

# Financing public transport: a spatial model based on city size

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## Abstract

**Purpose** – The purpose of this paper is to investigate the relationships between public transport services and the financial needs. Cities require to be equipped with public transport networks as they are primarily responsible for creation of wealth for countries and to ensure sustainability of urbanization. Once decisions have been taken to design, build and operate such networks, it is equally important to set rules for urban transport financing. Depending on the city size and other factors, authorities allocate resources. Nonetheless, is there a relationship between the size of the city and its public transport financial needs? This paper develops a model to explain such relationships.

**Design/methodology/approach** – The study develops a spatial model, while providing intuition through the use of graphs, to solve the question of the amount of resources allocated for financing the transport services.

**Findings** – It is verified that those financial needs are more than proportional to the size of the city; when a city grows in its number of boroughs, economic funds needed to support public transport have to increase in a greater proportion in comparison to the growth of boroughs growth. The model states a formula valid for explaining the financial needs.

**Originality/value** – The model is interesting as it explains why large metropolitan areas need special financial aid from authorities. Real life shows that big cities like Paris, Berlin or Madrid need extraordinary funds for this purpose, and in most of the cases, specific national laws are required for financing public transport networks in these large metropolitan areas.

**Keywords** Public transport, Finance, Subsidies, Transport infrastructures, Spatial model

**Paper type** Research paper

## 1. Introduction

There is no doubt cities require to be equipped with public transport networks as they are primarily responsible for creation of wealth for countries. Public transport improvements in any city enable the growth and densification of urban spaces. Therefore, it is necessary to develop infrastructures and urban transport services to solve mobility problems, especially to ensure sustainability (Banister, 2005), and in line with this principle, it is essential to include sustainability as a first-class variable in the planning and development of transport infrastructures (TRB, 2004). Once decisions have been taken to design, build and operate such networks, it is equally important to set rules for urban transport financing and make it likewise sustainable in the economic sphere, as financial sustainability has often been neglected (Buehler and Pucher, 2011). However, the following questions may arise: how many public transport services do cities require for them to be considered well connected? Are urban transport services' financial needs proportional to the number of cities' inhabitants? Is there any relationship between those financial needs and the size of the city?

Clearly the need to finance urban public transport is not alien to any country, including the developing countries. Among other reasons to finance it, one is citizens' demand it as a



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high priority, and as one of basic public services, authorities are compelled to provide. It is true that depending on the country, region or even city, public transport is financed in one way or another. There are countries where mobility policy consists in the provision of a very high percentage of the total cost towards transport network services. Contrary to that, other countries show little interest in financing buses, subways and trams using part of their public budget. But in every or almost every big city and metropolitan areas worldwide, there is to a greater or lesser extent a clear commitment towards modern transport systems, and accordingly, to cover part of the operating costs through subsidies. Moreover, the question that always arises is what percentage of urban transport costs should be subsidised or if those subsidies should be increased (Tscharaktschiew and Hirte, 2011).

The reason is simple: urban mobility has special social connotations, especially redistributing wealth, but it is also important that authorities be aware that the competitiveness of cities depends on the movement of number of inhabitants in the best possible way. In fact, the cities that do not anticipate urban growth and do not endow municipalities of the necessary transport infrastructures end up suffering traffic crashes that result in noticeable loss of productivity. Therefore, in one way or another, public authorities end up implementing certain approaches: first, setting up enough transport networks for providing services to the citizenship, and second, those networks to be financed in a certain way, i.e., a specific percentage of the cost subsidized by public funds, supplemented with fares paid by users.

