A critical review of wheel/rail high frequency vibration-induced vibration fatigue of railway bogie in China

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Abstract
Purpose – This review aims to give a critical view of the wheel/rail high frequency vibration-induced vibration fatigue in railway bogie.
Design/methodology/approach – Vibration fatigue of railway bogie arising from the wheel/rail high frequency vibration has become the main concern of railway operators. Previous reviews usually focused on the formation mechanism of wheel/rail high frequency vibration. This paper thus gives a critical review of the vibration fatigue of railway bogie owing to the short-pitch irregularities-induced high frequency vibration.

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including a brief introduction of short-pitch irregularities, associated high frequency vibration in railway bogie, typical vibration fatigue failure cases of railway bogie and methodologies used for the assessment of vibration fatigue and research gaps.

Findings – The results showed that the resulting excitation frequencies of short-pitch irregularity vary substantially due to different track types and formation mechanisms. The axle box-mounted components are much more vulnerable to vibration fatigue compared with other components. The wheel polygonal wear and rail corrugation-induced high frequency vibration is the main driving force of fatigue failure, and the fatigue crack usually initiates from the defect of the weld seam. Vibration spectrum for attachments of railway bogie defined in the standard underestimates the vibration level arising from the short-pitch irregularities. The current investigations on vibration fatigue mainly focus on the methods to improve the accuracy of fatigue damage assessment, and a systematical design method for vibration fatigue remains a huge gap to improve the survival probability when the rail vehicle is subjected to vibration fatigue.

Originality/value – The research can facilitate the development of a new methodology to improve the fatigue life of railway vehicles when subjected to wheel/rail high frequency vibration.

Keywords Wheel/rail high frequency vibration, Vibration fatigue, Railway bogie, Fatigue damage assessment

Paper type Literature review

1. Introduction
The railway bogie is a crucial component for rail vehicle and serves an important role in steering vehicle operating on the track and supporting loads from the car body. Therefore, the railway bogie is expected to be subjected to dynamic loads arising from wheel/rail interaction, car body motion, as well as traction and braking systems. These loads are usually characterized by the broadband frequency due to varying speeds, different track types and operational conditions, which pose a huge challenge for the vehicle design. In recent years, vibration fatigue, referred to as fatigue failure caused by random vibration especially when the excitation frequency is approaching the natural frequency of structural, has been widely reported by railway operators. These fatigue failures occurring in railway bogie are usually related to the wheel/rail high frequency impact and can pose highly adverse influences on operating safety. Therefore, this paper aims to elaborate the wheel/rail high frequency vibration, and the resulting vibration fatigue in railway bogie as well as the associated methodologies used to estimate vibration fatigue.

2. Wheel/rail high frequency vibration in railway bogie
The rail vehicle operates on two parallel rails through the wheel/rail rolling contact. The contact at the wheel/rail interface is characterized by large contact stiffness due to the steel-made wheel and rail, and the size of the contact patch is similar to the dimension of the thumb. Therefore, the contact force of wheel/rail interaction is very sensitive to irregularities at the wheel/rail interface. These irregularities could be from either the wheel or the rail interface. It has been reported that the short pitch irregularities of wheel/rail serve as the main resource of wheel/rail high frequency vibration (Tao, Wen, Jin, & Yang, 2020; Tao, Xie, Wang, Yang, Ding, & Wen, 2020), which are usually associated with the structural fatigue failure and wheel/rail noise emission, as shown in Figure 1. The short-pitch irregularities on the wheel and rail interface could be classified into discrete irregularities and periodic irregularities distributed on the wheel circumference and rail head. The discrete irregularities could consist of wheel flat, wheel shelling and wheel spalling on the wheel circumference, as well as the rail joint on the rail head. Those irregularities can give rise to a local impact at the wheel/rail interface. The wheel polygonal wear and rail corrugation are regarded as typically periodic irregularities at the wheel/rail interface, which can lead to high frequency and high magnitude impact for rail vehicles. Therefore, the wheel polygonal wear and rail corrugation-induced high frequency vibration are the main focus of this paper.
2.1 Formation mechanisms of short-pitch irregularities

(1) Wheel OOR

The irregularities on the wheel circumference are also referred to as wheel Out-Of-Round (OOR). Due to its highly adverse influence, huge efforts have been made to understand the formation mechanism and its associated influence on high-speed rail vehicles, metro cars and locomotives. The most recent review on the formation mechanisms of wheel polygonal wear for high-speed rail vehicles, metro cars and locomotive were made by Tao, Wen et al. (2020), Tao, Xie et al. (2020), Iwnicki, Nielsen, and Tao (2023). The frequency-fixed mechanism is regarded as a general law for the formation of wheel polygonal wear, although the specific driving force of wheel polygonal wear still remains controversy for specific vehicles. The proposed formation mechanism of wheel polygonal wear includes the bending of wheelset (Jin, Wu, Fang, Zhong, & Ling, 2012; Tao, Wen et al., 2020; Tao, Xie et al., 2020), P2 resonance of wheel/rail (Tao, Wen, Liang, Ren, & Jin, 2019; Cai, Chi, Tao, Wu, & Wen, 2019), rail localized bending between two wheelsets in a bogie (Wu, Rakheja, Cai, Chi, Ahmed, & Qu, 2019; Wu, Wu, Li, Shi, & Xu, 2019; Cai, Wu, Chi, Yang, & Huang, 2022; Qu, Zhu, Zeng, Dai, & Wu, 2020; Ma, Gao, Cui, & Xin, 2021), the structural resonance of bogie frame (Wu, Du, Zhang, Wen, & Jin, 2017), as well as the frictional self-excited vibration wheelset–track system (Zhao et al., 2019; Wu, Shang, Pan, Zhang, Shi, & Xiao, 2022; Wu, Xie, Liu, Wu, Wen, & Mo, 2022) and traction-induced wheelset torsional vibration and associated self-excited stick slip vibration (Spangenberg, 2020), as shown in Figure 2. In addition, the initial irregularities on the wheel circumference (Cai, Chi, Wu, Yang, & Huang, 2023; Ye, Shi, Krause, Tian, & Hect, 2020; Kang
et al., 2022), wheelset flexibility (Peng, Iwnicki, Shackleton, Crosbee, & Zhao, 2019; Peng, Han, Chu, Gao, Liu, & Xiao, 2019) and traction (Chen et al., 2023) can aggravate the formation of uneven wear on the wheel circumference.

It can be seen that the formation mechanisms of wheel polygonal wear are mainly attributed to the resonance of the natural vibration mode of the system, especially for the modes that affect the wheel/rail interaction. These suggest that the wheel polygonalization is a general wear process for rail vehicles, and a perfect round wheel could also tend to be an OOR wheel if the mitigation is not considered in the operation. Therefore, it is desirable to summarize the resulting excitation frequencies of polygonal wheel at the wheel/rail interface, which could facilitate the optimization of structure so as to avoid the exciting frequencies of polygonal wheels. Table 1 summarizes typical wheel polygonal wear and associated excitation frequencies for wheel/rail interaction reported by references. It can be seen the dominating excitation frequencies of polygonal wear for metro cars and locomotive mainly lies in the frequency band of less than 100 Hz approximately. Whereas for high-speed trains, the wheel polygonal wear-induced excitation frequency can reach up to 650 Hz, mainly ranging from 550 to 650 Hz.

