Accrual income statements and present value models

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Abstract

Purpose – This paper demonstrates that present value (PV) models can be viewed as multiperiod extensions of accrual income statements (AISs). Failure to include AIS details in PV models may lead to inaccurate estimates of earnings and rates of return on assets and equity and inconsistent rankings of mutually exclusive investments. Finally, this paper points out that rankings based on assets and equity earnings and rates of return need not be consistent, requiring financial managers to consider carefully the questions they expect PV models to answer.

Design/methodology/approach – AISs are used to guide the construction of PV models. Numerical examples illustrate the results. Deductions from AIS definitions demonstrate the potential conflict between asset and equity earnings and rates of return.

Findings – PV models can be viewed as multiperiod extensions of AISs. Mutually exclusive rankings based on assets and equity earnings and rates of return need not be consistent.

Research limitations/implications – PV models are sometimes constructed without the details included in AISs. The result of this simplified approach to PV model construction is that earnings and rates of return may be miscalculated and rankings based as asset and equity earnings and rates of return are inconsistent. Tax adjustments for asset and equity earnings may be miscalculated in applied models.

Practical implications – This paper provides guidelines for properly constructing PV models consistent with AISs.

Social implications – PV models are especially important for small to medium size firms that characterize much of agricultural. Providing a model consistent with AIS construction principles should help financial managers view the linkage between building financial statements and investment analysis.

Originality/value – This is the first paper to develop the idea that the PV model can be viewed as a multiperiod extension of an AIS.

Keywords Cash flow, Accrual income statements, Earnings and rates of return, Operating accounts, Present value models

Paper type Research paper

Introduction

This paper builds present value (PV) models consistent with an accrual income statement (AIS) In the process of developing PV models using an AIS as a guide, this paper makes three contributions. First, it demonstrates how to properly represent the financial characteristics of an investment in PV models. Second, it distinguishes between PV models by associating them with AIS earning and rates of return measures. And third, it clarifies the conditions required for earnings and rates of return on assets and equity to provide consistent rankings. These contributions are intended to help financial managers make better investment decisions.

We organize the remainder of the paper as follows. (1) We review the development and use of PV models. (2) We review AIS earnings and rates of return, note the differences and similarities between AISs and PV models and define PV models as extensions of AISs. (3) From AISs, we derive internal rate of return (IRR) on assets (IRR^A), equity (IRR^E), and after-tax return on equity $IRR^{E(1-T)}$. (4) We construct net present value (NPV) models that correspond to AIS earnings before interest and taxes are paid ($EBIT$), earnings after interest and before taxes are paid (EBT), and net income after interest and taxes are paid ($NIAT$). (5) Finally, we point out that AIS and PV model earnings and rates of return on assets and equity may not provide consistent rankings.

The development of PV models

The development of PV models has a long history, including early work by Stevin (1582) on loans, Wellington (1887) on locating railroads, Fisher (1930) and Grant (1930) on principles of present worth, equivalent annual cost and rates of return; and Boulding (1935) and Samuelson (1937) on the role of IRR versus NPV criteria.

Following a period of limited attention, PV model analysis became popular again in the 1950s following work by Lutz and Lutz (1951) and Dean (1951). Lorie and Savage (1955) identified the problem of multiple IRR values. Hirshleifer (1958, 1970) connected investment and disinvestment decisions and identified three areas of PV model applications: business and capital budgeting, public goods and cost-benefit analysis and national development or growth strategies. Johnson and Quance (1972) called attention to the need for a disinvestment to fund an investment. Perrin (1972) referred to the investment under consideration as a challenger and the investment considered for disinvestment as the defender, an approach adopted here. Since these early developments, the PV literature has become legion. Osborn (2010), Graham and Harvey (2001), Scott and Petty (1984) and a host of other authors have focused on the possible inconsistency between NPV and IRR rankings and how to resolve the possible conflict. One resolution to the ranking conflict focused on reinvesting cash flow, producing a new class of PV models that Lin (1976) and others have referred to as modified PV models. Related to modified PV models, Beaves (1988) and Shull (1994) describe implicit and explicit reinvestment rates. Magni (2013) proposed a weighted average IRR to resolve PV and IRR inconsistencies. Robison et al. (2015) listed homogenous size conditions that would guarantee IRR and NPV ranking consistency.

Recent studies have connected PV models to other disciplines. Magni (2020) linked PV models to accounting, finance and engineering. Robison et al. (2019) connected AIS from accounting to PV models by noting the need to account for changes in operating accounts and liquidations of capital accounts in PV models. This article emphasizes that by paying attention to AIS and PV model connections we can develop more accurate and transparent PV models and better understand the possible conflict in rankings, depending on whether the focus is on invested assets or equity.

AISs and PV models

Accrual income statements

AISs measure asset and equity earnings before and after taxes are paid. In addition, when combined with balance sheet data, AISs can estimate return on assets (ROA), equity (ROE) and after-tax return on equity ($ROE(1-T)$) where T is the average tax rate. AISs measure revenues and expenses when transactions occur, than relying exclusively on when cash payments are processed or received (see Harsh et al., 1981; Lazarus, 1987). To achieve this end, AISs include changes in operating and capital accounts that do not produce cash flow.

PV models defined
This paper defines PV models as multiperiod extensions of AISs. This definition applies because AIS earnings and rates of return have their corresponding measures in PV models. AIS derived *ROA*, *ROE* and *ROE*$(1-T)$ correspond to PV model derived IRR^A, IRR^E and $IRR^{E(1-T)}$. Likewise AIS *EBIT*, *EBT* and *NIAT* measures correspond to NPV for asset earnings (NPV^A), equity earnings (NPV^E) and after-tax equity earnings ($NPV^{E(1-T)}$).