Studies and research related to public transport subsidies have traditionally been focussed on two main aspects: the spatial network model the authorities have decided to implement and the effect subsidies exercise (Mohring, 1972; Martin, 2001; Parry and Small, 2009), or models which, in addition to taking into account the network concept, focus on aspects such as the social impact derived from transport subsidies in terms of its benefits to society and its impact on urban development (Vickrey, 1980; Zenou, 2000; Van Dender, 2003; Brueckner, 2005; Borck and Wrede, 2005, 2009; Su and DeSalvo, 2008). In summary, the bibliography regarding the effect of subsidies on public transport and financing is very diverse. Congestion is, undoubtedly, the externality caused by urban transportation and has attracted the attention of engineers and economists. Obviously, in most cities, people have a choice between using a car or public transportation, but authorities try to reduce the use of private cars by promoting and expanding the public transport networks, and consequently, financing them.

The objectives of this paper are the following. First, to propose a simple tractable way to study analytically how the level of resources grows when a city increases the number of boroughs. When that happens, authorities are obliged to provide more transport services, bus lines, for example, and then, economic funds needed to support those public transport lines have to be increased. For that purpose, a model is proposed and solved analytically, while providing intuition through the use of graphs. The second objective of this paper is to obtain explanations about a real fact: those economic funds for public transport grow in a greater proportion in comparison to the growth of boroughs. This evidence is well known in the sector and can be verified by simple comparison between budgets allocated for financing the subsidies in different cities. However, in the literature, this topic has not been considered in depth.

In conclusion, all these objectives are interesting to explain the way in which countries allocate public resources in the budget to finance the public transport, in terms of city size, especially for big cities. The results could be motivating as a topic of discussion not only academically but also professionally. Certainly, the scientific literature is not abundant at a time when the authorities and transport operators need to understand the mechanism for public subsidies in a theoretical way.

## 2. Financing public transport in terms of city size

Over the past two decades, most of the large cities worldwide have improved the quality of their public transport services. Bus lines, bus rapid transit, trams and undergrounds are now very popular, even in developing countries, where authorities try to reduce congestion and increase productivity. Engineering has provided multiple methods to guarantee that the needs of the people are met. However, when a city continues to grow, one of the most interesting issues is from view of economy, i.e., the financial sustainability.

The analysis of how and why public transport systems increased their financial needs is something that can be observed simply by reading the budgets of major cities. Certainly the complex interaction of government policies, agencies measures or simply the trend towards more financial efficiency is changing. For example, in countries like Germany, the approach to a transport system that is financially sustainable is certainly different (Buehler and Pucher, 2011). In any case, the costs to operate the transport networks have increased in the last years, and authorities are interested to understand such increases. Probably we are facing a new social phenomenon, and the financial sustainability of urban transport must be at the same level as city planning. As a matter of fact, it could be thought that the level of resources to finance city urban transport depends directly on the number of its inhabitants: the fewer the less and vice versa.

All this matter is especially interesting in big cities, due to the complexity of the networks, reaching extremely high levels of financing resources. For example, in Spain, the two big cities, Madrid and Barcelona, due to the increase of transport services and the global amount of financial resources to be allocated, authorities decided to provide them with a special status (Ruiz, 2014). By means of that agreement, those cities receive from the state an impressive sum, that is four times bigger than the total amount assigned for the rest of the country (Table I).