(2) Rail corrugation

Rail corrugation is a periodic wear on the rail head. Grassie classified rail corrugation into six types: heavy haul corrugation, light rail corrugation, corrugation related to track form, corrugation caused by P2 resonance, flange-type corrugation and rail Pinned-Pinned...
corrugation (Grassie, 2009). Zhai et al. and Wen et al. summarized the phenomenon, causes and countermeasures of rail corrugation in China’s high-speed lines and metro lines, respectively (Zhai, Jin, Wen, & Zhao, 2020; Wen, Tao, Zhao, Wei, & Jin, 2023), as listed in Table 2. As pointed out by Jin et al., rail corrugation exhibits diverse forms, and its formation mechanisms are highly complex (Jin, Li, Li, & Wen, 2016). The occurrence and development of rail corrugation are influenced by both the natural characteristics of the vehicle-track system, such as vehicle speed, un-sprung mass, rail pad spacing, fastener stiffness and damping and track slabs types, as well as the operating environmental conditions (e.g. straight track, curved track, roughness of the wheel–rail interface, wheel–rail friction eco-efficiency) and the type of vehicles (heavy haul, light rail, etc.). Due to this diversity and complexity, it is challenging to summarize the growth mechanism of all corrugation phenomena using a single theory. Most rail corrugation is due to the periodic fluctuations in the wheel–rail interface (such as wheel–rail normal force and creepages), then lead to periodic fluctuations in wheel/rail wear depth and gradually accumulate into rail corrugation. There are many reasons for these periodic wheel–rail vibrations, such as resonance of the wheelset itself

<table>
<thead>
<tr>
<th>Vehicle Order, vehicle speed and formation mechanisms</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metro car 9th order, 50–80km/h, First bending of wheelset</td>
<td>85 Hz</td>
</tr>
<tr>
<td>13th order, 120km/h, First bending of wheelset</td>
<td>78–96 Hz</td>
</tr>
<tr>
<td>5–8th order, 60 km/h, P2 resonance of wheel/rail</td>
<td>31–63 Hz</td>
</tr>
<tr>
<td>Locomotive 20th order, 40 km/h, resonance of a coupled traction motor pitching and axle torsional mode</td>
<td>60.4 Hz</td>
</tr>
<tr>
<td>High-speed train 18/19/24 order, 80 km/h, bending of wheelset and wheel disc</td>
<td>84 Hz, 122 Hz</td>
</tr>
<tr>
<td>18–20th order, 300 km/h, Localized bending of rail in the length range of wheelbase</td>
<td>550–580 Hz</td>
</tr>
<tr>
<td>25 order, 250 km/h, Localized bending of rail in the length range of wheelbase</td>
<td>595–650 Hz</td>
</tr>
</tbody>
</table>

Source(s): Authors own work

<table>
<thead>
<tr>
<th>Line</th>
<th>Wavelength, vehicle speed; track characteristics (reasons)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metro line 30–40 mm, 80–90 km/h; resilient fasteners (GJ-III)</td>
<td>556–833 Hz</td>
<td></td>
</tr>
<tr>
<td>50–63 mm, 55–70 km/h; resilient fasteners (Cologne-Egg)</td>
<td>243–389 Hz</td>
<td></td>
</tr>
<tr>
<td>40–50 mm, 83 km/h; resilient fasteners (Vanguard)</td>
<td>461–576 Hz</td>
<td></td>
</tr>
<tr>
<td>100–250 mm, 40–50 km/h; conventional fasteners, Curve Radius &lt;300 m</td>
<td>44–139 Hz</td>
<td></td>
</tr>
<tr>
<td>200–250 mm, 65–70 km/h; conventional fasteners, 600 m &lt; Curve Radius &lt;800 m</td>
<td>72–97 Hz</td>
<td></td>
</tr>
<tr>
<td>80–100 mm, 60–80 km/h; conventional fasteners, Rubber booted short sleeper, 400 m &lt; Curve Radius &lt;700 m</td>
<td>167–278 Hz</td>
<td></td>
</tr>
<tr>
<td>50 mm, 35–40 km/h; Type-II fasteners, Rubber booted short sleeper, Curve Radius = 350</td>
<td>194–222 Hz</td>
<td></td>
</tr>
<tr>
<td>125–200 mm, 60–80 km/h; conventional fasteners, Ladder-type sleeper, 400&lt; Curve Radius ≤600</td>
<td>83–178 Hz</td>
<td></td>
</tr>
<tr>
<td>60–100 mm, 35–40 km/h; Type-II fasteners, Ladder-type sleeper, Curve Radius = 350</td>
<td>97–185 Hz</td>
<td></td>
</tr>
<tr>
<td>High-speed line 60–80 mm, 300 km/h; pinned-pinned resonance</td>
<td>1,042–1,388 Hz</td>
<td></td>
</tr>
<tr>
<td>120–160 mm, 300 km/h; vertical bending mode of the rails</td>
<td>520–694 Hz</td>
<td></td>
</tr>
<tr>
<td>63–80 mm, 250 km/h; rail pre-grinding</td>
<td>868–1,102 Hz</td>
<td></td>
</tr>
</tbody>
</table>

Source(s): Authors own work, Wen et al. (2023)
resonance of the track itself (Jin, Wen, Wang, Zhou, & Liu, 2006; Li et al., 2016; Daniel, Horwood, Meehan, & Wheatley, 2008; Wu, 2011), wheel–rail contact resonance (Carson & Johnson, 1971), vehicle-track coupling resonance (Kurzeck, 2011; Wang & Wu, 2020) and self-excited vibrations of the wheel–rail system (Clark, Scott, & Poole, 1988; Chen et al., 2020; Cai et al., 2020), as shown in Figure 3. Similar to the wheel polygonal wear, the rail corrugation developed by dynamic cause also follows the frequency-fixed mechanism.

2.2 Wheel/rail high frequency vibration

Through the above-mentioned investigations, the wheel/rail short-pitch irregularities can lead to high frequency and high magnitude impact at the wheel/rail interface near the passing frequency, thereby intensified vibration level for rail vehicles. The wheel polygonal wear-induced impact can affect the whole wheel re-profiling cycle until the reprofiling process is performed. The rail corrugation usually affects the local track section when the vehicle passes through, especially for the tight curve section. Considering the highly adverse influence on the rail vehicles, huge efforts have been made to explore the short-pitch irregularities-induced high frequency vibration through both the tests and numerical simulations.

(1) Experimental investigations

The short-pitch irregularities-induced high frequency impact load can reach up to 1,000 Hz, which poses a huge challenge to measure impact loads arising from the wheel polygonal wear and rail corrugation through the instrumented wheelset. Therefore, the axle box acceleration is usually employed to quantify the influences of short-pitch irregularities on the vehicle in the tests. To capture the evolution of axle box acceleration from the new wheel to the worn wheel conditions, the long-term field test campaigns were usually conducted. Cai et al. conducted a long-term field test for a high-speed train operating at a speed of 250 km/h (Cai et al., 2020). In the test, the tested vehicle was subjected to severe wheel polygonal wear with the harmonic order of 22–25th, which gives rise to obvious influences on the axle box.

