=1}^{n} u_{lR}\left(\varepsilon_{\theta_i}\right)^{w_i}, \prod_{i=1}^{n} u_{uR}\left(\varepsilon_{\theta_i}\right)^{w_i} \right], \right.\right.$$

$$\left.\left. \left[1-\prod_{i=1}^{n}\left(1-v_{lR}\left(\varepsilon_{\theta_i}\right)\right)^{w_i}, 1-\prod_{i=1}^{n}\left(1-v_{uR}\left(\varepsilon_{\theta_i}\right)\right)^{w_i} \right] \right) \right\rangle, \qquad (49)$$

where the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$. $(\theta_1, \theta_2, \ldots, \theta_n)$ is any permutation of $(1, 2, \ldots, n)$, such that $\tilde{\varepsilon}_{\theta_{i-1}} \geqslant \tilde{\varepsilon}_{\theta_i}$ for all $(i = 1, 2, \ldots, n)$.

It is easy to prove that the above operators have the properties of commutativity, idempotency, boundedness and monotonicity.

In addition, based on the IL weighted arithmetic mean operator, Wang *et al.* (2014) developed intuitionistic linguistic ordered weighted mean (ILOWM) operator and the intuitionistic linguistic hybrid operator. Su *et al.* (2014) presented the intuitionistic linguistic ordered weighted mean distance operator, quasi-arithmetic intuitionistic linguistic ordered weighted mean distance operator and multi-person intuitionistic linguistic ordered weighted mean distance operator.

3.2 The extended MCDM approaches for IUFS

(1) The ETOPSIS approaches for IUFS.

In general, the standard TOPSIS approach can only process the real value and cannot deal with fuzzy information, such as IUFS. Wei (2014) introduced an ETOPSIS approach to process the IUFS in real decision-making circumstance.

Du and Zuo (2011) developed an extended technique for TOPSIS in which the criteria values are in the form of IULVs and the criteria weights are unknown.

Joshi *et al.* (2018) combined the TOPSIS and IVIULVs by redefining the basic operation rules and distance measure to solve the MCGDM problems.

Wei (2011) used the ETOPSIS approach to solve the MAGDM problems with 2TIULVs.

(2) The ETODIM approaches for IUFS.

We all know that TODIM approach can take into account the bounded rationality of experts based on prospect theory in MCDM. The classical TODIM can only process the MCDM problems where the criteria values are exact numbers. Liu (Liu and Teng, 2015) developed an ETODIM to deal with MCDM problems with IULVs. Yu *et al.* (2016) presented an interactive MCDM approach based on TODIM and NLP with IULVs. Wang and Liu (2017) proposed TODIM for IL (ILTODIM) approach and TODIM for IUL (IULTODIM) approach by improving the distance measure to deal with the MADM problems with the forms of ILV and IULV.

(3) The EVIKOR approach for IULVs.

The VIKOR approach is a very useful tool to dispose decision-making problems by selecting the best alternative based on the maximizing "group utility" and minimizing "individual regret." At present, a number of researchers pay more and more attention to VIKOR approach. Li *et al.* (2017) extended the VIKOR approach to deal with IULVs and presented the EVIKOR for MADM problems with IULVs. Furthermore, Liu and Qin (2017) developed the EVIKOR by using the Hamming distance to deal with the IVIULVs and presented the EVIKOR approach for MADM problems with IVIULVs.

3.3 Some intuitionistic linguistic fuzzy AOs considering the interrelationships between criteria

In some real decision-making problem, we should take into account the interrelationships between criteria because of existing the situation of mutual support in some criteria. Liu, Chen and Chu (2014) presented an IULBOWM operator, WIULBOWM operator. Liu, Liu and Zhang (2014) proposed the IULAHM operator, IULGHM operator, WIULAHM operator, WIULGHM operator. Ju *et al.* (2016) developed the IULMSM operator and WIULMSM operator.

Definition 12. (Liu, Chen and Chu, 2014) Let $\tilde{\varepsilon}_i = \langle [s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}], (u(\varepsilon_i), v(\varepsilon_i)) \rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by IULBOWM operator is an IULV, and:

$$\mathrm{IULBOWM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n) =$$

$$\left\langle \left[s_{\left(\frac{1}{n}\sum_{i=1}^{n}\left(\varphi_{(i)}\sum_{j=1}^{n-1} \mathrm{w}_j \varphi_{\xi(j)}\right)\right)^{1/2}}, s_{\left(\frac{1}{n}\sum_{i=1}^{n}\left(\vartheta_{(i)}\sum_{j=1}^{n-1} \mathrm{w}_j \vartheta_{\xi(j)}\right)\right)^{1/2}} \right] \right.$$

$$\left(1-\left(\prod_{i=1}^{n}\left(1-u_i\left(1-\prod_{i=1}^{n}\left(1-u_{\xi(j)}\right)^{\mathrm{w}_j}\right)\right)\right)^{1/n}\right)^{1/2},$$

$$\left. 1-\left(1-\left(\prod_{i=1}^{n}\left(1-v_i\left(1-\prod_{i=1}^{n}\left(1-v_{\xi(j)}\right)^{\mathrm{w_j}}\right)\right)\right)^{1/n}\right)\right\rangle, \qquad (50)$$

where $\xi(i)$ is the ith largest element in the tuple $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$, and w_i is the OWA weighted vector of dimension n with the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$.