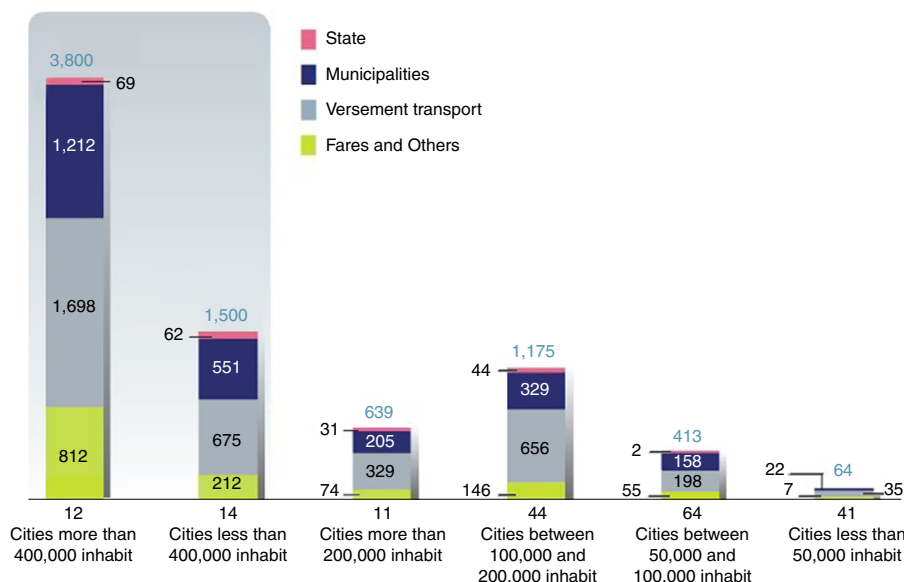
According to the system established by the Ministry of Economy, the subsidies provided by the State for the public transport in Spain are mainly assigned to the two main cities, while the rest, 89 cities, receives only 17.3 per cent of the total budget. As it is shown, there is no relationship between the subsidy per inhabitant and the size of the city.

Another example is France, where the "Île de France", the Parisian region, through different methods and taxes, like the *versement transport*, receives impressive figures for financing the public transport, undoubtedly higher than other areas in the country[1]. In general terms, the financial resources for the French cities, excluding the capital of the country and its metropolitan area, reach up to 7,591 million euro, as it is shown in Figure 1 (loans not included). That budget, coming from at least four main sources – the French state,

	Number of cities	Inhabitants (millions)	Total subsidies (millions €)	Subsidy/inhabitant
<i>Big cities</i>				
Madrid		6.3	167.5	26.7
Barcelona		4.9	152.0	30.9
Total		11.2	319.5	
<i>Rest of the Spanish cities</i>				
500,000-1 million inhabitants	4	2.7	28.6	10.5
100,000-500,000 inhabitants	36	7.3	31.5	4.3
50,000-100,000 inhabitants	40	2.8	6.1	2.2
20,000-50,000 inhabitants	9	0.3	0.7	2.3
Total	89	13.2	66.9	

**Table I.**  
State subsidies for public transport in Spain

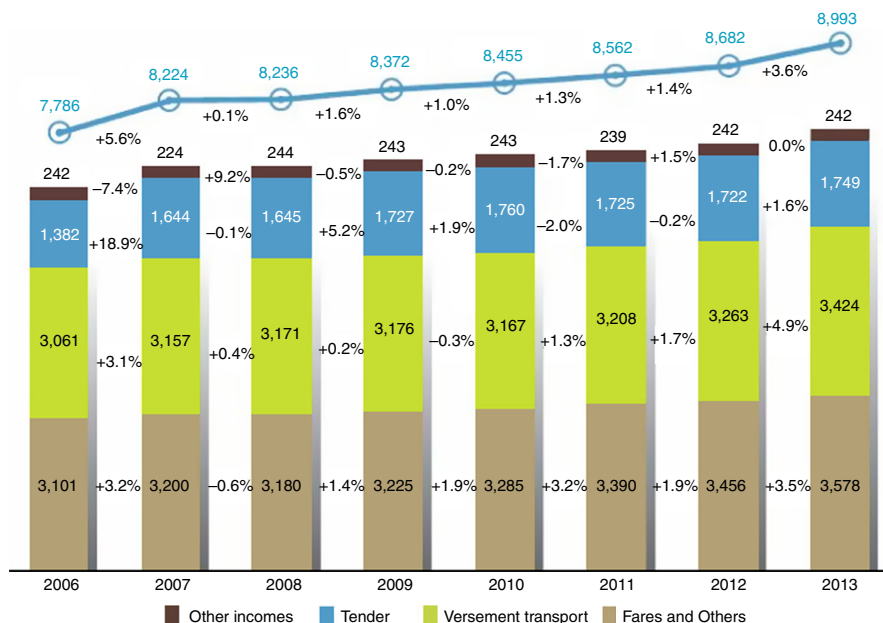
**Source:** Ministry of Economy (2008)



**Figure 1.** Financial resources for public transport in French cities, excluding Paris (2013)

the local governments, the versement transport tax and the fares and others commercial incomes – is distributed among 186 cities.