AIS and PV model differences
Despite the correspondence between AISs and PV models, there are some important differences. Consider two. First, AISs are constructed to measure a firm's financial performance. As a result, they are often ex-post in their focus. PV models consider the financial advisability of an investment whose profitability depends on future cash flows. As a result, PV models are often *ex ante* in their focus.

Second, AISs are constructed to measure rates of return and earnings on assets and equity before and after taxes are paid in one period. PV models can be constructed to measure rates of return and earnings on assets and equity before and after tax are paid for investments of several periods. As a result, AISs report earnings at the end of the first period. PV models report the PV of cash flow earned over several periods and the liquidated value of operating and capital accounts at the end of the analysis.

Details included in AISs and PV models
AISs and PV models correspond to and are consistent with each other. This consistency requires that we include the same detail and distinctions in both PV models and AISs. First, we first need to determine if we are investigating return on assets or equity. Second, we need to account for changes in accounts receivable, inventories, accounts payable, accrued liabilities and capital accounts in both AISs as well as in PV models. If we fail to include these details in our PV models, we risk misrepresenting the firm's earnings and rates of return on its investments.

Finally, to calculate rates of return on assets and equity requires asset and equity balances besides earnings data. AISs require beginning assets and equity data from balance sheets. PV models also require assets and equity data to determine how investments are supported.

AIS earnings and rates of return on assets and equity
Earnings and rate of return on assets
AIS earnings and rates of return on a firm's beginning assets equals:

(1) the difference between cash receipts and the sum of cash cost of goods sold (*COGS*) and cash overhead expenses (*OE*) and

(2) changes in the value of the firm's operating and capital accounts.

A numerical example
We illustrate how to find AIS earnings and rates of return on assets using data that describes the fictional firm Hi-Quality Nursery (HQN) described in Robison, Hanson and Black. We report the AIS for HQN in Table 1.

The AIS reported in Table 1 organizes cash flow and changes in operating and capital accounts into total revenue and total expenses and reports the difference as *EBIT*. The *EBIT* calculation for HQN is summarized in equation (1). Total revenue equals cash receipts (*CR*), plus the change in accounts receivable (ΔAR), plus the change in inventory (ΔInv), plus realized capital gains (losses) (*RCG*). Total expenses equal the sum of cash COGS, plus the change in accounts payable (ΔAP), plus the change in cash overhead expenses (ΔOE), plus

Date		2018
"+"	Cash receipts	$38,990
"+"	Change in accounts receivable	($440)
"+"	Change in inventories	$1,450
"+"	Realized capital gains (losses)	$0
Total revenue		$40,000
"+"	Cash cost of goods sold	$27,000
"+"	Change in accounts payable	$1,000
"+"	Cash overhead expenses	$11,078
"+"	Change in accrued liabilities	($78)
"+"	Depreciation	$350
Total expenses		$39,350
Earnings before interest and taxes (EBIT)		$650
"–"	Less interest costs	$480
Earnings before taxes (EBT)		$170
"–"	Less taxes	$68
Net income after taxes (NIAT)		$102
"–"	Less dividends and owner draw	$287
Addition to retained earnings		($185)

Table 1.
HQN's 2018 accrual
income statement

the change in accrued liabilities (ΔAL), plus the change in the book value of capital assets or depreciation (*Dep*). *EBIT* represents HQN's earnings from its beginning assets that include assets supported by its liabilities or debt.

$$EBIT = \text{total revenue} - \text{total expense}$$
$$= (CR + AR + \Delta Inv + RCG) - (COGS + \Delta AP + OE + \Delta AL + Dep)$$
$$= \$40,000 - \$39,350 = \$650 \tag{1}$$

We find HQN's *ROA* by dividing HQN's *EBIT* of $650 by its beginning assets (A_0) of $10,000 reported in Table 2. HQN's *ROA* equals:

$$ROA = \frac{EBIT}{A_0} = \frac{\$650}{\$10,000} = 6.5\% \tag{2}$$

Earnings and rate of return on equity
We find HQN's earnings on its beginning equity by subtracting from *EBIT* interest costs (*Int*) that represent payments for the use of debt and other liabilities and refer to the result as *EBT*, earnings after interest and before taxes are paid. We find *ROE* for HQN by dividing *EBT* by the firm's beginning equity (E_0) of $2,000 reported in Table 2. HQN's *ROE* equals:

$$ROE = \frac{EBIT - Int}{E_0} = \frac{EBT}{E_0} = \frac{\$170}{\$2,000} = 8.5\% \tag{3}$$

Earnings and changes in beginning assets and equity
EBIT and *EBT* calculate changes in the firm's beginning assets and equity respectively. However, these estimates may not equal the actual changes in assets and equity between periods reported in HQN's balance sheets. To explain, the change in equity between periods reported in Table 2 equals ($185), ($1,815 − $2,000). However, this value is not equal to *EBT* of $170 estimated from HQN's AIS in Table 1. The difference between the change in equity and

Date	12/31/2017	12/31/2018
Cash and marketable securities	$930	$600
Accounts receivable	$1,640	$1,200
Inventory	$3,750	$5,200
Notes receivable	$0	$0
Total current assets	$6,320	$7,000
Depreciable assets	$2,990	$2,710
Non-depreciable assets	$690	$690
Total long-term assets	$3,680	$3,400
Total assets	$10,000	$10,400
Notes payable	$1,500	$1,270
Current portion long-term debt	$500	$450
Accounts payable	$3,000	$4,000
Accrued liabilities	$958	$880
Total current liabilities	$5,958	$6,600
Non-current long-term debt	$2,042	$1,985
Total liabilities	$8,000	$8,585
Contributed capital	$1,900	$1,900
Retained earnings	$100	($85)
Total equity	$2,000	$1,815
Total liabilities and equity	$10,000	$10,400