Definition 13. (Liu, Chen and Chu, 2014) Let $\tilde{\varepsilon}_i = \langle [s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}], (u(\varepsilon_i), v(\varepsilon_i)) \rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by WIULBOWM operator is an IULV, and:

$$\mathrm{WIULBOWM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\left(\sum_{i=1}^{n}\left(w_i\varphi_{(i)}\sum_{j=1}^{n-1}w_j\varphi_{\xi(j)}\right)\right)^{1/2}}, s_{\left(\sum_{i=1}^{n}\left(w_i\vartheta_{(i)}\sum_{j=1}^{n-1}w_j\vartheta_{\xi(j)}\right)\right)^{1/2}} \right] \right.$$

$$\left(1-\left(\prod_{i=1}^{n}\left(1-u_i\left(1-\prod_{i=1}^{n}\left(1-u_{\xi_{(j)}}\right)^{w_j}\right)\right)\right)^{w_i}\right)^{1/2},$$

$$\left. 1-\left(1-\left(\prod_{i=1}^{n}\left(1-v_i\left(1-\prod_{i=1}^{n}\left(1-v_{\xi_{(j)}}\right)^{w_j}\right)\right)\right)^{w_i}\right)\right\rangle, \tag{51}$$

where $\xi(i)$ is the ith largest element in the tuple $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$, and w_i is the OWA weighted vector of dimension n with the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

Obviously, the above IULBOWM and WIULBOWM operators have the desirable properties of commutativity, idempotency, monotonicity and boundedness.

Furthermore, Liu and Liu (2017) introduced IUL partitioned BM (IULPBM) operator, weighted IUL partitioned BM operator, geometric IUL partitioned BM operator and weighted geometric IUL partitioned BM because they consider that in some time the interrelationships between criteria do not always exist and we can take the criteria into some part based on the different categories and the interrelationships between criteria in same part exist.

At the same time, the DOWM operator has the advantage of relieving the impact of biased criteria values. Liu *et al.* (2017) combined the DOWM operator and BM operator to present the intuitionistic linguistic dependent BM operator and weighted intuitionistic linguistic dependent BM operator.

Definition 14. (Liu, Liu and Zhang, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by IULAHM operator is an IULV, and:

$$\mathrm{IULAHM}^{a,b}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\left(2/(n(n+2))\sum_{i=1}^{n}\sum_{j=i}^{n}\varphi_i{}^a\varphi_j{}^b\right)^{1/(a+b)}}, s_{\left(2/(n(n+2))\sum_{i=1}^{n}\sum_{j=i}^{n}\vartheta_i{}^a\vartheta_j{}^b\right)^{1/(a+b)}} \right] \right.$$

$$\left(\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-u_i{}^a u_j{}^b\right)\right)^{2/n(n+1)}\right)^{1/(a+b)},\right.$$

$$\left. 1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(1-v_i)^a\left(1-v_j\right)^b\right)^{\frac{2}{n(n+1)}}\right)^{1/(a+b)}\right)\right)\right\rangle. \tag{52}$$

It is easy to know that the IULGHM operator has the properties of monotonicity, idempotency and boundedness.

Definition 15. (Liu, Liu and Zhang, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by WIULAHM operator is an IULV, the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$

is $\mathrm{w}=(\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$, n is a balance parameter, and:

$$\mathrm{IULAHM}^{a,b}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\left(2/(n(n+2)) \sum_{i=1}^{n} \sum_{j=i}^{n} (n\mathrm{w}_i \varphi_i)^a (nw_j \varphi_j)^b\right)^{1/(a+b)}}, s_{\left(2/(n(n+2)) \sum_{i=1}^{n} \sum_{j=i}^{n} (n\mathrm{w}_i \vartheta_i)^a (n\mathrm{w}_j \vartheta_j)^b\right)^{1/(a+b)}} \right] \right.$$

$$\left(\left(1 - \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - (1 - (1-u_i)^{n\mathrm{w}_i})^a \left(1 - (1-u_j)^{n\mathrm{w}_j}\right)^a \right) \right)^{2/(n(n+1))} \right)^{1/(a+b)}, \right.$$

$$\left. \left. 1 - \left(1 - \left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(1 - (1 - v_i^{n\mathrm{w}_i})^a \left(1 - v_j^{n\mathrm{w}_j}\right)^b \right)^{2/(n(n+1))} \right)^{1/(a+b)} \right) \right) \right\rangle. \tag{53}$$

It is easy to prove that the WIULAHM operator has not the property of idempotency, but it has the property of monotonicity.