On the other side, the Parisian region receives almost 9,000 million euro. This metropolitan area includes the city of Paris, the region and seven departments (Figure 2).



**Figure 2.** Financial resources for public transport in the île-de-France (Paris metropolitan area)

As result of these figures, it is easily understood the huge difference when financing the public transport in the Paris metropolitan area and the rest of cities in the country (Table II).

Consequently, the interest of this paper is to analyse why big cities demand such large financial resources, in comparison with medium and small urban places. For that purpose, a simple method is proposed, based on graphs, in order to determine the transport lines needed. The city model, either monocentric or polycentric, could affect the result of studies, but in the model described in this paper, a principle of interconnection is established between all neighbourhoods or districts chosen for the purpose of the calculation of the financing required, as the maximum number of public transport lines a city needs to implement.

Regardless, in agreement with the articles consulted and provided, each city is different; the urban spatial structure studies (Anas *et al.*, 1998) determine that cities are strongly shaped by agglomeration economies, especially external scale economies. Cities teem with positive and negative externalities, all acting with different strengths, among different agents, at different distances. Particularly interesting is debate about transport in monocentric and policentric cities. Subsidies in monocentric small towns could be non-effective because they distort the rational use of land. In fact, in a great number of cities, transport networks may have contributed in making citizens choose the outskirts (suburbs) as an alternative to living in the city centre. However, it should be the opposite. As a city grows it becomes polycentric, with different nuclei of interest citizens wish to visit such as large shopping centres, etc. That is the reason why in this study the question alluding to the maximum number of public transport lines possible among cities interesting points is simplified. Of course, the study can be completed with more inputs that appear in cities. Faster and cheaper travel may change where firms locate, and where people could decide to live. Cities or neighbourhoods that gain higher accessibility may also increase in size and productivity. Thus, a transportation investment can cause a spatial concentration of firms seeking larger market areas that enable the realization of internal economies of scale in production (Chatman and Noland, 2011). Moreover, for households, reduced transport costs could make job searching easier and commuting cheaper, increasing employment participation and hours worked, and again, increasing productivity. Consequently, good transportation may also help cities grow and diversify. Whether improvement in transport does any of these things depends on its spatial, modal, and temporal characteristics, in addition to a number of other economic factors. For example, transportation might increase employment in cities, by increasing firm access to labour and by increasing links between companies (Venables, 2007). There is a substantial empirical literature quantifying the relationship between city size and productivity, but for the purposes of this paper, and for the model proposed, the matter considering the maximum number of transport lines that can operate in a city will be simplified. So, it is possible to determine the upper limit of

France: global financial resources for public transport (million euro)	Paris	Rest of French cities (186)
Paris (Île de France)	8,993	–
12 cities with more than 400,000 inhabitants	–	3,800
14 cities with less than 400,000 inhabitants	–	1,500
11 cities with more than 200,000 inhabitants	–	639
44 cities between 100,000 and 200,000 inhabitants	–	1,175
64 cities between 50,000 and 100,000 inhabitants	–	413
41 cities with less than 50,000 inhabitants	–	64
Total	8,993	7,591

**Table II.**  
Global financing  
resources in  
France 2013

transport infrastructures operating in a city, and consequently, the financial needs, in order to propose a simple tractable way to study analytically how the level of resources grows when a city increases the number of boroughs. Unquestionably, authorities are obliged to provide more and more transport services, and the economic funds needed to support those services have to increase.