Figure 3.
Typical formation mechanisms for rail corrugation

Source(s): Authors own work
acceleration (especially for the passing frequency band of 550–650 Hz), as shown in Figure 4. In the lateral stage of the wheel re-profiling cycle, the acceleration of the axle box showed a rapid increase with the vehicle mileages due to the wheel polygonal wear. The results also showed that the frequently varying speed can lower the increase rate of wheel polygonal wear. Wu et al. reported the evolution of wheel polygonal wear for a high-speed train operating at 300 km/h and pointed out that the maximum vertical acceleration of the axle box can reach up to 400 g due to severe wheel polygonal wear, and the associated impact can excite some high frequency vibration mode (up to around 1,000 Hz) near the double passing frequency of wheel polygonal wear (Wu, Rakheja, & Wu, 2018; Wu, Rakheja, & Qu, 2018). Moreover, the wheel polygonal wear-induced high frequency vibration is obviously greater than those defined in IEC 61373 (Wu et al., 2023), especially for the dominating frequency band of wheel polygonalization (550–650 Hz), as shown in Figure 5. This suggests that the axle box vibration level given in IEC 61373 could underestimate the vibration level of the axle box in the presence of wheel polygonalization. These could significantly shorten the fatigue life of rail vehicles, thereby the occurrence of in-service fatigue failures.

Alternatively, some researchers investigated the wheel polygonal wear-induced high frequency vibration through a roller test rig. China Academy of Railway Sciences Corporation Limited developed a full-size roller test rig for a single wheelset so as to

![Figure 4](image)  
**Figure 4.** Evolution of RMS value for vertical axle box acceleration in terms of vehicle mileage

![Figure 5](image)  
**Figure 5.** Comparison of axle box ASD spectra for new- and worn-wheel conditions

Source(s): Wu et al. (2023)
investigate the high frequency vibration of wheel/rail interaction (Chang, Cai, Chen, Li, & Lin, 2022), as shown in Figure 6. This test rig was employed to explore the wheel flat-induced impact at the wheel/rail interface (Wu, Chi, Liu, Hu, Liang, & Wen, 2020; Wu, Xie, Liu, Wu, Wen, & Mo, 2020) and the formation process of wheel polygonal wear (Wu, Shang et al., 2022; Wu, Xie et al., 2022). Similarly, Liu et al. utilized a full-scale roller test rig for a single wheelset incorporating with a numerical model to study the combined vibration of the wheelset and gear box arising from the wheel polygonal wear and track irregularity (Liu, Yang, & Liu, 2022), as shown in Figure 7. Although the considered wheel polygonal wear is limited to the

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**Figure 6.**
Experimental investigation on the wheel/rail interaction due to wheel flat, (a) roll test rig, (b) wheel flat-induced wheel/rail impact forces, and (c) axle box acceleration caused by wheel flat

**Source(s):** Chang *et al.* (2022)

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**Figure 7.**
Full-scale roller test rig for a single wheelset

**Source(s):** Liu *et al.* (2022)
first-order OOR, the wheel polygonal wear-induced vibration is still identified as the main contributor for the vibration of wheelset. Similarly, Zhang et al. designed the roller as polygonal wheel with high-order polygonal wear to simulate the high frequency vibration of wheelset (Zhang, Ping, & Wu, 2018). To better simulate the high frequency vibration of bogie frame, Wu et al. developed a roller test rig for the whole railway bogie rather than a single wheelset (Wu, Chi et al., 2020; Wu, Xie et al., 2020), as shown in Figure 8, which enables us to identify the transfer path of high frequency vibration from the wheel/rail interaction to the bogie frame. Considering the limitations of hydraulic actuators for the high frequency loading, the above-mentioned test rigs serve as an alternative method to simulate the high frequency of wheel/rail interaction, although these test rigs are still subjected to limitations, such as regardless of the effects of track support stiffness which could overestimate wheel/rail impact load and axle box acceleration with respect to the real operating condition.

Rail corrugation is discretely distributed in the local region of track, especially in tight curves. It is known that the inner rail is subjected to larger wheel/rail creepage in the curve, therefore, the inner rail is much more vulnerable to rail corrugation with respect to the outer rail. The characteristics of rail corrugation are highly related to track properties, and the resulting dynamic responses could vary substantially in a single railway line due to different track types (Wu, Chi et al., 2020; Wu, Xie et al., 2020), as shown in Figure 9. The rail corrugation-induced axle box acceleration is significantly greater than those observed in the track section without rail corrugation. Moreover, the short sleeper track and rubber booted sleeper track yield different wavelengths for the rail corrugation, thereby different dominating frequencies (70 Hz for short sleeper track and 200 Hz for rubber booted sleeper track). In China, a railway line could consist of different damping track types (especially for metro lines) for different purposes of vibration reduction, thereby different characteristics for rail corrugation and exciting frequency for wheel/rail interaction, as listed in Table 2. The variations in the excitation frequencies of rail corrugation for different tracks pose huge challenges for the modal matching for the vehicle and track system, and the natural frequencies of rail vehicles cannot avoid all the exciting frequencies arising from the wheel/rail interaction, thereby leading to the vibration fatigue in the service.

(2) Theoretical investigations

In the theoretical study, the wheel polygonal wear and rail corrugation are generally modelled as a harmonic variation superposing on the wheel circumference or rail head (Li et al., 2011; Liu & Zhai, 2014; Zhang et al., 2014), which serves as a basic input for the simulation of vehicle and track model. From the interested frequency point of view, the vehicle and track system...
can be modelled in different frequency ranges, such as the mid-frequency range and the high
frequency range. A review of modelling the vehicle and track system in the mid-frequency
range 50–500 Hz was given by Popp et al., in which the rail corrugation, deterioration of
ballast or Out-Of-Round (OOR) wheel have been considered as main contributors to the mid-
frequency oscillation in the railway system (Popp, Kruse, & Kaiser, 1991). Regarding to the
wheel/rail noise problem, the high frequency vibration of the system up to 5,000 Hz is the
interest. The related methods used to build the vehicle and track system in the high frequency
range were reviewed by Knothe and Grassie (1993). In order to characterize the high
frequency vibration arising from short-pitch irregularities, a vast number of models were
developed on the basis of the theory of vehicle/track coupled dynamics (Zhai, Wang, & Cai,
2009) and the theory of rigid/flexible coupled dynamics. Dependent upon different purposes,
the proposed models show varying complexities, including the flexibility in the rail, slab
track, wheelset, axle box, coil spring, gear box and bogie frame, so as to consider the desired
high frequency vibration (see Figure 10).

Through the simulation of a vehicle/track coupled dynamic model together with flexible
models of interest components, the influences of short pitch irregularities have been widely
investigated, in terms of wheel/rail normal force, wheel/rail creepage force, vibrations in the
track, axle box, wheelset, gearbox, roller bearing, coil spring and bogie frame, as well as