Definition 16. (Liu, Liu and Zhang,(2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by IULGHM operator is an IULV, and:

$$\mathrm{IULAHM}^{a,b}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{1/(a+b)\left(\prod_{i=1}^{n} \prod_{j=i}^{n} (a\varphi_i + b\varphi_j) \right)^{2/(n(n+2))}}, s_{(1/a+b)\left(\prod_{i=1}^{n} \prod_{j=i}^{n} (a\vartheta_i + b\vartheta_j) \right)^{2/(n(n+2))}} \right] \right.$$

$$\left(\left(1 - \left(\prod_{i=1}^{n} \prod_{j=1}^{n} (1 - (1-u_i)^a (1-u_j)^a) \right)^{2/(n(n+1))} \right)^{1/(a+b)}, \right.$$

$$\left. \left. \left(1 - \left(\prod_{i=1}^{n} \prod_{j=1}^{n} (1 - v_i^a v_j^b)^{2/(n(n+1))} \right)^{1/(a+b)} \right) \right) \right\rangle. \tag{54}$$

It is easy to know that the IULGHM operator has the properties of monotonicity, idempotency and bounded.

Definition 17. (Liu, Liu and Zhang, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)} \right], (u(\varepsilon_i), v(\varepsilon_i)) \right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by WIULGHM operator is an IULV, the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w}=(\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$, n is a balance parameter, and:

$$\mathrm{WIULAHM}^{a,b}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{1/(a+b)\left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(a(\varphi_i)^{n\mathrm{w}_i} + b(\varphi_j)^{n\mathrm{w}_j} \right) \right)^{2/(n(n+2))}}, s_{1/(a+b)\left(\prod_{i=1}^{n} \prod_{j=i}^{n} \left(a(\vartheta_i)^{n\mathrm{w}_i} + b(\vartheta_j)^{n\mathrm{w}_j} \right) \right)^{2/(n(n+2))}} \right] \right.$$

$$\left(1-\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(1-(1-u_i^{nw_i}))^a\left(1-\left(1-u_j^{nw_j}\right)\right)^a\right)\right)^{2/(n(n+1))}\right)^{1/(a+b)},$$

$$\left(1-\left(\prod_{i=1}^{n}\prod_{j=i}^{n}\left(1-(1-(1-v_i)^{nw_i})^a\left(1-(1-u_j)^{nw_j}\right)^a\right)\right)^{2/(n(n+1))}\right)^{1/(a+b)}\Bigg)\Bigg\rangle. \quad (55)$$

Obviously, the WIULGHM operator has not the property of idempotency, but it has the property of monotonicity.

In addition, Peng *et al.* (2018) proposed weighted intuitionistic linguistic fuzzy Frank improved Heronian mean operator to construct the coal mine safety evaluation. Zhang *et al.* (2017) investigated the generalized ILHM operator and weighted GILHM operator.

Definition 18. (Ju *et al.*, 2016) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i))\right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs and $r = 1, 2, \ldots, n$. The value aggregated by IULMSM operator is an IULV.

It is easy to demonstrate that the IULMSM operator has the properties of idempotency, monotonicity, boundedness and commutativity.

Definition 19. (Ju *et al.*, 2016) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i))\right\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs and $r = 1, 2, \ldots, n$. The value aggregated by WIULMSM operator is an IULV, and:

$$\mathrm{WIULMSM}^{(r)}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$

$$= \left\langle \left[s_{\left(\left(\sum_{1\leqslant i_1<i_2<\ldots i_r\leqslant n}\prod_{i=1}^{r} w_{i_j}\varphi\left(\varepsilon_{i_j}\right)\right)/C_n^r\right)^{1/r}}, s_{\left(\left(\sum_{1\leqslant i_1<i_2<\ldots i_r\leqslant n}\prod_{i=1}^{r} w_{i_j}\vartheta\left(\varepsilon_{i_j}\right)\right)/C_n^r\right)^{1/r}}, \right]\right.$$

$$\left(\left(1-\left(\prod_{1\leqslant i_1<i_2<\ldots i_r\leqslant n} 1-\prod_{i=1}^{r}\left(1-\left(1-u\left(\varepsilon_{i_j}\right)\right)^{w_{i_j}}\right)\right)^{1/C_n^r}\right)^{1/r},\right.$$

$$\left.\left. 1-\left(1-\left(\prod_{1\leqslant i_1<i_2<\ldots i_r\leqslant n}\left(1-\prod_{i=1}^{r}\left(1-v\left(\varepsilon_{i_j}\right)\right)^{w_{i_j}}\right)\right)^{1/C_n^r}\right)^{1/r}\right)\right\rangle. \quad (56)$$

The WILMSM has the property of monotonically in the case of parameter *r*.

In some time, for the sake of selecting the best alternative, we not only take into account the criteria values, but also consider the interrelationships between the criteria. Power average (PA) operator introduced first by Yager (Yager, 2001, 2015; Xu and Yager, 2010) can overcome the above weakness by setting different criteria weights. Recently, based on the PA and BM operator, Liu and Liu (2017) presented ILF power BM and weighted ILF power BM operator.

3.4 Generalized intuitionistic linguistic fuzzy (GILF) AOs

The desirable characteristic of GILF is that they can take into account as many as possible circumstances by setting different parameter values. Liu (2013a) introduced a GILDOWM operator and a GILDHWM operator.