Table 2.
HQN's beginning and
ending balance sheets

EBT can be attributed to sum of taxes paid equal to $68 and owner draw equal to $287. If we subtract taxes and owner draw from *EBT*, we find the change in equity between periods of ($185) equal to the actual change in equity reported in Table 2.

$$\Delta \text{Equity} = EBT - \text{taxes–owner draw} = \$170\text{–}\$68\text{–}\$287 = (\$185) \qquad (4)$$

Table 2 reports a change in HQN's assets of $400, ($10,400−$10,000). Meanwhile, HQN's AIS reports *EBIT* equal to $650. We can explain part of the difference between *EBIT* and the actual change in assets by accounting for interest and taxes paid and owner draw. These describe how operating activities can explain the difference between beginning and ending assets. Then if we add the effect of increased liabilities of $585, ($8,585−$8,000) and the corresponding increase in assets, we explain the discrepancy. We summarize these results in Table 3.

The main point is that while rates of return on assets and equity reflect some changes in beginning assets and equity—they do not necessarily equal the differences between beginning and ending assets and equity reported in balance sheets. Therefore, we cannot measure rates of return on assets and equity as percentage changes in ending and beginning assets and equity reported in balance sheets.

EBIT	$650
−Interest paid	$480
−Taxes paid	$68
−Owner draw	$287
=Change in retained earnings	($185)
+Changes in total liabilities	$585
=Change in total assets	$400

Table 3.
HQN's *EBIT* and
change in assets

IRR models

IRR model definition

To build an *IRR* model, we reorganize an AIS into cash flow and changes in operating and capital accounts. This reorganization allows us to extend an AIS into an *n* period IRR model by separating *n* periods of cash flow from the liquidation of operating and capital accounts in the nth period.

Table 4 divides cash flow into cash receipts (*CR*) and cash expenses (*CE*). *CR* include cash sales from operations, reductions in accounts receivable ($\Delta AR < 0$), reductions in inventories held for sale ($\Delta Inv < 0$) and realized capital gains (*RCG*). *CE* include cash *COGS*, cash *OE*, reductions in accounts payable ($\Delta AP < 0$) and reductions in accrued liabilities ($\Delta AL < 0$).

Table 5 records changes in operating accounts and depreciation. Changes in operating accounts include ΔAR, ΔInv, ΔAP and ΔOE. Note that we include negative changes in operating accounts that produce *CR* and *CE* cash flow. We include changes in operating accounts regardless of their sign in Table 5 to assure that we are measuring returns and expenses when they occur.

To summarize the calculations included in Tables 4 and 5 we express HQN's EBIT as the sum of cash flow and changes in operating and capital accounts:

$$EBIT = (CR - CE) + (\Delta AR + \Delta INV - \Delta AP - \Delta AL - Dep) = \$912 + (\$262) = \$650 \tag{5}$$

Notice that the sum of cash flow (*CR–CE*) recorded in Table 4 of \$912 plus changes in operating and capital accounts ($\Delta AR + \Delta INV - \Delta AP - \Delta AL - Dep$) recorded in Table 5 of (\$262) equal EBIT of \$650 reported in Table 1. Were the capital assets sold and their liquidation value not equal to their book value, the difference in capital accounts would be recorded as realized capital gains or losses (*RCG*) and included in our cash flow measure.

Finally, the *EBIT* estimate of change in assets minus interest costs equals *EBT*, the estimate of HQN's change in equity:

$$EBT = EBIT - Int = \$650 - \$480 = \$170 \tag{6a}$$

+	Cash receipts from operations (*CR*)	\$38,990
+	Realized capital gains (*RCG*)	\$0
=	*Cash receipts (CR)*	*\$38,990*
+	Cash cost of goods sold (*COGS*)	\$27,000
+	Cash operating expenses (*OE*)	\$11,078
=	*Cash expenses (CE)*	*\$38,078*
	CR–CE	*\$912*

Table 4.
HQN 2018 cash flow (cash receipts minus cash expenses)

+	Change in accounts receivable (ΔAR)	(\$440)
+	Change in inventories (ΔInv)	\$1,450
−	Change in accounts payable (ΔAP)	\$1,000
−	Change in accrued liabilities (ΔAL)	(\$78)
=	*Changes in operating accounts*	*\$88*
−	Depreciation (*Dep*)	\$350
=	*Changes in capital accounts*	*\$350*
=	*Changes in operating and capital accounts*	*(\$262)*

Table 5.
HQN 2018 changes in operating and capital accounts

IRR^A models

Single period IRRA models

We found ROA and ROE from an AIS by dividing $EBIT$ and EBT by beginning assets A_0 and equity E_0 respectively. We follow a similar procedure when we build PV models. We must account for the beginning value of assets and equity as well as relevant changes in their ending values, including only those changes that affect $EBIT$ or EBT. We are not interested in explaining total changes in equity and assets over the periods of analysis, but only those changes that we can attribute to operating, investing, and financing activities. To that end, we rearrange equation (2) and write:

$$A_0 ROA = EBIT \tag{6b}$$

Now suppose that we add A_0 to both sides of equation (6b) and after factoring, divide both sides of equation (6b) by $(1+ROA)$ to obtain:

$$A_0 = \frac{A_0 + EBIT}{(1+ROA)} = \frac{A_0 + (CR_1 - CE_1) + (\Delta AR_1 + \Delta Inv_1 - \Delta AP_1 - \Delta AL_1 - Dep_1)}{(1+ROA)} \tag{7}$$

We simplify equation (7) by substituting for A_0, the value of capital accounts V_0 plus the value of current asset accounts AR_0 and Inv_0 plus beginning cash balance Csh_0.