**Figure 9.**
Rail corrugation-induced high frequency vibration

**Source(s):** Wu, Chi, *et al.* (2020) and Wu, Xie, *et al.* (2020)
dynamic stress arising from the high frequency vibration. Through simulation of a vertical coupled vehicle/track model along with the measured wheel polygonal wear, Liu and Zhai concluded that the damping effects of the wheel/rail interaction can impose a significant influence on the fluctuations of wheel/rail contact forces in the presence of wheel polygonalization, and the peak wheel/rail contact force always occur at the raising slopes of radius (Liu & Zhai, 2014), which was comparable with observed by Morys (1998). Considering a 20th-order wheel polygonal wear, Wu et al. investigated the influences of vehicle speed and polygonal wear amplitude on the wheel/rail normal force, axle box acceleration and stress of wheelset axle (Wu, Rakheja, & Wu, 2018; Wu, Rakheja, & Qu, 2018). The results demonstrated a nearly linear increase in the wheel/rail contact forces with wear amplitude. The high magnitude and high frequency impact loads owing to wheel polygonalization can excite some bending modes of vehicle and track subcomponents and further contribute considerably to the reduction of fatigue lifetime. Wu et al. investigated the dynamic stress developed in the wheelset axle shaft arising from the impact loads due to the wheel flat and wheel polygonalization through a coupled vehicle/track dynamic model along with a rotating flexible wheelset (Wu, Chi, & Wu, 2015). The results suggested that the wheel polygonalization can impose a significant influence on the dynamic stress in the wheelset axle shaft, and the secondary bending vibration mode of wheelset axle shaft could be excited by the wheel polygonalization-induced impact load. Yang et al. investigated the effects of wheel polygonal wear of locomotive on the wheel/rail creep forces and concluded that the anti-slip controller could contribute to the development of wheel polygonal wear (Yang, Xu, Ling, & Zhai, 2022). Moreover, a number of investigations focused on the effects of wheel polygonal wear on dynamic responses of roller bearing of axle box (Liao et al., 2022; Wang et al., 2019), torsional vibration of transmission system (Wang, Cheng, Mei, Zhang, Huang, & Yin, 2020; Wang, Xie, Jiang, Song, Sun, & Wang, 2020; Wang, Bai, Wu, Zheng, & Zhou, 2020) and gear box (Wu, Rakheja et al., 2019; Wu, Wu et al., 2019).

Regarding the rail corrugation-induced impact, Wu et al. developed a flexible bogie model incorporating an antenna beam and concluded that rail corrugation-induced high frequency vibration near 78 Hz can excite the coupled vibration of bogie frame and antenna beam thereby the intensified stress level (Wu, Shang et al., 2022; Wu, Xie et al., 2022). Zhou et al. reported the failure mechanism of coil spring through the simulation of vehicle/track model incorporating a flexible coil spring model and pointed out that the natural vibration mode of coil spring can be excited by the rail corrugation (Zhou et al., 2020). In the above
investigations, the track was invariably considered as a tangent line. Considering a curved track, Bethel et al. developed a tram/track coupled model to study the effect of wheel polygonal wear on the dynamics of vehicle when negotiating a curve (Bethel et al., 2022).

3. Wheel/rail high frequency vibration-induced vibration fatigue of railway bogie

The intensified vibration of railway bogie is expected to deteriorate the loading conditions thereby shortening the fatigue life of rail vehicles. Table 3 and Figure 11 summarized some typical fatigue failures of railway bogie reported by references, including the high-speed railway vehicles and metro vehicles. The well-known fatigue failure caused by the wheel OOR is the failure of the resilient wheel for IEC train, and the identified main causal factor is the low-order wheel polygonal wear-induced impact at the wheel/rail interface. After that, the wheel polygonal wear-induced vibration became a main concern for the structural integrity. In China, Zhang, Tan, and Lin (2016) and Hu, Liu, Liu, and Hai (2017) reported the fatigue failure of gear box in high-speed rail car and concluded that the failure of gear box was highly related to the structural resonance caused by wheel OOR. The experimental results showed that the wheel circumferential roughness was predominated by 20th-order wheel polygonal wear, and the resulting high frequency impact at the wheel/rail interface near 580 Hz can excite the vibration modes of the gear box, thereby the reduced fatigue life for the gear box. Similarly, the brake disc mounted on the wheelset axle have been also reported to be subjected to fatigue failure due to the combined effects of high frequency vibration and thermal loading in the braking process (Jin et al., 2020). Through the field test, Peng et al. pointed out that the failure of the vertical block was related to the high order wheel polygonal wear-induced high frequency impact at the wheel/rail interface (Peng, Iwnicki et al., 2019; Peng, Han et al., 2019). Besides, the high-level vibration arising from the wheel polygonal wear can also give rise to the non-linear vibration in the axle box front cover of high-speed train and further contribute to the failure of connecting bolts (Feng, Qu, Li, Dai, & Shu, 2023). Dong et al. reported the failure of the wire bracket installed on the axle box through the field test and numerical simulation and concluded that the crack initiated from the weld defect and the combined impacts due to wheel polygonal wear and rail corrugation serve as the main driving force for fatigue failure (Dong, Wang, Dai, & Li, 2023).

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Failed components</th>
<th>Causal factors</th>
<th>Characteristic frequency band</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-speed train</td>
<td>Gearbox</td>
<td>20th-order wheel polygonal wear</td>
<td>580 Hz</td>
</tr>
<tr>
<td></td>
<td>Brake disc</td>
<td>20th-order wheel polygonal wear</td>
<td>580 Hz</td>
</tr>
<tr>
<td></td>
<td>Axle box bolts</td>
<td>20th-order wheel polygonal wear</td>
<td>580 Hz</td>
</tr>
<tr>
<td></td>
<td>Vertical block of axle box</td>
<td>25th- to 27th-order wheel polygonal wear</td>
<td>510 Hz</td>
</tr>
<tr>
<td></td>
<td>Wire bracket</td>
<td>15th to 22nd-order wheel polygonal wear</td>
<td>400–800 Hz</td>
</tr>
<tr>
<td>Metro cars</td>
<td>Motor installation of bogie frame</td>
<td>8th-order wheel polygonal wear and rail</td>
<td>56.6–62.5 Hz</td>
</tr>
<tr>
<td></td>
<td>Side frame of bogie</td>
<td>corrugation with the wavelength of 206–417 mm</td>
<td>71, 89, 94 Hz</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14th- to 16th-order wheel polygonal wear and rail</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>corrugation</td>
<td></td>
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<tr>
<td></td>
<td>Coil spring</td>
<td>Rail corrugation</td>
<td>60 Hz</td>
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<tr>
<td></td>
<td>Antenna beam</td>
<td>Rail corrugation (wavelength 125–200 mm)</td>
<td>60–80 Hz</td>
</tr>
<tr>
<td></td>
<td>Safety hanger of axle box</td>
<td>Rail corrugation (wavelength 61.5 mm)</td>
<td>258 Hz</td>
</tr>
<tr>
<td></td>
<td>Lifeguard</td>
<td>Rail corrugation</td>
<td>63 Hz</td>
</tr>
</tbody>
</table>

Table 3. Typical fatigue failures of rail vehicles arising from high frequency vibrations

Source(s): Authors own work
Regarding metro cars, Wang, Cheng et al. (2020), Wang, Xie et al. (2020) and Wang, Bai et al. (2020) reported the fatigue failures of bogie frame through the field test, and the combined impacts due to the wheel polygonal wear and the rail corrugation were identified as the main contributor of fatigue failure of bogie frame. Zhou et al. (2020) and Ling et al. (2017) studied the fatigue failures of coil springs in a metro car and pointed out that the passing frequency of rail corrugation near 60 Hz can excite some vibration modes of coil spring. Through the field test and numerical simulation, Wu et al. concluded that the coupled bending mode between the bogie frame and antenna beam can be excited by the high frequency impact near 78 Hz arising from the rail corrugation (Wu, Chi et al., 2020; Wu, Xie et al., 2020). Li et al. studied the vibration fatigue of lifeguard in a metro car and concluded that the rail corrugation-induced
high frequency impacts excite the natural frequency of lifeguard thereby reducing fatigue lifetime (Li et al., 2019). Shi et al. pointed out that the fatigue failure of the safety hanger is mainly attributed to the rail corrugation with the dominating wavelength of 61.5 mm (Shi, Wang, Dai, & Wu, 2019). Based on the aforementioned investigations, it is concluded that the axle-mounted components are much more vulnerable to fatigue failures compared with other components. The fatigue crack usually initiates from the defect of weld seam, and the wheel/rail high frequency impact-induced structural resonance serves as the main driving force of the failure of vehicle sub-components.