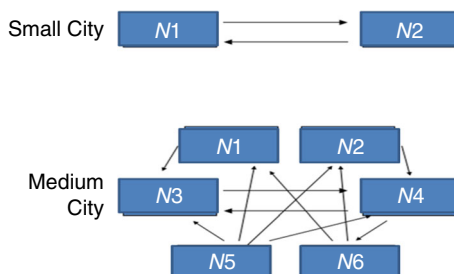
### 3. The proposed spatial model

All transport systems in cities operate as networks. Although we could use more complex theories about networks, the proposed spatial model can explain, while providing intuition through the use of graphs, the question of the amount of resources allocated for financing the services.

To draw conclusions valid to the model proposed, we can begin by assuming that we operate urban transport in a small town with only two districts or boroughs: districts 1 and 2. Considering this, the city should be run only by one urban transport line, with little buses and regular two-way service between the two: districts 1 and 2. This system obviously shows itself as not expensive because it requires little resources and also because of the little demand expected. Rail services such as trams or similar are not to be considered (Figure 3).

If we move to a medium-sized city with six districts, citizens' mobility is very different. As more districts get involved, new needs develop in moving terms among them, all in such a way that citizens in district 1 would like to travel to district 2 or 3, or 4, or 5, or 6. Regarding the other districts, the same will happen considering all possible combinations. Therefore, urban transport network supply becomes more complex with this kind of city. Obviously, we should have more bus lines, at least one per possible combination so districts keep connected in the best way with a network of relationships. It is even possible to sequentially run a circle line among the districts establishing bilateral relations between districts because it is not necessary to go through a third district when travelling between the other two, especially the one considered as central, city centre or down town.

If we consider a medium-sized city with development prospects it is possible to incorporate a sort of tram system or light rail transit besides buses that could add an ability to move for people in those areas with greater flow. This new rail mode would increase financing needs bearing in mind that it is more expensive than those transport systems based on buses. In fact, this is a recently common phenomenon in most cities over 300,000 inhabitants in Spain and Europe. These growth tendency cities due to a dispersed urbanism and with many residential neighbourhoods have chosen trams to provide services between population centres. Curiously, the implementation of these tram systems has increased the financial needs of transport systems as their technical rates and passengers total (real) transport costs are higher than those based on buses.



**Figure 3.**  
Sketch map of  
transport lines  
in towns

Variations of different transport lines from one district to another would be mathematically expressed in the following way:

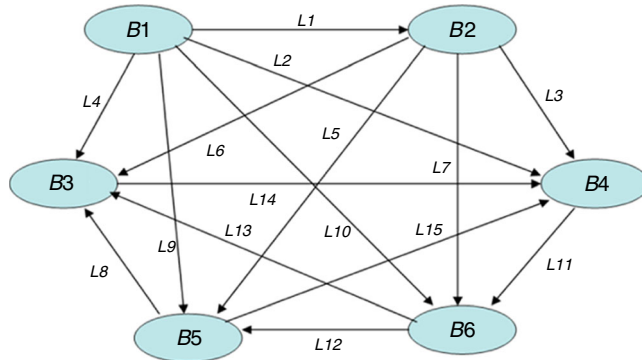
$$P(n, r) = {}^n P_r = \frac{n!}{(n-r)!}$$

To transport people from one neighbourhood to another, we choose the number of districts in the city ( $n$ ) and mobility possible choices ( $r$ ). Of course, this last variable is always  $r=2$ , because between two districts, transport lines have two choices: one-way ticket and return.