$$A_0 = \frac{V_0 + AR_0 + Inv_0 + Csh_0 + (CR_1 - CE_1) + (\Delta AR_1 + \Delta Inv_1 - \Delta AP_1 - \Delta AL_1 - Dep_1)}{(1+ROA)} \tag{8}$$

We simplify equation (8) still more by recognizing that the value of capital assets V_0 less depreciation, Dep_1, equals the book value of capital assets V_1^{book} at the end of the period. However, if the capital assets are actually liquidated, then the liquidation value of capital assets can be written as $V_1^{liquidation} = V_0^{book} - Dep_1 + RCG$. Furthermore, $AR_0 + \Delta AR_1 = AR_1$, and $Inv_0 + \Delta Inv_1 = Inv_1$. Now we can rewrite equation (8) as:

$$A_0 = \frac{V_1^{liquidation} + AR_1 + Inv_1 + Csh_0 + (CR_1 - CE_1) - (\Delta AP_1 + \Delta AL_1)}{(1+ROA)} \tag{9}$$

Multiperiod IRR^A models

To write the multiperiod equivalent of equation (9), we allow time subscripts to range over $t = 1, \ldots, n$ periods. To convert cash flow and liquidated values of noncash operating and capital accounts to their PV, we discount them by $(1+ROA)$. However, the discount rate in the multiperiod equation is not the ROA derived from the one-period AIS but IRR^A, a multiperiod average internal rate of return on assets, that we substitute for ROA. We summarize our results in equation (10):

$$
\begin{aligned}
A_0 &= V_0 + AR_0 + Inv_0 + Csh_0 \\
&= \frac{CR_1 - CE1}{(1+IRR^A)} + \cdots + \frac{CR_n - CE_n}{(1+IRR^A)^n} \\
&\quad + \frac{V_n^{liquidation} + AR_n + Inv_n + Csh_0 - (AP_n - AP_0) - (AL_n - AL_0)}{(1+IRR^A)^n}
\end{aligned} \tag{10}
$$

To demonstrate equation (10) with data from HQN, we set $n = 1$, replace IRR^A with ROA and write:

$$A_0 = \frac{CR_1 - CE_1}{(1 + ROA)} + \frac{V_1^{\text{liquidation}} + (AR_1 + Inv_1 + Csh_0) - (AP_1 - AP_0) - (AL_1 - AL_0)}{(1 + ROA)}$$

$$= \frac{\$38,990 - \$38,078}{(1.065)} + \frac{(\$3,400 - \$70) + (\$1,200 + \$5,200 + \$930) - \$1,000 - \$78)}{(1.065)}$$

$$= \frac{\$10,650}{1.065} = \$10,000$$

(11)

To explain equation (11), we compare the result with HQN's AIS. We observe CR_1 less CE_1 ($COGS + OE$) produces $\$38,990 - \$38,078 = \$912$ (see Table 1). Ending period long-term assets (LTA) equal $\$3,400$ (see Table 2) from which we subtract purchases minus sales of LTA ($\$100 - \$30 = \$70$). Ending account balances $AR_1 + Inv_1$ equal $\$1,200 + \$5,200$, and the beginning cash balance is $\$930$. Next, we subtract changes in accounts payable of $\$1,000$ and changes in accrued liabilities of ($\$78$).

IRR^E models
We computed ROE by subtracting from $EBIT$ interest paid for the use of debt and divided the result by beginning equity, E_0. To find the multiperiod IRR for equity, $IRRE$, we subtract in each period t interest cost iD_{t-1} where D_{t-1} equals the firm's debt at the end of the previous period and i equals the average cost of debt. To find the amount of equity invested, we subtract from initial assets initial debt D_0. Outstanding debt during the period of analysis collects interest. No changes in debt occur in the last period and debt at the end of the $t-1$st period, $Dn-1$, is retired in the last period. Revising equation (10) to account for interest costs and debt and replacing ROE with IRR^E, we can find the multiperiod equivalent of ROE. We write:

$$E_0 = V_0 + AR_0 + Inv_0 + Csh_0 - D_0$$
$$= \frac{(CR_1 - CE_1 - iD_0)}{(1 + IRR^E)} + \cdots + \frac{(CR_n - CE_n - iD_{n-1})}{(1 + IRR^E)^n}$$
$$+ \frac{V_n^{\text{liquidation}} + AR_n + Inv_n + Csh_0 - (AP_n - AP_0) - (AL_n - AL_0) - D_0}{(1 + IRR^E)^n} \quad (12)$$

To illustrate equation (12) with data from HQN, we set $n = 1$, replace IRR^E with ROA and write:

$$E_0 = \frac{CR_1 - CE_1 - iD_0}{(1 + ROE)}$$
$$+ \frac{V_1^{\text{liquidation}} + AR_1 + Inv_1 + Csh_0 - (AP_1 - AP_0) - (AL_1 - AL_0) - D_0}{(1 + ROE)}$$
$$= \frac{\$38,990 - \$38,078 - \$480}{(1.085)} + \frac{(\$3,400 - \$70) + \$1,200 + \$5,200 + \$930}{(1.085)}$$
$$- \frac{\$1,000 + (\$78) + \$8,000}{(1.085)}$$
$$= \frac{\$2,170}{1.085} = \$2,000$$

(13)

Intertemporal investments and borrowings

The multiperiod IRR^A and IRR^E models described in equations (10) and (12) follow AIS construction principles. Investing and borrowing recorded in AISs occur at the beginning of the period. Liquidation of investments and repayments occur at the end of the period. When we extend single period AISs to multiperiod PV models, we must allow for multiperiod repayments and borrowings and investing and disinvesting. We can easily extend equations (10) and (12) to account for these possibilities. However, wanting to maintain the focus of this paper on AIS and PV model connections, we omit these complications for now.