4. Methodologies used for assessment of vibration fatigue

The fatigue damage is usually employed to quantify the influences of service loads on the fatigue life of considered components. Therefore, this part focuses on methodologies used to assess the fatigue damage in both the design and service phases, especially focusing on load definitions and fatigue damage assessment models, as shown in Figure 12.

Figure 12.
Basic flowchart of fatigue assessment

Source(s): Authors own work
4.1 Load definitions

(1) Main load for railway bogie frame

A number of international standards have been developed to specify the load spectrum of railway bogie, such as UIC515-4, UIC615-4, EN 13749, JIS E 4207 and JIS E 4208. UIC615-4 and UIC515-4 were defined by the International Union of Railways for motor bogie and trailer bogie of passenger rolling stock, respectively. Based on the track conditions in Europe, European countries developed EN 13749 according to the loads given in UIC standards. However, EN standard gives a more detail definition for longitudinal loads of bogie frame, such as the longitudinal lozenging force and potential shock. In those standards, the loads are classified into three types, such as exceptional loads, particular in-service loads and fatigue loads. Considering different operation conditions, the loads given in each standard are different. Japanese standards association developed JIS E 4207 and JIS E 4208 standards to verify the strength of bogie frame. To demonstrate the effects of difference of load definition, a number of investigations compared the fatigue results obtained from different methods. An et al. investigated the methodologies of fatigue assessment given in the JIS standard and further compared with UIC and EN standards and concluded that the JIS standard gave a more detail definition for the operational loads of bogie frame compared with other standards, whereas the exceptional load conditions are not defined in JIS standard (An, Li, Huang, Fu, & Yu, 2009). Liu et al. investigated the fatigue strength of a metro bogie frame and concluded that the JIS standard yields more conservative results for the fatigue strength of the weld seam of bogie frame compared with the UIC standard (Liu, Yang, Xiao, Yang, & Zhu, 2019).

It is known that the load spectrum is the basic input for verification of fatigue strength of railway bogie, and it is significantly affected by vehicle conditions and interfaces, operational characteristics, line characteristics and environmental conditions, and so on. The loads given in the current standards thus could be subjected to limitations due to differences in the operation condition of China railway. Therefore, it is of great interest to explore the load spectrum of bogie through field tests. In China, huge efforts have been made to understand the load spectrum of railway bogie, especially for new modes of high-speed rail vehicles, through the development of load cells for bogie frame, coil spring, axle box, traction rad, braking and wheelset, as shown in Figure 13.

Zhang et al. proposed a methodology to establish the load spectrum of bogie frame for a high-speed train based on the criteria of damage consistency method, which was used to measure the load spectrum of bogie frame operating at a speed of 350 km/h on the Jin-Jin high-speed railway line (Zhang, 2008). Ren et al. developed an instrument coil spring to measure dynamic loads of coil springs through a field test and pointed out that the maxima load mainly occurs at the entrance of the depot and railway connection section (Ren, Sun, & Li, 2010). Wang et al. developed the coil spring and axle box as load sensors with a validated frequency of up to 50 Hz, so as to measure the vertical and lateral forces acting on the bogie frame in the operation condition (Wang et al., 2015). The obtained loads were further employed to calculate the bounce, roll, torsion and lateral load spectrum of bogie frame. The results concluded that the wheel reprofiling process can effectively reduce lateral loads of axle box, up to 50% and 40% for the tangent and curve sections, respectively. Ren et al. proposed a method to identify the traction and braking loads and concluded that the traction and braking forces were highly related to the traction and braking process, irrespective of curve negotiation (Ren, Zhao, Li, Wang, & Wu, 2022). Zou et al. conducted a comprehensive load calibration experiment for a bogie frame to identify the service load conditions of bogie frame (Zou et al., 2016, 2021). The damage consistency criterion was subsequently used to develop the load spectrum of bogie frame.
In the aforementioned investigations, a linear relationship between the load and the stress was usually established through the laboratory test based on the quasi-static method, so as to identify the load through the stress developed on the structure. In the operation, the bogie frame is usually subjected to multiple dynamic forces. Therefore, the relationship between the stress and force can be further derived as:

\[
S_1 = F_1k_{11} + F_2k_{12} + \ldots + F_nk_{1n} \\
S_2 = F_1k_{21} + F_2k_{22} + \ldots + F_nk_{2n} \\
\vdots \\
S_m = F_1k_{m1} + F_2k_{m2} + \ldots + F_nk_{mn}
\]

\[
\Rightarrow S_m = K_{mn}F_n
\]  

(1)

where \(S_m\) is the stress matrix which is a measurement obtained in field tests, \(K_{mn}\) is the force–stress transfer matrix obtained in the calibration experiment by applying the unit force for each loading location, and \(F_n\) is the desired dynamic forces. In this method, the location of stress gauge has to be selected carefully so as to omit the coupling effect of multiple forces, and the wheatstone bridge is usually employed in the calibration. This method is capable of identifying low frequency dynamic loads of bogie frame because the effects of mode vibration...
of bogie frame on the stress are not taken into consideration. The main loads of bogie frame are mainly induced by the low frequency motion of car body, therefore, the quasi-static based instrument bogie frame could be considered acceptable for main load identification for a railway bogie frame. The methodologies used to identify the high frequency loads of railway bogie frame under the service condition still remain the challenge for the load definition of the railway bogie frame.

(2) Vibration spectrum for attachments of railway bogie

Unlike the main structure of a railway bogie, the attachments of bogie frame are mainly subjected to inertial loads arising from random vibration. EN 13749 defined the inertial accelerations for attachments installed on the axle box and bogie frame, respectively, as listed in Table 4. The proposed inertial is mainly limited to the magnitude of acceleration irrespective of exciting frequency, which could underestimate influences arising from the vibration fatigue. IEC 61373 standard defined a flat spectrum of ASD for bogie frame- and axle box-mounted components for both functional- and the simulated long-life vibration test, which has been employed to verify the structural integrity due to the random vibration. Figure 14 illustrates the typical load spectrum for axle box-mounted components given in IEC 61373. The cut-off frequency $f_2$ is taken as 500 Hz for the mass less than 50 kg, and 200 Hz for the mass greater than 125 kg. When the mass lies in the range from 50 to 125 kg, the cut-off

<table>
<thead>
<tr>
<th>Direction</th>
<th>Axle box-mounted equipment</th>
<th>Bogie frame-mounted equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical+</td>
<td>Exceptional acceleration</td>
<td>Fatigue acceleration</td>
</tr>
<tr>
<td></td>
<td>±70g</td>
<td>±25g</td>
</tr>
<tr>
<td>Lateral+</td>
<td>±10g</td>
<td>±5g</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>±10g</td>
<td>±5g</td>
</tr>
</tbody>
</table>

**Note(s):**

* The values in the table apply to the bogie frame above the primary suspension. They may be reduced linearly to half the value at the bogie centre and should be extrapolated to higher values outboard of the primary suspension.

* The value to be used depends on the type of bogie and application and should be consistent with the longitudinal shunt cases.