Definition 20. (Liu, 2013a; Liu and Wang, 2014) Let $\tilde{\varepsilon}_i = \langle s_{\varphi(\varepsilon_i)}, (u(\varepsilon_i), v(\varepsilon_i))\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by GILDOWM operator is an IULV, and:

$$\begin{aligned}&\mathrm{GILDHWM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)\\&= \Bigg\langle s_{\left(\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})(\varphi(\tilde{\varepsilon}_i))^{\gamma}\right)/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)\right)^{1/\gamma}},\\&\left(\left(1-\left(\prod_{i=1}^{n}(1-u(\tilde{\varepsilon}_i)^{\gamma})^{(s(\tilde{\varepsilon}_i,\bar{\varepsilon}))/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)}\right)\right)\right)^{1/\gamma}\\&1-\left(1-\prod_{i=1}^{n}(1-v(\tilde{\varepsilon}_i)^{\gamma})^{(s(\tilde{\varepsilon}_i,\bar{\varepsilon}))/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)}\right)^{1/\gamma}\Bigg)\Bigg\rangle, \qquad (57)\end{aligned}$$

where $\bar{\varepsilon} = \langle s_{\varphi(\bar{\varepsilon})}, (u(\bar{\varepsilon}), v(\bar{\varepsilon}))\rangle$ is the average of $\tilde{\varepsilon}_i = \langle s_{\varphi(\varepsilon_i)}, (u(\varepsilon_i), v(\varepsilon_i))\rangle$ $(i = 1, 2, \ldots, n)$, $d(\tilde{\varepsilon}_i, \bar{\varepsilon})$ is the normalized Hamming distance between $\tilde{\varepsilon}_i$ and $\bar{\varepsilon}$, denoted by:

$$d(\tilde{\varepsilon}_i, \bar{\varepsilon}) = \frac{\left(\left|\varphi(\tilde{\varepsilon}_i)(1+u(\tilde{\varepsilon}_i)-v(\tilde{\varepsilon}_i))-\varphi(\bar{\varepsilon})(1+u(\bar{\varepsilon})-v(\bar{\varepsilon}))\right|\right)}{2t},$$

$s(\tilde{\varepsilon}_i, \bar{\varepsilon})$ is the similarity degree between $\tilde{\varepsilon}_i$ and $\bar{\varepsilon}$, denoted by:

$$s(\tilde{\varepsilon}_i, \bar{\varepsilon}) = \frac{d(\tilde{\varepsilon}_i, \bar{\varepsilon})}{\sum_{i=1}^{n} d(\tilde{\varepsilon}_i, \bar{\varepsilon})}.$$

Definition 21. (Liu (2013a; Liu and Wang, 2014) Let $\tilde{\varepsilon}_i = \langle s_{\varphi(\varepsilon_i)}, (u(\varepsilon_i), v(\varepsilon_i))\rangle$ $(i = 1, 2, \ldots, n)$ be a collection of IULVs. The value aggregated by GILDHWM operator is an IULV, and:

$$\begin{aligned}&\mathrm{GILDHWM}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)\\&= \Bigg\langle s_{\left(\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})(n\mathrm{w}_i\varphi(\tilde{\varepsilon}_i))^{\gamma}\right)/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)\right)^{1/\gamma}},\\&\left(\left(1-\left(\prod_{i=1}^{n}\left(1-(1-u(\tilde{\varepsilon}_i)^{m\mathrm{w}_i})^{\gamma}\right)^{(s(\tilde{\varepsilon}_i,\bar{\varepsilon}))/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)}\right)\right)\right)^{1/\gamma}\\&1-\left(1-\prod_{i=1}^{n}\left(1-(1-v(\tilde{\varepsilon}_i)^{m\mathrm{w}_i})^{\gamma}\right)^{(s(\tilde{\varepsilon}_i,\bar{\varepsilon}))/\left(\sum_{i=1}^{n} s(\tilde{\varepsilon}_i,\bar{\varepsilon})\right)}\right)^{1/\gamma}\Bigg)\Bigg\rangle, \qquad (58)\end{aligned}$$

where the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, $\mathrm{w}_i \in [0, 1]$ and $\sum_{i=1}^{n} \mathrm{w}_i = 1$, $\bar{\varepsilon} = \langle s_{\varphi(\bar{\varepsilon})}, (u(\bar{\varepsilon}), v(\bar{\varepsilon}))\rangle$ is the average of $\tilde{\varepsilon}_i = \langle s_{\varphi(\varepsilon_i)}, (u(\varepsilon_i), v(\varepsilon_i))\rangle$ $(i = 1, 2, \ldots, n)$, $d(\tilde{\varepsilon}_i, \bar{\varepsilon})$ is the normalized Hamming distance between $\tilde{\varepsilon}_i$ and $\bar{\varepsilon}$, denoted by:

$$d(\tilde{\varepsilon}_i, \bar{\varepsilon}) = \frac{\left(\left|\varphi(\tilde{\varepsilon}_i)(1+u(\tilde{\varepsilon}_i)-v(\tilde{\varepsilon}_i))-\varphi(\bar{\varepsilon})(1+u(\bar{\varepsilon})-v(\bar{\varepsilon}))\right|\right)}{2t},$$

$s(\tilde{\varepsilon}_i, \bar{\varepsilon})$ is the similarity degree between $\tilde{\varepsilon}_i$ and $\bar{\varepsilon}$, denoted by:

$$s(\tilde{\varepsilon}_i, \bar{\varepsilon}) = \frac{d(\tilde{\varepsilon}_i, \bar{\varepsilon})}{\sum_{i=1}^{n} d(\tilde{\varepsilon}_i, \bar{\varepsilon})}.$$

3.5 IL fuzzy AOs based on CI

The CI is a very available method of measuring the expected utility of an uncertain incident and can be utilized to present some IL fuzzy AOs.