AIS earnings and NPV models

In the previous sections we derived multiperiod IRR^A and IRR^E that correspond to ROA and ROE derived from a one period AIS and beginning assets and equity. Now we introduce multiperiod NPV models that correspond to one period AIS earnings, $EBIT$, EBT and $NIAT$. We begin by emphasizing the main difference between IRR and NPV models. IRR models find the rate of return earned by the investment or equity supporting the investment. NPV models measure the earnings realized by transferring funds from a defending investment, the investment in place, to a challenging investment, the investment being considered to replace the defending investment. Thus, NPV models convert multiperiod future cash flow and changes in operating and capital accounts from a challenging investment for present dollars at the rate of one plus the defender's IRR, $(1+IRR)$.

EBIT and NPV^A earnings on assets

We write NPV for asset earnings by rearranging equation (10) and by reinterpreting IRR^A as the internal rate of return on a defending investment.

$$NPV^A = -A_0 + \frac{CR_1 - CE_1}{(1 + IRR^A)} + \cdots + \frac{CR_n - CE_n}{(1 + IRR^A)^n}$$
$$+ \frac{V_n^{\text{liquidation}} + AR_n + Inv_n + Csh_0 - (AP_n - AP_0) - (AL_n - AL_0)}{(1 + IRR^A)^n} \quad (14)$$

To demonstrate equation (14) with HQN data, we set $n = 1$, replace IRR^A with ROA, and write:

$$NPV^A = -A_0 + \frac{CR_1 - CE_1}{(1 + ROA)}$$
$$+ \frac{V_1^{\text{liquidation}} + AR_1 + Inv_1 + Csh_0 - (AP_1 - AP_0) - (AL_1 - AL_0)}{(1 + ROA)}$$
$$= -\$10,000 + \frac{\$38,990 - \$38,078}{(1.065)} + \frac{(\$3,400 - \$70) + \$1,200 + \$5,200 + \$930}{(1.065)}$$
$$- \frac{\$1,000 + (\$78)}{(1.065)}$$
$$= -\$10,000 + \frac{\$10,650}{1.065} = \$0$$
$$(15)$$

Notice that the NPV^A after exchanging funds from a defender with an identical challenger is zero. But if we found NPV at the end of one period, $(IRR^A)(A_0)$, the product would equal $EBIT$: $(0.065)(\$10,000) = \$650$ (see equation 6). These results emphasize that one important difference between AIS and NPV earnings is that AISs value earnings at the end of the period while PV models value earnings in the present. NPVs value earnings in the present because

the present is where we live and make decisions. It should be obviously that if the defender's *IRR* were not equal to the challenger's *IRR*, then NPV^A would not equal zero. For example, suppose that in equation (15), the defender's *IRR* were 6%. Then NPV^A would equal \$47.17.

EBT *and* NPV^E *earnings on equity*
We write the *NPV* for equity earnings by rearranging equation (14) and by recognizing that IRR^E is the internal rate of return on equity for a defending investment.

$$NPV^E = -(A_0 - D_0)$$
$$= -(V_0 + AR_0 + Inv_0 + Csh_0) + D_0 + \frac{(CR_1 - CE_1 - iD_0)}{(1 + IRR^E)} + \cdots$$
$$+ \frac{(CR_n - CE_n - iD_{n-1})}{(1 + IRR^E)^n}$$
$$+ \frac{V_n^{\text{liquidation}} + AR_n + Inv_n + Csh_0 - (AP_n - AP_0) - (AL_n - AL_0) - D_0}{(1 + IRR^E)^n} \quad (16)$$

To illustrate equation (16) with data from HQN, we set $n = 1$, replace IRR^E with *ROE*, and write:

$$NPV^E = -E_0 + \frac{CR_1 - CE_1 - iD_0}{(1 + ROE)}$$
$$+ \frac{V_1^{\text{liquidation}} + AR_1 + Inv_1 + Csh_0 - (AP_1 - AP_0) - (AL_1 - AL_0) - D_0}{(1 + ROE)}$$
$$= -\$2,000 + \frac{\$38,990 - \$38,078 - \$480}{(1.085)} + \frac{(\$3,400 - \$70) + \$6,400 + \$930}{(1.085)}$$
$$- \frac{\$1,000 + (\$78) + \$8,000}{(1.085)}$$
$$= -\$2,000 + \frac{\$2,170}{1.085} = \$0 \quad (17)$$

Like the results obtained for NPV^A, NPV^E from exchanging funds from a defender with an identical challenger is zero. But if we found NPV^E at the end of one period, $(IRR^E) \times (E_0)$, the product would equal *EBT*: $(0.085) \times (\$2,0000) = \$170$ (see equation 3). It should be obviously that if the defender's *IRR* were not equal to the challenger's *IRR*, that NPV^E would not equal zero. For example, suppose that in equation (17), the defender's *IRR* were 8%. Then NPV^E would equal \$9.26.

After-tax *ROE* and *ROA*
PV models often focus on after-tax cash flow because it represents what firms/investors keep after paying all their expenses including taxes. In what follows we present tax obligations in a simplified form to illustrate their impact on earnings and rates of return. Our goal is to find the average tax rate T that adjusts *ROE* to *ROE*$(1–T)$ and T^* that adjusts *ROA* to *ROA*$(1–T^*)$. We do not try to duplicate the complicated processes followed by taxing authorities to find T and T^*. Instead we suggest that the firm pays an average tax rate T or T^* on *EBT* and *EBIT* respectively.