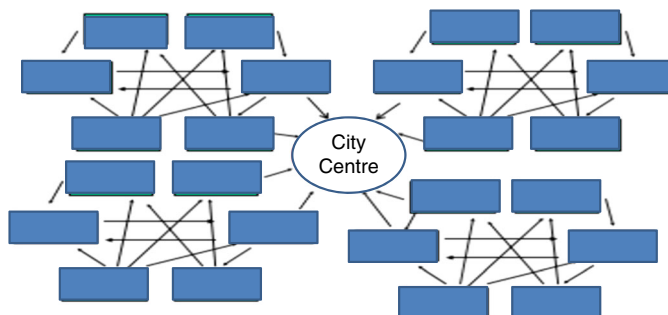
This simple formula would give the variations related to the number of lines a city would require to fully meet the needs of mobility of its inhabitants from one neighbourhood to another. For example, in small towns as mentioned above, with only two districts:  $P(2, 2) = 2!/(2-2)! = 2$ . Logically, to communicate two neighbourhoods, there are two transport streams that meet: "district 1" to "2", and neighbourhood "2" to "1". This is accomplished using a single back and forth bus line. For that reason, the result should always be divided by 2. Total number of lines between two neighbourhoods  $L(2) = P(2, 2)/2 = 2/2 = 1$ . In the following six districts city example,  $P(6, 2) = 6!/(6-2)! = 6!/4! = 6 \times 5 = 30$ , and the number of lines among six neighbourhoods:  $L(6) = P(6, 2) = 30/2 = 15$ .

That is to say, in a six districts city, if there is a wish to connect all districts among them, 15 bus lines are necessary, as can be seen in Figure 4. If we now find ourselves in a large design city, these characteristics would be represented in Figure 5.

In this figure, we now represent a 24 districts big city with a centre of town that behaves as main point of trips' attraction. Using the formula shown above this "big-sized city", implementation would yield the following results:  $P(25, 2) = 25!/(25-2)! = 600$ . Then, the number of lines would reach a theoretical dimension to serve all districts would be



**Figure 4.**  
Public transport lines for a city with six boroughs



**Figure 5.**  
Public transport lines for a large city

$L(25) = 600/2 = 300$ . To express it clearly, this great city of 25 nodes needs 300 public transport lines to cover in full 100 per cent of mobility needs of its inhabitants, being able to transport them directly from one district to another with no transfers. Evidently, with the above formula we have tried to draw an overall conclusion in the mathematical field but far from the usual practice of most municipalities for several reasons. First, because transport operators size is very different from one city to another. That is to say, not all cities of the same size have equal public transport fleets (number of buses, underground lines, trams, etc.). Second, relations between districts develop in an uneven way in those cities with a strong monocentric character in which citizens prefer to go mostly to the same point which is usually the city centre. However, the current trend in developed countries is the “polycentric” model, with new points citizens wish to move to (e.g. macro shopping centres and multimodal transport nodes). Finally, cities solve mobility problems with public transport lines that meet citizens’ demands but in most cases with transfer requirements to other lines. In the foregoing theoretical model it has been considered that all boroughs or districts have direct lines to move to the others, but in reality, it is very different. In most big cities, transfer from one line to another is necessary to reach the chosen destination. Furthermore, transfer is not only necessary in railway systems (undergrounds, trams etc.) but also in city buses. Therefore, most transport networks include transfer free of charge, so that citizens do not have to pay for taking two buses, provided you do so during a certain period of time.

The conclusion we can draw from what has been theoretically considered is that as cities grow public transport systems’ needs become more and more expensive. Financing needs will grow vertiginously as the city grows. It is possible to calculate the sequence as shown in Table III.

Considering the number of transport lines necessary depending on the number of districts the sequence would be as shown in Table IV.

This is a sequence using the terms:  $a_1 = 1, a_2 = 3, a_3 = 6, a_4 = 10, a_5 = 15, a_6 = 21$ . As shown, these series respond to the following pattern: each term is equal to the previous adding the order number. The formula of the sequence that corresponds to the number of lines to be implanted in a city based on the number of districts is as follows:

$$a_n = a_{n-1} + n$$

Number of neighbourhoods	Variations	Number of public transport lines needed
1	No need for public transport	0
2	$P(2, 2) = 2$	$L(2) = 1$
3	$P(3, 2) = 6$	$L(3) = 3$
4	$P(4, 2) = 12$	$L(4) = 6$
5	$P(5, 2) = 20$	$L(5) = 10$
6	$P(6, 2) = 30$	$L(6) = 15$
7	$P(7, 2) = 42$	$L(7) = 21$
8	...	...