Definition 22. (Meng *et al.*, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of IVIULVs, and ζ be a fuzzy measure on $E = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n\}$. The aggregation value by interval-value intuitionistic uncertain linguistic set Choquet averaging (IVIULCA) operator is also an IVIULV, expressed by:

$$\mathrm{IVIULCA}_{\zeta}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n) = \left\langle \left[s_{\sum\limits_{i=1}^{n} \varphi(\varepsilon_i)(\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1}))}, s_{\sum_{i=1}^{n} \vartheta(\varepsilon_i)(\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1}))} \right], \right.$$
$$\left(\left[1-\prod_{i=1}^{n} (1-u_{lR}(\varepsilon_i))^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})}, 1-\prod_{i=1}^{n} (1-u_{uR}(\varepsilon_i))^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})} \right], \right.$$
$$\left. \left. \left[\prod_{i=1}^{n} v_{lR}(\varepsilon_i)^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})}, \prod_{i=1}^{n} v_{uR}(\varepsilon_i)^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})} \right] \right) \right\rangle, \tag{59}$$

where $\cdot_{(i)}$ represent as a permutation on E, which such that $\tilde{\varepsilon}_{(1)f,g}\tilde{\varepsilon}_{(2)f,g}\cdots{}_{f,g}\tilde{\varepsilon}_{(n)}$, and $\mathrm{E}_{(i)} = \left\{\tilde{\varepsilon}_{(i)}, \ldots, \tilde{\varepsilon}_{(n)}\right\}$ with $\mathrm{E}_{(n+1)} = \varnothing$.

Definition 23. (Meng *et al.*, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of IVIULVs, and ζ be a fuzzy measure on $\mathrm{E} = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n\}$. The aggregation value by interval-value intuitionistic uncertain linguistic set Choquet geometric averaging (IVIULCGA) operator is also an IVIULV, expressed by:

$$\mathrm{IVIULCGA}_{\zeta}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n) = \left\langle \left[s_{\prod\limits_{i=1}^{n} \varphi(\varepsilon_i)^{\left(\zeta(\mathrm{E}_i)-\zeta\left(\mathrm{E}_{i+1}\right)\right)}}, s_{\prod\limits_{i=1}^{n} \vartheta(\varepsilon_i)^{\left(\zeta(\mathrm{E}_i)-\zeta\left(\mathrm{E}_{i+1}\right)\right)}} \right], \right.$$
$$\left(\left[\prod_{i=1}^{n} u_{lR}(\varepsilon_i)^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})}, \prod_{i=1}^{n} u_{uR}(\varepsilon_i)^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})} \right], \right.$$
$$\left. \left. \left[1-\prod_{i=1}^{n} (1-v_{lR}(\varepsilon_i))^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})}, 1-\prod_{i=1}^{n} (1-v_{uR}(\varepsilon_i))^{\zeta(\mathrm{E}_i)-\zeta(\mathrm{E}_{i+1})} \right] \right) \right\rangle, \tag{60}$$

where $\cdot_{(i)}$ represent as a permutation on E, which such that $\tilde{\varepsilon}_{(1)f,g}\tilde{\varepsilon}_{(2)f,g}\cdots{}_{f,g}\tilde{\varepsilon}_{(n)}$, and $\mathrm{E}_{(i)} = \left\{\tilde{\varepsilon}_{(i)}, \ldots, \tilde{\varepsilon}_{(n)}\right\}$ with $\mathrm{E}_{(n+1)} = \varnothing$.

From definition (Meng *et al.*, 2014), we can find the above two operators only take into account the correlation between the E_i and $\mathrm{E}_{i+1} (i = 1, 2, \ldots, n)$ when there exist interrelated characteristics between elements. For manifesting the correlation between elements, we utilize CI and generalized Shapley function introduced firstly by Marichal to define IVIULV operators, denoted as follows:

$$\phi_f(\zeta,J) = \sum_{K \subseteq J/F} \frac{(j-k-f)!t!}{(j-k+1)!}(\zeta(J \cup K) - \zeta(K)) \quad \forall F \subseteq J \tag{61}$$

where j, k and f indicate the cardinalities of the coalitions J, K and F, respectively.