AISs report taxes paid by the firm and subtracts them from *EBT* to obtain *NIAT*. We calculate interest costs by multiplying the average interest rate i times beginning period debt

D_{t-1} (iD_{t-1}) and subtract them from earnings to reduce tax obligations. As a result, $NIAT$ represents changes in equity after both interest and taxes have been paid. In 2018, HQN paid \$68 in taxes. To find the average tax rate HQN paid on its changes in equity we set taxes equal to the average tax rate T times EBT:

$$\text{Taxes} = (T)(EBT) = \$68 \tag{18}$$

Solving for the average tax rate T that HQN paid on its earnings we find:

$$T = \frac{\text{Taxes}}{EBT} = \frac{\$68}{\$170} = 40\% \tag{19}$$

Finally, we adjust ROE for taxes and find HQN's after-tax ROE to be:

$$ROE(1 - T) = \frac{NIAT}{E_0} = \frac{\$102}{\$2,000} = 5.1\% \tag{20}$$

AISs and after-tax ROA*s*
An AIS computes taxes paid by the firm on its return to equity but not on its return to assets. They record only one value for taxes paid and these estimates account for tax saving resulting from interest payments. As a result, we cannot use the average tax rate T calculated for taxes paid on equity earnings to adjust ROA for taxes. To find the average tax rate T^* that adjusts ROA to $ROA(1-T^*)$, we calculate taxes "as if" there were no interest costs to reduce the average tax rate to T^*. We find $ROA(1-T^*)$ in equation (21) as:

$$ROA(1 - T^*) = \frac{(EBIT - \text{Taxes})}{A_0} = \frac{(\$650 - \$68)}{\$10,000} = 5.8\% \tag{21}$$

Solving for T^* we find:

$$T^* = 1 - \frac{(EBIT - \text{Taxes})}{EBIT} = 1 - \frac{\$650 - \$68}{\$650} = 10.5\% \tag{22}$$

Equation (22) emphasizes an important point: adjusting ROE and ROA for taxes nearly always requires different average tax rates. The only time that $T = T^*$ is when interest costs are zero. In that case, we can easily demonstrate that $T^* = T$ since $EBIT = EBT$:

$$T = T^* = \frac{\text{Taxes}}{EBIT} = \frac{\text{Taxes}}{EBT} = 10.5\% \tag{23}$$

After-tax multiperiod *IRRE*(1−*T*) model
We are now prepared to introduce taxes into the IRR^E model described in equation (12). We begin by solving for $NIAT$ in equation (20) and replacing $ROE(1-T)$ with $IRR^E(1-T)$:

$$NIAT = E_0 ROE(1 - T) = E_0 IRR^E(1 - T) \tag{24}$$

Next, we write $NIAT$ as $EBIT$ adjusted for both interest costs and taxes:

$$NIAT = (EBIT - Int)(1 - T) \tag{25}$$

Then, we substitute for $EBIT$ the right-hand side of equation (5) and for $NIAT$, the right-hand side of equation (23) and add time subscripts. The result is equation (26).

$$E_0 ROE(1 - T) = [(CR_1 - CE_1 - Int_1) + (\Delta AR_1 + \Delta Inv_1 - \Delta AP_1 - \Delta AL_1 - Dep_1)](1 - T) \tag{26}$$

Finally, we add E_0 to both sides of equation (26) and after factoring $[1 + ROE(1 - T)]$ and dividing both sides of equation (26) by the factor, we obtain:

$$E_0 = \frac{E_0 + [(CR_1 - CE_1 - Int_1) + (\Delta AR_1 + \Delta Inv_1 - \Delta AP_1 - \Delta AL_1 - Dep_1)](1 - T)}{[1 + ROE(1 - T)]} \tag{27}$$

Replacing E_0 with $Csh_0 + AR_0 + Inv_0 + V_0 - D_0$ in the numerator of (27), we can write:

$$E_0 = \frac{Csh_0 + AR_0 + Inv_0 + V_0 - D_0 + [(CR_1 - CE_1 - Int_1) + (\Delta AR_1 + \Delta Inv_1 - \Delta AP_1 - \Delta AL_1 - Dep_1)](1 - T)}{[1 + ROE(1 - T)]} \tag{28}$$

Finally, we simplify equation (28) by recognizing that

$$V_0 - Dep_1(1 - T) = V_1^{\text{book}} + TDep_1, \tag{29}$$

$$Int_1 = iD_0, \text{ and} \tag{30}$$

$$AR_0 + Inv_0 + (\Delta AR_1 + \Delta Inv_1)(1 - T) = T(AR_0 + Inv_0) + (1 - T)(AR_1 + Inv_1) \tag{31}$$

These simplifications allow us to rewrite equation (28) as:

$$E_0 = \frac{Csh_0 - D_0 + T(AR_0 + Inv_0) + (1 - T)(AR_1 + Inv_1) + V_1^{\text{book}} + TDep_1}{[1 + ROE(1 - T)]}$$
$$+ \frac{[(CR_1 - CE_1 - iD_0) - (\Delta AP_1 + \Delta AL_1)](1 - T)}{[1 + ROE(1 - T)]} \tag{32}$$

To verify our results, we substitute HQN numerical values into equation (29) and find:

$$E_0 = \frac{\$930 - \$8,000 + [0.4(\$1,640 + 3,750)] + [0.6(\$1,200 + \$5,200)] + (\$3,400 - \$70) + [0.6(\$350)]}{(1.051)}$$
$$+ \frac{[(\$38,990 - \$38,078 - \$480) - (\$1,000 - \$78)]0.6}{(1.051)}$$
$$= \$2,000 + \frac{[(\$38,990 - \$38,078 - \$480) - (\$1,000 - \$78)]0.6}{(1.051)} = \$2,000 \tag{33}$$

To write the multiperiod equivalent of equation (33) we discount n periods of operating income and in the nth period we liquidate operating and capital accounts and replace $ROE(1 - T)$ with $IRR^E(1 - T)$.