**Source(s):** Authors own work

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**Figure 14.**
ASD spectrum of axle box-mounted components defined in IEC 61373 standard

**Table 4.**
Typical accelerations for equipment mounted on bogie frame and axle box in EN 13749

**Source(s):** Wu et al. (2023)
frequency $f_2$ is calculated though $125 \times 200 \text{ Hz/mass}$. The vibration spectrum is taken as a flat spectrum in the frequency range of $20 \sim 100 \text{ Hz}$. Whereas for the frequency range of 10 to $20 \text{ Hz}$ and $100 \sim f_2 \text{ Hz}$, the slopes of $9 \text{ dB/octave}$ and $-6 \text{ dB/octave}$ are considered for the ASD spectrum, respectively.

The functional vibration spectrum is the minimum vibration level to demonstrate that the equipment is capable of functioning when subjected to conditions that are likely to occur in service. The $RMS$ values of the functional vibration spectrum in the vertical, transverse and longitudinal directions given by IEC 61373 are 38, 34 and $17 \text{ m/s}^2$, respectively. Whereas the simulated long-life test aims at demonstrating the mechanical integrity of equipment at an increased vibration level, which is usually determined based on the vibration level of the functional vibration spectrum together with an acceleration ratio. In IEC 61373, the acceleration ratio is taken as 3.78 for the axle-mounted components, which permits us to simulate 25 years of fatigue lifetime for equipment in 15 h considering 5 h for each direction. Therefore, the $RMS$ values of the simulated long-lifetime test are determined as 144, 129 and $64.3 \text{ m/s}^2$ for the vertical, transverse and longitudinal directions, respectively.

It can be seen that the axle box ASD spectrum given in IEC 61373 mainly consists of three interest frequency bands, and the vibration level in the frequency band of $20 \sim 100 \text{ Hz}$ is taken as a flat spectrum. This suggests that the vibration spectrum of axle box defined in IEC 61373 aims to simulate the vibration level of a rail vehicle in the frequency band of $20 \sim 100 \text{ Hz}$ and considers that this frequency band predominates the vibration level of axle-mounted components in the operation. However, in the operation, the vibration of axle box is predominated by multiple frequency bands rather than a single frequency band alone (Cai et al., 2021; Wu et al., 2017; Wu, Shang et al., 2022; Wu, Xie et al., 2022). Through analysing field test measurements obtained from nine high-speed railway lines, Wu et al. pointed out that the ASD spectrum in IEC61373 mainly focuses on the low frequency range of $20 \sim 100 \text{ Hz}$ (Wu et al., 2023). This can represent the overall vibration level of $20 \sim 100 \text{ Hz}$, whereas still underestimates the vibration level near the frequency of wheel/rail coupled mode ($P_2$ force). The worst underestimations were observed for the frequency band of $500 \sim 700 \text{ Hz}$, especially for the high-speed train in the presence of wheel polygonal wear. Therefore, a multi-characteristic frequency bands-based axle box ASD vibration spectrum was proposed based on the distribution of wheel/rail coupled vibration modes in order to better characterize the vibration level of rail vehicles, as shown in Figure 15. The vibration level for each concerned frequency band is determined depending upon the distribution of $rms$ values.

![Figure 15. Multi-characteristic frequency bands-based axle box ASD spectrum](image_url)

**Source(s):** Wu et al. (2023)
A few investigations have been made to synthesize the vibration spectrum based on the field test measurement through statistical synthesis methods, such as the vibration test specifications for flight vehicles (Klein & Piersol, 1965) developed by NASA, and the associated criteria for dynamic vibration test (Keegan, 2001; Himelblau, Piersol, Wise, & Grundvig, 1994; Angeli, 2016; MIL-STD-810F, 2008). Wange et al. conducted a comprehensive review of induction methods used in the aircraft vibration test data (Wang, Bai, Wan, & Yan, 2011). GJB/Z 126–99 defines a detailed procedure to synthesize the vibration spectrum for military vehicles, as shown in Figure 16. This method could be employed in railways for the induction of vibration spectrum. Upon the method

Source(s): Authors own work

![Figure 16. Basic procedure for vibration induction given in GJB/Z 126–99](image-url)
given in the standard, Ding et al. performed a vibration spectral induction for the onboard equipment of rail vehicles (Ding, Zhang, & Wang, 2016). Considering the strong non-normality of data in the railway vehicles, Deng et al. used the Johnson’s law to improve the standard method of vibration induction (Deng et al., 2019). A similar investigation was also conducted by Han et al. (2021). In the abovementioned investigations, the vibration data are inevitably considered as a Gaussian distribution; however, the vibration of railway vehicles expresses strong non-Gaussian characteristics, especially for the vibration of axle box due to random excitations at the wheel/rail interface.

4.2 Evaluation models of vibration fatigue

(1) Endurance strength method

In the current standards, the verification of the strength of railway bogie is based on the endurance strength method. The Goodman–Smith diagram and Goodman–Haigh diagram are usually employed to verify the fatigue strength of railway bogie. ERRI B 12/RP 17 defined the permissible stress for three types of steel using Goodman–Smith diagram in terms of the tensile strength (as shown in Figure 17), including the tensile strength of at least 370, 420 and

![Figure 17. Goodman–Smith diagram of steel given in ERRI B 12/RP 17](source(s): Authors own work)
520 N/mm². Three types of curves are used to specify the permissible stress for butt-weld, fillet weld and non-welded area, respectively. JIS E 4207 defined the Goodman–Haigh diagram, as shown in Figure 18. Three regions are taken into consideration to include the effects of weld grinding on the fatigue limit.

In the abovementioned Goodman diagram, the maximum- and minimum-stresses for all considered fatigue load cases have to be determined through the principal stress along the direction of maximum principal stress in the considered fatigue load cases. The amplitude- and mean-stress can be determined as:

\[
\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}; \quad \sigma_m = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2}
\]

where \(\sigma_a\) is the amplitude of the stress cycle of considered fatigue load cases, \(\sigma_m\) is the mean stress for considered fatigue load cases, \(\sigma_{\text{max}}\) is the maximum principal stress for all considered fatigue load cases and \(\sigma_{\text{min}}\) is the minimum stress determined by resolving the

Source(s): Authors own work
principal stress in the direction of maximum principal stress. Alternatively, DVS 1612 utilizes the normal stresses longitudinal and lateral to the seam direction ($\sigma_{II}$ and $\sigma_{\perp}$) and shearing stresses longitudinal to the seam direction ($\tau$) to evaluate the fatigue strength through MKJ diagram, as shown in Figure 19. In the comparison of Goodman–Smith and Goodman–Haigh diagram, MKJ diagram gives a more detail description of the permissible stress depending upon the type of weld joint. In the assessment of fatigue, the concerned stress range has to lie in the envelope of the fatigue limit diagram in order to demonstrate the infinite life for the considered structure.

(2) Damage accumulative method

In the endurance strength method, the stress amplitude of cyclic loading has to be less than the fatigue limit defined by the fatigue strength diagram. Whereas in the operation, the service loading-induced stress could exceed the fatigue limit and then result in fatigue damage. Therefore, the fatigue damage assessment is usually conducted on the basis of the damage accumulation rule. A comprehensive review of time- and frequency-domain methods for fatigue damage estimation was given by Muñiz-Calvente et al. (2022). Fatigue damage estimation can be classified into two groups, including the time-domain methods and frequency-domain methods, as listed in Table 5. The first fatigue damage assessment method was proposed by Palmgren (1924) and Miner (1945) based on the ratio between the applied cycles and total cycles to failures, which considers the fatigue damage accumulative as a simple linear process and neglects the effects of the load-level and load-sequence and lack of load-interaction accountability. Accordingly, a huge number of investigations have been made to improve the Palmgren–Miner model, which results in several fatigue models, such as the double-linear model considering both crack initiation and propagation stages (Langer et al., 1937), the nonlinear damage rules developed by Marco and Starkey (1954), energy damage models (including Leis, 1988 and Niu, Li, & Lee, 1987), continuum damage models (such as Chaboche & Lesne, 1988), as well as the probabilistic damage model proposed by Fernández-Canteli et al. (2014) and Fernández-Canteli (1982). A more detailed discussion of those models can be found in Fatemi and Yang (1998).