It is easy to know that the above equation produces to be the Shapley function when there is only one element in F:

$$\phi_i(\zeta,J) = \sum_{K \subseteq J/i} \frac{(j-k-1)!t!}{j!}(\zeta(i \cup K) - \zeta(K)) \quad \forall i \subseteq J \tag{62}$$

Definition 24. (Meng *et al.*, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of IVIULVs, and ζ be a fuzzy measure on $\mathrm{E} = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n\}$. The aggregation value by generalized Shapley interval-value intuitionistic uncertain linguistic set Choquet averaging (GSIVIULCA) operator is also an IVIULV, expressed by:

$$\begin{aligned}
&\mathrm{GSIVIULCA}_{\phi}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n) \\
&= \left\langle \left[s_{\sum_{i=1}^{n} \varphi(\varepsilon_i)\left(\phi_{\mathrm{E}_{(i)}}(\zeta,\mathrm{E}) - \phi_{\mathrm{E}_{(i+1)}}(\zeta,\mathrm{E})\right)}, s_{\sum_{i=1}^{n} \vartheta(\varepsilon_i)\left(\phi_{\mathrm{E}_{(i)}}(\zeta,\mathrm{E}) - \phi_{\mathrm{E}_{(i+1)}}(\zeta,\mathrm{E})\right)} \right], \right. \\
&\left(\left[1 - \prod_{i=1}^{n} (1 - u_{lR}(\varepsilon_i))^{\phi_{(\mathrm{E}_i)}(\zeta,\mathrm{E}) - \phi_{(\mathrm{E}_{i+1})}(\zeta,\mathrm{E})}, \right. \right. \\
&\left. 1 - \prod_{i=1}^{n} (1 - u_{uR}(\varepsilon_i))^{\phi_{(\mathrm{E}_i)}(\zeta,\mathrm{E}) - \phi_{(\mathrm{E}_{i+1})}(\zeta,\mathrm{E})} \right], \\
&\left. \left. \left[\prod_{i=1}^{n} v_{lR}(\varepsilon_i)^{\phi_{(\mathrm{E}_i)}(\zeta,\mathrm{E}) - \phi_{(\mathrm{E}_{i+1})}(\zeta,\mathrm{E})}, \prod_{i=1}^{n} v_{uR}(\varepsilon_i)^{\phi_{(\mathrm{E}_i)}(\zeta,\mathrm{E}) - \phi_{(\mathrm{E}_{i+1})}(\zeta,\mathrm{E})} \right] \right) \right\rangle,
\end{aligned} \tag{63}$$

where $\cdot_{(i)}$ represent as a permutation on E, which such that $\tilde{\varepsilon}_{(1)f_g}\tilde{\varepsilon}_{(2)f_g}\cdots{}_{f_g}\tilde{\varepsilon}_{(n)}$, and $\mathrm{E}_{(i)} = \left\{\tilde{\varepsilon}_{(i)}, \ldots, \tilde{\varepsilon}_{(n)}\right\}$ with $\mathrm{E}_{(n+1)} = \varnothing$.

Definition 25. (Meng *et al.*, 2014) Let $\tilde{\varepsilon}_i = \left\langle \left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], ([u_{lR}(\varepsilon_i), u_{uR}(\varepsilon_i)], [v_{lR}(\varepsilon_i), v_{uR}(\varepsilon_i)]) \right\rangle$ be a collection of IVIULVs, and ζ be a fuzzy measure on $\mathrm{E} = \{\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n\}$. The aggregation value by generalized Shapley interval-value intuitionistic uncertain linguistic set Choquet geometric averaging (GSIVIULCGA) operator is also an IVIULV, expressed by:

$$\begin{aligned}
&\mathrm{GSIVIULCGA}_{\phi}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n) \\
&= \left\langle \left[s_{\prod_{i=1}^{n} \varphi(\varepsilon_i)\left(\phi_{\mathrm{E}_{(i)}}(\zeta,\mathrm{E}) - \phi_{\mathrm{E}_{(i+1)}}(\zeta,\mathrm{E})\right)}, s_{\prod_{i=1}^{n} \vartheta(\varepsilon_i)\left(\phi_{\mathrm{E}_{(i)}}(\zeta,\mathrm{E}) - \phi_{\mathrm{E}_{(i+1)}}(\zeta,\mathrm{E})\right)} \right], \right.
\end{aligned}$$

$$\left(\left[\prod_{i=1}^{n} u_{lR}(\varepsilon_i)^{\phi(\mathrm{E}_i)(\zeta,\mathrm{E})-\phi\left(\mathrm{E}_{i+1}\right)(\zeta,\mathrm{E})}, \prod_{i=1}^{n} u_{uR}(\varepsilon_i)^{\phi(\mathrm{E}_i)(\zeta,\mathrm{E})-\phi\left(\mathrm{E}_{i+1}\right)(\zeta,\mathrm{E})}\right],\right.$$
$$\left[1-\prod_{i=1}^{n}(1-v_{lR}(\varepsilon_i))^{\phi(\mathrm{E}_i)(\zeta,\mathrm{E})-\phi\left(\mathrm{E}_{i+1}\right)(\zeta,\mathrm{E})},\right.$$
$$\left.\left.1-\prod_{i=1}^{n}(1-v_{uR}(\varepsilon_i))^{\phi(\mathrm{E}_i)(\zeta,\mathrm{E})-\phi\left(\mathrm{E}_{i+1}\right)(\zeta,\mathrm{E})}]\right)\right\rangle, \tag{64}$$

where $\cdot_{(i)}$ represent as a permutation on E, which such that $\tilde{\varepsilon}_{(1)f,g}\tilde{\varepsilon}_{(2)f,g}\cdots{}_{f,g}\tilde{\varepsilon}_{(n)}$, and $\mathrm{E}_{(i)} = \left\{\tilde{\varepsilon}_{(i)},\ldots,\tilde{\varepsilon}_{(n)}\right\}$ with $\mathrm{E}_{(n+1)} = \varnothing$.