$$E_0 = \frac{(CR_1 - CE_1 - iD_0)(1 - T) + TDep_1}{[1 + IRR^E(1 - T)]} + \cdots + \frac{(CR_n - E_n - iD_{n-1})(1 - T) + TDep_n}{[1 + IRR^E(1 - T)]^n}$$
$$\frac{Csh_0 - D_0 + T(AR_0 + Inv_0) + (1 - T)(AR_n + Inv_n) + V_n^{\text{book}} - [AP_n - AP_0 + AL_n - AL_0](1 - T)}{[1 + IRR^E(1 - T)]^n} \tag{34}$$

Capital gains (losses) and taxes
At the end of the analysis, PV models value their capital assets at their book value or if they liquidate them, they value them at their liquidation value $V_n^{\text{liquidation}}$. For tax purposes, if the

difference between the liquidation and book value of capital assets is positive, $(V_n^{\text{liquidation}} - V_n^{\text{book}}) > 0$, the firm or the investment has realized capital gains whose after-tax value is $(1-T)(V_n^{\text{liquidation}} - V_n^{\text{book}}) > 0$. On the other hand, if the difference is negative $(V_n^{\text{liquidation}} - V_n^{\text{book}}) < 0$, then the firm has realized a capital loss and earned tax credits whose after-tax value loss is $(1-T)(V_n^{\text{liquidation}} - V_n^{\text{book}}) < 0$. To simplify the tax discussion, we ignore the tax rate differences between income, capital gains and depreciation recapture and apply only one tax rate T, the average of all tax rates. Finally, to adjust capital accounts for taxes, we replace V_n^{book} in equation (32) with what follows:

$$(1 - T)(V_n^{\text{liquidation}} - V_n^{\text{book}}) + V_n^{\text{book}} = (1 - T)V_n^{\text{liquidation}} + TV_n^{\text{book}} \tag{35}$$

Now we can write the after-tax IRR^E model for changes in equity consistent with AIS construction principles.

$$
\begin{aligned}
E_0 = {} & \frac{(CR_1 - CE_1 - iD_0)(1 - T) + TDep_1}{[1 + IRR^E(1 - T)]} + \cdots + \frac{(CR_n - CE_n - iD_{n-1})(1 - T) + TDep_n}{[1 + IRR^E(1 - T)]^n} \\
& + \frac{Csh_0 - D_0 + T(AR_0 + Inv_0) + (1 - T)(AR_n + Inv_n)}{[1 + IRR^E(1 - T)]^n} \\
& + \frac{(1 - T)V_n^{\text{liquidation}} + TV_n^{\text{book}} - [AP_n - AP_0 + AL_n - AL_0](1 - T)}{[1 + IRR^E(1 - T)]^n}
\end{aligned}
\tag{36}
$$

After-tax multiperiod $IRR^A(1 - T^*)$ model

There is no explicit measure for T^* that can be used to find $ROA(1 - T^*)$. This peculiar result occurs because taxes must account for interest costs that we do not consider when finding $EBIT$. Yet, many applied IRR models solve for after-tax return on assets that assume we can measure $ROA(1 - T^*)$. Still, we can find such a measure from an AIS allowing us to write:

$$
\begin{aligned}
A_0 = {} & \frac{(CR_1 - CE_1)(1 - T^*) + T^*Dep_1}{[1 + IRR^A(1 - T^*)]} + \cdots + \frac{(CR_n - CE_n)(1 - T^*) + T^*Dep_n}{[1 + IRR^A(1 - T^*)]^n} \\
& \frac{Csh_0 + T^*Accts_0 + (1 - T^*)Accts_n + (1 - T^*)V_n^{\text{liquidation}} + T^*V_n^{\text{book}} - [AP_n - AP_0 + AL_n - AL_0](1 - T^*)}{[1 + IRR^A(1 - T^*)]^n}
\end{aligned}
\tag{37}
$$

The main difference between equations (36) and (37) is that T is replaced with T^*, interest charges are not subtracted from periodic cash flow, and initial liabilities are no longer subtracted. All these changes are required so that earnings can be attributed to beginning assets rather than beginning equity.

Although there is no explicit AIS measure corresponding to equation (37), we do know the value of beginning assets A_0 and $IRR^A(1 - T^*)$ so we can write the one period HQN numerical equivalent of (32) assuming capital assets are valued at their book value:

$$
\begin{aligned}
A_0 & = \frac{Csh_0 + T^*Accts_0 + (1 - T^*)Accts_1 + V_1^{\text{book}} + T^*Dep_1 + [(CR_1 - CE_1) - (\Delta AP_1 + \Delta AL_1)](1 - T^*)}{[1 + ROA(1 - T^*)]} \\
& = \frac{\$930 + \$565.95 + \$5,728 + \$3,330 + \$36.75 + [(\$38,990 - \$38,078) - (\$1,000 - \$78)]0.895}{(1.058)} \\
& = \$10,000
\end{aligned}
\tag{38}
$$

Rates of return on assets and equity

Miller and Bradford (2000) reviewed and compared rates of return on assets and equity. We agree with their conclusion that the two measures should be viewed as complementary. To describe the relationship between ROE and ROA, we begin with equations (2) and (3) that employ AIS definitions of ROA and ROE. From these two equations we deduce the rates of return identity:

$$ROE = \frac{EBIT - Int}{E_0} = \frac{(ROA)(A_0) - (i)(D_0)}{E_0} = \frac{(ROA - i)D_0}{E_0} + ROA \tag{39}$$

Note that in equation (39) if $i = ROA$ or if $D_0 = 0$ then $ROE = ROA$. Note also that ROE and ROA are positively related. Furthermore, if we solve for ROA as a function of ROE, we find the familiar weighted cost of capital (WCC) equation that we illustrate using HQN data:

$$ROA = ROE\left(\frac{E_0}{A_0}\right) + i\left(\frac{D_0}{A_0}\right) = 8.5\%\left(\frac{\$2,000}{\$10,000}\right) + 6\%\left(\frac{\$8,000}{\$10,000}\right) = 6.5\% \tag{40}$$

Of course, we are less confident about the relationships in equations (39) and (40) when measured in multiperiod settings where ROA is replaced with IRR^A and ROE is replaced with IRR^E and interest rates and asset and debt levels may vary over time.