**Table III.**  
Transport lines in terms of the number of neighbourhoods

Public transport lines	Term of the sequence
$L(2) = 1$	$a_1$
$L(3) = 3$	$a_2$
$L(4) = 6$	$a_3$
$L(5) = 10$	$a_4$
$L(6) = 15$	$a_5$

**Table IV.**  
Terms of the sequence depending on the number of transport lines



From this, it can be concluded that when in a city the number of districts in need of urban transport services increases, the number of lines to be implemented grows in a greater proportion than the number of districts. It is clear that having a greater number of transport lines funding needs will grow even more. Consequently, financing needs of public transport in cities grow at a higher rate than that of their own neighbourhoods or districts. This makes the funding of transport in large cities reach considerably higher values in comparison to those of medium and small cities.

All this can be represented graphically, as shown in Figure 6. For example, if a city has three districts, we have seen that it needs six transport lines. If a district grows up to four districts, then it will need  $a_n = a_{n-1} + n$ , that is, ten public transport lines. Going from three to four districts, public transport increases from six to ten lines.

In short, large cities require greater financing needs, and they do it at a rate that grows according to the formula shown above.

#### 4. Conclusions

It is certain that public transport improvements lead to benefits to the economy of a country. Although the potential impacts will depend on the specific projects, authorities allocate funds in their budgets for financing the public transport networks. Each country has specific rules for estimating, allocating and distributing the total amount of money for public transport, including different methods for the income of such funds, for example, the case of France, with the *versement transport* tax, or Germany, with the *Mineralölsteuer*. But in all cases, the cost benefits analysis shows that it is expected large effects of investments in big cities, and virtually no effects in smaller cities. Although in fact most of the cities demand public transport services, the result is that buses, trams and metros provide such services to the population in almost all medium and big cities. At the end, authorities dedicate more and more resources, and the financial sustainability of public transport is an important omission in several countries.

In this paper, a model has been formulated to explain why big cities require huge amounts of money for financing public transport, compared to medium or small cities. Although the method is simple, real life shows that big cities like Paris, Berlin or Madrid need extraordinary funds for this purpose, and in most of the cases, specific national laws are required for financing public transport networks in those large metropolitan areas.

The simple tractable way studied analytically in this paper shows that the level of resources grows more than proportionally when a city increases the number of boroughs. When that happens, authorities are obliged to provide more transport services, and then, economic funds needed to support those public transport lines have to increase. As a result,

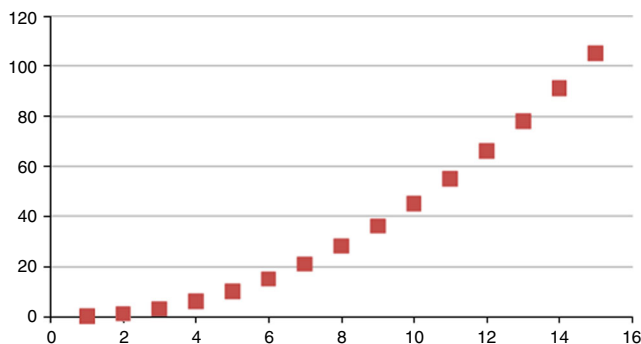


Figure 6.  
Transport lines  
in terms of  
number of boroughs

it is obtained that those economic funds grow in a greater proportion in comparison to boroughs growth itself.

Finally, in the opinion of the author, this analysis opens ways for future research, for several reasons, and at least, two main. First, this is a topic deeply treated in the relationships between transport operators and public authorities. The services required by citizens have no limits, although the public money for paying those services is strictly limited. As an example, in this paper, the cases of Spanish and French cities have been studied. These results are interesting to understand the enormous budgets countries allocate to finance the public transport in big cities that sometimes, like the case of Madrid or Paris, are huge, really higher than the budget for the rest of the country. As this is a matter still not solved, the inquisitiveness of the sector, sometimes large companies operating in several European cities, will attract the attention of universities and academics to study more in deep this issue.