The IVIULCA, IVIULCGA, GSIVIULCA and GSIVIULCGA operators satisfy the commutativity, idempotency and boundedness.

3.6 Induced IL fuzzy AOs

Now, a type of induced AOs has been a hot topic in a lot of research literatures, which take criteria as pairs, in which the first element denoted order induced variable is used to induce an ordering over the second element which is the aggregated variables. Illuminated by Xu's work (Xu, 2006; Xu and Xia, 2011; Xu, 2007; Xian and Xue, 2015) introduced IFLIOWM operator, IFLIOWGM operator.

Definition 26. (Xian and Xue, 2015; Meriglo *et al.*, 2012) Let $\tilde{\varepsilon}_i = \left\langle\left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i))\right\rangle$ $(i = 1,2,\ldots,n)$ be a collection of IULVs and $r = 1, 2, \ldots, n$. The value aggregated by IFLIOWA operator is an IULV, the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, satisfies $\mathrm{w}_i \in [0, 1], \sum_{i=1}^{n} \mathrm{w}_i = 1$, and:

$$\mathrm{IFLIOWA}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$
$$= \left\langle\left[s_{\sum_{i=1}^{n}\mathrm{w}_i\varphi_{\rho_{(i)}}}, s_{\sum_{i=1}^{n}\mathrm{w}_i\vartheta_{\rho_{(i)}}}\right], \left(1-\prod_{i=1}^{n}\left(1-u_{\tilde{\varepsilon}_{\rho(i)}}\right)^{\mathrm{w}_i}, \prod_{i=1}^{n}\left(v_{\tilde{\varepsilon}_{\rho(i)}}\right)^{\mathrm{w}_i}\right)\right\rangle, \tag{65}$$

where $\tilde{\varepsilon}_{\rho(i)} = \left\langle[s_{\varphi_{\rho_{(i)}}}, s_{\vartheta_{\rho_{(i)}}}], \left(u_{\tilde{\varepsilon}_{\rho(i)}}, v_{\tilde{\varepsilon}_{\rho(i)}}\right)\right\rangle$, ρ: $(1, 2, \ldots, n) \rightarrow (1, 2, \ldots, n)$is a permutation.

Definition 27. (Xian and Xue, 2015; Meriglo *et al.*, 2012) Let $\tilde{\varepsilon}_i = \left\langle\left[s_{\varphi(\varepsilon_i)}, s_{\vartheta(\varepsilon_i)}\right], (u(\varepsilon_i), v(\varepsilon_i))\right\rangle$ $(i = 1,2,\ldots,n)$ be a collection of IULVs and $r = 1, 2, \ldots, n$. The value aggregated by IFLIOWGA operator is an IULV, the weighted vector of $\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n$ is $\mathrm{w} = (\mathrm{w}_1, \mathrm{w}_2, \ldots, \mathrm{w}_n)^T$, satisfies $\mathrm{w}_i \in [0, 1], \sum_{i=1}^{n} \mathrm{w}_i = 1$, and:

$$\mathrm{IFLIOWGA}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots, \tilde{\varepsilon}_n)$$
$$= \left\langle\left[s_{\prod_{i=1}^{n}\left(\varphi_{\rho_{(i)}}\right)^{\mathrm{w}_i}}, s_{\prod_{i=1}^{n}\left(\vartheta_{\rho_{(i)}}\right)^{\mathrm{w}_i}}\right], \left(\prod_{i=1}^{n}\left(u_{\tilde{\varepsilon}_{\rho(i)}}\right)^{\mathrm{w}_i}, 1-\prod_{i=1}^{n}\left(1-v_{\tilde{\varepsilon}_{\rho(i)}}\right)^{\mathrm{w}_i}\right)\right\rangle, \tag{66}$$

where $\tilde{\varepsilon}_{\rho(i)} = \left\langle[s_{\varphi_{\rho_{(i)}}}, s_{\vartheta_{\rho_{(i)}}}], \left(u_{\tilde{\varepsilon}_{\rho(i)}}, v_{\tilde{\varepsilon}_{\rho(i)}}\right)\right\rangle$, ρ: $(1, 2, \ldots, n) \rightarrow (1, 2, \ldots, n)$ is a permutation.

The IFLIOWA and IFLIOWGA operators satisfy the commutativity, idempotency, monotonicity and boundedness.

More specifically, Meriglo *et al.* developed two new induced operators of IULVs, such as weighted intuitionistic linguistic induced ordered mean operator and generalized weighted intuitionistic linguistic induced ordered mean operator. Xian *et al.* (2018) proposed a generalized IVIULV induced hybrid aggregation (GIVIULIHG) operator with entropic order inducing variable and TOPSIS approach by redefining IULVs.