We emphasize that both ROA and ROE provide interesting and important information. Financial managers should be interested in what firms and investments can earn independent of how they are financed. Then, if the difference between return on assets and the cost of debt matters, as it should, ROE provides important information for choosing between alternative financing options.

Conflicting asset and equity earnings and rates of return

Suppose we are comparing two mutually exclusive challengers, 1 and 2, funded by the same defender and earning rates of return on invested assets of $ROA^1 > ROA^2$. Do these results imply that $ROE^1 > ROE^2$? That NPV earnings from assets invested in challengers 1 and 2 satisfy $NPV^{A1} > NPV^{A2}$? Or, that NPV earnings from equity invested in challengers 1 and 2 satisfy $NPV^{E1} > NPV^{E2}$? The answer is that earnings and rates of return on assets and equity are consistent only under limited conditions. These include, A_0 and E_0 must satisfy homogeneity of size conditions and the average interest cost i must be the same for both investments. We demonstrate that if the homogeneity and average interest rate conditions are satisfied, then $ROA^1 > ROA^2$ implies $ROE^1 > ROE^2$, $NPV^{A2} > NPV^{A1}$, and $NPV^{E1} > NPV^{E2}$. To begin, recall equation (40) that allows us to write:

$$ROA^1 - ROA^2 = ROE^1\left(\frac{E_0}{A_0}\right) + i\left(\frac{D_0}{A_0}\right) - ROE^2\left(\frac{E_0}{A_0}\right) - i\left(\frac{D_0}{A_0}\right)$$

$$= (ROE^1 - ROE^2)\left(\frac{E_0}{A_0}\right) \tag{41}$$

Therefore, if $ROA^1 > ROA^2$, then $ROE^1 > ROE^2$. Next, if $ROA^1 > ROA^2$, then from equation (2), it follows that $EBIT^1 > EBIT^2$ and $NPV^{A1} > NPV^{A2}$ since:

$$NPV^{A1} - NPV^{A2} = \frac{EBIT^1 - EBIT^2}{(1 + ROA)} > 0 \tag{42}$$

Finally, if $EBIT^1 > EBIT^2$ and interest costs are the same for both investments, then $EBT^1 > EBT^2$ and $NPV^{E1} > NPV^{E2}$ since:

$$NPV^{E1} - NPV^{E2} = \frac{EBIT^1 - EBIT^2}{(1 + ROE)} \tag{43}$$

Conflicting rankings may occur when interest rates or debt levels financing the two challengers differ. To illustrate, suppose we decided to rank challengers 1 and 2 that satisfied homogeneity of size conditions for assets and equity and whose ROA^1 and ROA^2 were equal. Now assume that interest costs for the two investments differed. Then we would rank the two investments based on their asset earnings and rates of return as equal. But for rankings based on equity earnings and rates of return, the investment with the lower interest cost would be preferred. The consequence is that asset-based rankings would be equal and equity-based rankings would be unequal and asset and equity-based rankings would be inconsistent.

To make clear that asset and equity earnings and rates of return may produce conflicting rankings, consider HQN's one-period ROA of 6.5% ($650/$10,000) and its one-period ROE of 8.5% ($170/$2,000) respectively. Let HQN's beginning assets and $EBIT$ describe both investments 1 and 2. Now suppose that interest costs for investments 1 and 2 differed. For example, let the average interest rate charged on investment 1 be 6% and 0% for investment 2. As a result, the IRR^E and NPV^E rankings would no longer be consistent with IRR^A and NPV^A rankings for investments 1 and 2. We summarize these results in Table 6.

One can imagine other less extreme cases in which asset and equity rankings could be inconsistent simply because interest cost influence earnings and rates of return on equity but not for assets.

Summary and conclusions

We now make explicit the main point of this paper. PV models should be constructed as multiperiod extensions of AISs. Otherwise, they may misrepresent the financial characteristics of investments and lead to less than optimal investment decisions. Furthermore, different AIS earnings and rates of return help us distinguish between different NPV models and IRRs. These distinctions are important since rates of return and earnings measures on assets and equity may lead to different recommendations.

We emphasize that AISs help us recognize the conditions required for asset and equity earnings and rates of return rankings to be consistent. These insights that we learn from AISs and multiperiod extensions of AISs, we believe will help financial managers better understand, build and interpret PV models. However, these results, also task financial managers with the responsibility to carefully decide whether to base their recommendation on asset or equity earnings and rates of return.

Using the PV models developed in this paper, we can imagine financial managers building Excel templates or similar computerized support systems to solve applied investment problems that include more details than we were able to include in our demonstrations. These details may include more complete description of taxes and allow more investments and

	Investment 1	Investment 2	
Asset earnings and rates of return (rankings)			
NPV^As (rankings)	$EBIT$ = $650 (1)	$EBIT$ = $650 (1)	**Table 6.**
IRR^As (rankings)	$650/$10,000 = 6.5% (1)	$650/$10,000 = 6.5% (1)	HQN's Inconsistent rankings based on asset and equity earnings and rates of return
Equity earnings and rates of return (rankings)			
NPV^Es (rankings)	EBT = $170 (2)	EBT = $650 (1)	
IRR^Es (rankings)	$170/$2,000 = 8.5% (2)	$650/$2,000 = 32.5% (1)	

disinvestments to occur during the analysis. We wish you all success in this effort—to develop and apply PV models that represent multiperiod extensions of AISs.

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