Precisely, the second reason to motivate future research, academically, is the following. There is interest to develop theories in the field of sustainability, and specifically, in the financial sustainability, which is a major topic today, not only in Europe, but worldwide. Increasing public transport's financial sustainability provides great opportunity to employ funds more efficiently. Obviously, sustainable mobility has a central role to play in the future of sustainable cities.

#### Note

1. Figures content in the Annual report by GART: L'année 2013 des transport urbains. GART: Groupement des autorités responsables de transport.

#### References

- Anas, A., Arnott, R. and Small, K.A. (1998), "Urban spatial structure", *Journal of Economic Literature*, Vol. 36 No. 3, pp. 1426-1464.
- Banister, D. (2005), *Unsustainable Transport: City Transport in the New Century*, Routledge, London and New York, NY.
- Borck, R. and Wrede, M. (2005), "Political economy of commuting subsidies", *Journal of Urban Economics*, Vol. 57 No. 3, pp. 478-499.
- Borck, R. and Wrede, M. (2009), "Subsidies for intracity and intercity commuting", *Journal of Urban Economics*, Vol. 63 No. 1, pp. 25-32.
- Brueckner, J.K. (2005), "Transport subsidies, system choice, and urban sprawl", *Regional Science and Urban Economics*, Vol. 35 No. 6, pp. 715-733.
- Buehler, R. and Pucher, J. (2011), "Making public transport financially sustainable", *Transport Policy*, Vol. 18 No. 1, pp. 126-138.
- Chatman, D.G. and Noland, R.B. (2011), "Do public transport improvements increase agglomeration economies? A review of literature and an agenda for research", *Transport Reviews*, Vol. 31 No. 6, pp. 725-742.
- Martin, R.W. (2001), "Spatial mismatch and costly suburban commutes: can commuting subsidies help?", *Urban Studies*, Vol. 38, pp. 1305-1318.
- Ministry of Economy (2008), "Ley 51/2007, de 26 de diciembre, de Presupuestos Generales del Estado".
- Mohring, H. (1972), "Optimization and scale economics in urban bus transportation", *American Economic Review*, Vol. 62 No. 4, pp. 591-604.
- Parry, I.W.H. and Small, K.A. (2009), "Should urban transit subsidies be reduced?", *American Economic Review*, Vol. 92 No. 3, pp. 1276-1289.
- Ruiz, M. (2014), "La financiación del transporte urbano: un reto para las ciudades españolas del siglo XXI", *Investigaciones Europeas de Dirección y Economía de la Empresa*, Vol. 20 No. 1, pp. 1-4.

- 
- Su, Q. and DeSalvo, J.S. (2008), "The effect of transportation subsidies on urban sprawl", *Journal of Regional Science*, Vol. 48 No. 3, pp. 567-594.
- TRB (2004), "Integrating sustainability into surface transportation process. transportation research board, national academies", *Committee for the Conference on Introducing Sustainability into Surface Transportation Planning, Maryland, 11-13 July*.
- Tscharaktschiew, S. and Hirte, G. (2011), "Should subsidies to urban passenger transport be increased? A spatial CGE analysis for a German metropolitan area", *Transport Research*, Vol. 46 No. 2, pp. 285-309.
- Van Dender, K. (2003), "Transport taxes with multiple trip purposes", *Scandinavian Journal of Economics*, Vol. 105 No. 2, pp. 295-310.
- Venables, A.J. (2007), "Evaluating urban transport improvements: cost-benefit analysis in the presence of agglomeration and income taxation", *Journal of Transport Economics and Policy*, Vol. 41 No. 2, p. 173-188.
- Vickrey, W. (1980), "Optimal transit subsidy policy", *Transportation*, Vol. 9 No. 2, pp. 389-409.
- Zenou, Y. (2000), "Urban unemployment, agglomeration and transportation policies", *Journal of Public Economics*, Vol. 77 No. 1, pp. 97-133.

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