4. The applications about the AOs of IULVs

In this section, we give an overview of some practical applications of the IULVs AO and approach in the domain of different types of MCDM and MCGDM. Based on IULWGM, OIULWGM, GIULWGM, GOIULWGM, IULBOWM, WIULBOWM, IULAHM, IULGHM, WIULAHM, WIULGHM, IULMSM, WIULMSM, GILDOWM and GILDHWM operator and so on, the corresponding MCDM or MCGDM methods were developed to solve the real MCDM or MCGDM problems, such as human resource management, supply-chain management, project investment (PI) and benefit evaluation:

(1) PI.

Liu and Jin (2012) applied the MCDM methods based on IULHG, WIULGA and WIULOG operators to solve investment problems, in which an investment company wants to invest a sum of money in the best selection. Liu and Wang (2014) developed MCGDM methods based on GWILPA and GWILPOA operators to deal with investment evaluate problems. Wang *et al.* (2014) proposed a MCGDM approach based on the ILHA and WILAA operator to disposal MCGDM problem involving a PI. Wang *et al.* (2015) proposed the weighted trapezium cloud arithmetic mean operator, ordered weighted trapezium cloud arithmetic mean operator and the trapezium cloud hybrid arithmetic operator, and then used them to solve PI problems. Xian *et al.* (2018) gave a real example about selecting the best investment strategy for an investment company by applying GIVIFLIHA operator to aggregate IVIFLVs. Yu *et al.* (2018) gave an illustrated example about investment selection by developing IU2TL continuous extend BM (IU2TLCEBM) operator. Xia *et al.* (2017) presented a novel IFL hybrid aggregation operator to deal with an investment risk evaluation problem in the circumstance of IFLI.

(2) Suppler selection.

In many literature, researchers have attempted to dispose the suppler selection problems by using the AOs to aggregate intuitionistic linguistic fuzzy information (ILFI). For example, Liu and Chen (2018) presented a MAGDM method based on I2LGA by extending the Archimedean TN and TC to select the best suppler for manufacturing company' core competition. Krishankumar *et al.* (2017) applied a novel approach based on IL AOs to select the best applier from the four potential suppliers. Wang *et al.* (2017) developed an IVIFLI-MCGDM approach based on the IV2TLI and applied it to the practice problem about a purchasing department want to select a best supplier. Liu *et al.* (2017) presented an IL multiple attribute decision making with ILWIOWA and ILGWIOWA operator and its application to low carbon supplier selection.

(3) Some other applications.

Zhang *et al.* (2017) gave two IL MCDM based on HM approaches and their application to evaluation of scientific research capacity. Imanov *et al.* (2017) analyzed thoroughly the impact of external elements to economic state, social consequences and government responses by applying IFLI. Beg and Rashid (2016) built an I2TLI model to solve the problem about a family to purchase a house in best locality. Kan *et al.* (2016) presented an approach based on induced IVIULOWG operator to evaluating the knowledge management performance with IVIULFI. Wan (2016) built a model for evaluating the design patterns of the Micro-Air vehicle under interval-valued intuitionistic uncertain linguistic environment.

5. Further research directions

Although the approach and theory of IUL have gained abundant research achievements, a number of works on IUL fuzzy information should be further done in the future.

First, some new operational rules, such as Einstein and interactive operational rule (Zhao and Wei, 2013), Schweizer – Sklar TC and TN (Liu and Wang, 2018), Dombi operations (Liu *et al.*, 2018), Frank TC and TN (Tang *et al.*, 2018), Archimedean TC and TN (Xia, 2017) and so on, should be extended and applied in the process of aggregation of ILFI.

Moreover, some other AOs, such as cloud distance operators (Yu and Liao, 2016), prioritized weighted mean operator (Garg and Arora, 2018), geometric prioritized weighted mean operator (Liu and Liu, 2018), power generalized AO, evidential power AO (Jiang and Wei, 2018), induced OWA Minkowski distance operator (Liu and Teng, 2018a), continuous OWGA operator (Rashid *et al.*, 2018), Muirhead mean operator, and so on should be developed to aggregation ILFI.

Finally, the applications in some real and practical fields, such as online comment analysis, smart home, Internet of Things, precision medicine and Big Data, internet bots, unmanned aircraft, software robots, virtual reality and so on, are also very interesting, meaningful and significance in the future. After doing so, we will propose a much more complete and comprehensive theory knowledge system of ILFI.

6. Conclusions

IULVs, characterized by linguistic terms and IFSs, can more detailed and comprehensively express the criteria values in the process of MCDM and MCGDM. Therefore, lots of researchers pay more and more attention to the MCDM or MCGDM methods with IULVs. In this paper, we primarily give an overview of AOs of ILFI. First, some meaningful AOs have been discussed. Then, we summarize and analyze the applications about the AOs of IULVs. Finally, we point out some possible directions for future research.

Acknowledgments

This paper is supported by the National Natural Science Foundation of China (Nos 71771140 and 71471172), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), Shandong Provincial Social Science Planning Project (Nos 17BGLJ04,16CGLJ31 and 16CKJJ27), the Natural Science Foundation of Shandong Province (No. ZR2017MG007), Key Research and Development Program of Shandong Province (No. 2016GNC110016) and the Science Research Foundation of Heze University (No. XY16SK02).

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Further reading

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