In the time-domain methods, the loading in terms of stress-time history observed in real operation conditions is needed to determine load cycles based on the rain flow counting method, which serves as basic input for fatigue damage assessment. However, due to economic and feasible reasons, the loading history with the limited time interval can be only obtained. This could be subjected to limitation when the load is a typical random because the limited load cycles cannot describe all load distributions in the service. For this reason, the

![Figure 19. Permissible normal and shear stress for S355 and S235 given in DVS 1612](source(s): Authors own work)
The statistical information of spectral density to assess fatigue damage together with the S/N curve and Palmgren–Miner rule. In these methods, the random load histories are generally treated as a stationary Gaussian process, represented by the power spectral density (PSD). A random process can be characterized by the autocorrelation function \( R_X(\tau) \), defined as follows:

\[
R_X(\tau) = E[X(t)X(t + \tau)]
\]

(3)

where \( E[\cdot] \) operator is the probabilistic expected value. Similarly, this process can be expressed in the frequency domain with the two-sided PSD function, which is the Fourier transform of the autocorrelation function,

\[
S_X(w) = \int_{-\infty}^{\infty} R_X(\tau) e^{-i\omega \tau} d\tau
\]

(4)

The statistical information of spectral density \( S_X(w) \) can be described through the means of the \( m \)th spectral moments \( \lambda_m \):

<table>
<thead>
<tr>
<th>Fatigue damage model</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-domain</td>
<td></td>
</tr>
<tr>
<td>Palmgren–Miner</td>
<td>( D = \sum n_i/N )</td>
</tr>
<tr>
<td>Marko–Starkey - Leis</td>
<td>( D = \sum (n_i/N)^{0.5} )</td>
</tr>
<tr>
<td>Niu</td>
<td>( D = (n/N)^{1/(\delta+d)} )</td>
</tr>
<tr>
<td>Fernández–Cantelli</td>
<td>( p )</td>
</tr>
<tr>
<td>Chaboche–Lesne</td>
<td>( D = 1 - [1 - (n/N)^{1/(\delta+\rho)}]^{1/(1+\rho)} )</td>
</tr>
<tr>
<td>Frequency-domain</td>
<td></td>
</tr>
<tr>
<td>Narrow–band</td>
<td>( \mathcal{D} = \mathcal{D}_0 + \mathcal{D}_0 + \mathcal{D}_1 )</td>
</tr>
<tr>
<td>approximation</td>
<td></td>
</tr>
<tr>
<td>Range-mean</td>
<td></td>
</tr>
<tr>
<td>approximation</td>
<td></td>
</tr>
<tr>
<td>Wirsching–Light</td>
<td></td>
</tr>
<tr>
<td>method</td>
<td></td>
</tr>
<tr>
<td>( \alpha_{0,b} ) method</td>
<td></td>
</tr>
<tr>
<td>Jiao–Moan method</td>
<td></td>
</tr>
<tr>
<td>Gao–Moan method</td>
<td></td>
</tr>
<tr>
<td>Dirlik method</td>
<td></td>
</tr>
<tr>
<td>Zhao–Baker method</td>
<td></td>
</tr>
<tr>
<td>Tovo–Benasciutti</td>
<td></td>
</tr>
<tr>
<td>method</td>
<td></td>
</tr>
<tr>
<td>Petruci–Zucarello</td>
<td></td>
</tr>
<tr>
<td>method</td>
<td></td>
</tr>
<tr>
<td>Zalaznik–Nagode</td>
<td></td>
</tr>
</tbody>
</table>

Source(s): Palmieri et al. (2017)

| Table 5. Fatigue damage estimation models for both time- and frequency-domains |
The even moments correspond to the variance $\sigma^2_X$ of the random process $X$ and its derivatives $\dot{X}(t)$ and $\ddot{X}(t)$:

$$\lambda_0 = \sigma^2_X, \lambda_2 = \sigma^2_X, \lambda_4 = \sigma^4_X$$

Based on the spectrum parameter, the expected peak occurrence frequency $\nu_0$ and expected positive zero-crossing rate $\nu_p$ can be obtained as:

$$\nu_0 = \frac{1}{2\pi} \sqrt{\frac{\lambda_2}{\lambda_0}}, \nu_p = \frac{1}{2\pi} \sqrt{\frac{\lambda_4}{\lambda_2}}$$

These parameters are usually used to determine the total load cycles in the fatigue damage estimation for the frequency-domain methods. Additionally, the irregularity factor $\alpha_1, \alpha_2$ for the spectral density PSD can be expressed as:

$$\alpha_1 = \frac{\lambda_1}{\sqrt{\lambda_0 \lambda_2}}, \alpha_2 = \frac{\lambda_2}{\sqrt{\lambda_0 \lambda_4}}$$

where $\alpha_2$ is the most commonly used parameter. When the random process tends to be a narrow-band process, the parameter $\alpha_2$ is close to 1, otherwise, tends to zero. This parameter can be further employed to define the bandwidth parameter $\epsilon = \sqrt{1 - \alpha^2}$. When $\epsilon$ is close to 0, the signal tends to be a narrow band process, otherwise, the broadband process. Based on the derivation of above-mentioned spectral parameters, the distribution of rainflow cycle can be obtained, which is considered the main difference compared to time-domain methods. Rice (1944) established the probability density function (PDF) of the peak amplitude for a general broadband process $X(t)$:

$$P_p(x) = \frac{1 - \alpha^2_2}{\sqrt{2\pi\sigma^2_x}} e^{-\frac{x^2}{2\sigma^2_x}} + \frac{\alpha^2 x}{\sigma^2_x} e^{-\frac{x^2}{2\sigma^2_x}} \Phi\left(\frac{\alpha^2 x}{\sigma^2_x \sqrt{1 - \alpha^2}}\right)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2} dt$$

Based on the S–N curve, the relationship between the stress and fatigue life is

$$S^k N = C$$

where $N$ is the number of cycles to failure at the stress amplitude $S$, $C$ and $k$ are the fatigue–strength coefficient and fatigue–strength exponent, respectively. In the time-domain method, $S$ is usually determined through rainflow counting method and is considered a deterministic parameter. Whereas, for a random process, the stress cycle amplitudes are non-deterministic parameters, which are calculated through the PDF $p(\tilde{S})$. The number of cycles $n_i$ for stress range $(S_i, S_{i+1})$ in the time period $T$ can be determined as:
Therefore, based on the Palmgren–Miner method, the fatigue damage can be expressed as:

\[
D = \sum D_i = \sum \frac{n_i}{N_i} = \sum \frac{v_p T_p(S_i) \Delta S}{N_i} = v T \int_0^\infty \frac{p(S_i)}{N(S_i)} dS
\]  

(13)

\[
D = \frac{v_p}{C} \int_0^\infty S^k p(S) dS
\]  

(14)

and the damage per unit time thus can be defined as:

\[
d = \frac{D}{T} = \frac{v_p}{C} \int_0^\infty S^k p(S) dS
\]  

(15)

It can be seen that the main work for frequency-domain method is to formulate a PDF for stress–cycle amplitude distribution. Therefore, huge works have been made to develop the rainflow PDF approximation methods, as listed in Table 6. The pioneering work was conducted by Dirlik (1985) modelling the rainflow PDF through combining one exponential and two Rayleigh probability densities. Zhao and Baker (1992) proposed a rainflow PDF through the linear combination of the Weibull and Rayleigh PDF. Based on the Rayleigh, a standard Rayleigh and a half-Gaussian distribution, Park, Choung, and Kim (2014) developed another rainflow PDF. Similarly, Jun and Park (2020) further corrected the rainflow PDF through a Rayleigh, a standard Rayleigh, a half-Gaussian and an additional exponential distribution.

Apart from the rainflow PDF approximation methods, the narrowband formulation together with the narrowband correction factor is also used to estimate the fatigue damage for a broadband random process, such as Wirsching–Light method (Wirsching & Light, 1980), Ortiz–Chen method (Ortiz & Chen, 1987) and Tovo–Benasciutti method (Tovo et al., 2002). To deal with the multimodal in the broadband process, a spectral density is usually considered as a superposition of two or more narrowband processes, and then each narrowband process-induced damage is estimated by the related approximation method, such as Jiao–Moan method (Jiao & Moan, 1990), Sakai–Okamura method (Sakai & Okamura, 1995), Fu–Cebon method (Fu & Cebon, 2000), Low method (Low, 2014) and Gao–Moan

<table>
<thead>
<tr>
<th>Models</th>
<th>Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrow band (Miles et al., 1954)</td>
<td>( p_0(s) = \frac{e^{-s^2}}{\sqrt{\pi}} )</td>
</tr>
<tr>
<td>Dirlik method (Dirlik, 1985)</td>
<td>( p_0(z) = \frac{1}{\sqrt{2 \pi}} \left( \frac{G_1}{\sqrt{2 \pi}} + \frac{G_2}{\sqrt{2 \pi}} e^{z^2} + \frac{G_3}{\sqrt{2 \pi}} e^{-z^2} \right) )</td>
</tr>
<tr>
<td>Zhao and Baker method (Zhao &amp; Baker, 1992)</td>
<td>( p_0(z) = \omega \alpha z^{\alpha-1} e^{-\alpha z^2} + (1-\omega) Ze^{-z^2} )</td>
</tr>
<tr>
<td>Park method (Part et al., 2014)</td>
<td>( p_0(z) = c_{G1} e^{z^2} + c_{G2} Ze^{-z^2} + c_{G3} \sqrt{\frac{2}{\pi}} e^{-z^2} )</td>
</tr>
<tr>
<td>Jun–Park (Jun &amp; Park, 2020)</td>
<td>( p_0(z) = Q \left( D_1 e^{z^2} + D_2 \frac{z^2}{n_{th}} + D_3 Ze^{-z^2} + D_4 \sqrt{\frac{2}{\pi}} e^{-z^2} \right) )</td>
</tr>
</tbody>
</table>

Source(s): Authors own work

Table 6. Rainflow cycle PDF approximation methods
method (Gao & Moan, 2008). Alternatively, some other researchers directly used the narrow band formulation to estimate fatigue damage for each narrow band process and studied the combination methods for individual damage obtained, such as the Lotsberg method (Lotsberg, 2005), Huang–Moan method (Huang & Moan, 2006), Han–Ma (Han, Ma, Qu, Yang, & Qin, 2016) and Bands method (Braccesi, Cianetti, & Tomassini, 2015). Considering 104 different PSD spectra and three different fatigue–strength exponents, AlesZorman, Slavić, and Boltezar (2023) compared the frequency-domain methods to the time-domain method using the numerical simulation and concluded that Ortiz–Chen, $a_0, 75$, Tovo–Benasciutti 2, Dirlik, Park and Jun–Park methods can give an acceptable engineering accuracy for small values of the $S–N$ curve slope ($k = 3.324$). For a larger fatigue strength exponent ($k = 7.3$), Ortiz–Chen, $a_0, 75$, Park, Jun-Park and Huang–Moan methods can yield a relatively smaller error below 25%. Table 1 lists the most cited frequency–domain-based fatigue damage models. Dirlik and Benasiutti (2021) performed a comprehensive review of the spectral methods, especially focusing on the review of comparison investigations for spectral methods, and concluded that Dirlik and TB methods generally give a better estimation.

The aforementioned spectral methods invariably treated the random vibration as a stationary Gaussian process. However, due to local irregularities on the track, the vibration in railway vehicle usually expresses non-stationary and non-Gaussian characteristics. A number of scholars investigated how the non-stationary and non-Gaussian affect the vibration fatigue. Rizzi, Przekop, and Turner (2011) and Kihm, Rizzi, Ferguson, and Halfpenny (2013) pointed out that for a stationary excitation the non-Gaussian signal results in a Gaussian stress response, and the non-stationarity is the origin of non-Gaussian response. Through studying the vibration fatigue life of Y-shaped specimens considering the non-Gaussianity and non-stationarity loading, Palmieri, Cesnik, Slavić, Cianetti, and Boltezar (2017) concluded that the Gaussian theoretical-based methods could be questionable when the excitation is non-stationary and non-Gaussian excitation, and the fatigue life was found to be significantly impacted due to characteristics of non-Gaussianity and non-stationarity in the vibration loading. Capponi, Česnik, Slavić, Cianetti, and Boltezar (2017) defined an index to quantify the non-stationarity of a signal and investigated the effects of non-stationary on fatigue life. The obtained results further show that the non-stationary can have significantly shorter fatigue life compared with those caused by stationary excitations.

To assess the fatigue damage arising from the non-Gaussian loading, the Gaussian-based methods were further modified. Benasiutti and Tovo (2006) utilized a nonlinear transformation to transform the Gaussian load to a non-Gaussian load and further proposed the non-Gaussian-based frequency domain method for the narrow-band approximation and TB method, as shown in Figure 20. Wolfsteiner (2017) proposed a new methodology to estimate fatigue damage of non-stationary random vibration through the decomposition of a non-stationary vibration signal into several stationary vibrations with Gaussian distributions. The essential of this method is to estimate the distribution of statistical moments over the frequency band of the given non-stationary signal. This method was further employed to assess the fatigue damage of rail guard (Wolfsteiner & Breuer, 2013). Jiang et al. investigated on the non-Gaussian random vibration fatigue analysis and accelerated test (Jiang, Tao, & Chen, 2021).

In both time- and frequency-domain methods, the $S–N$ curve is the essential part of the fatigue damage estimation, which defines the relationship between the stress range and the number of cycles to failure. This relationship is subjected to variations considering different materials and weld joint details. Therefore, a huge number of tests have been performed to establish the $S–N$ curve for the interested material or design detail. The mathematical expression of $S–N$ curves can be classified into deterministic models and probabilistic models. Castillo and Fernandez–Canteli summarize the typical $S–N$ curve models (Castillo & Fernández-Canteli, 2009), as listed in Table 7. In most industrial standards, the $S–N$ curve is
represented by the Basquin equation \( \text{OH, 1910} \), such as IIW standard, BS7608, JIS 4207, and Eurocode 3. Figure 21 illustrates the typical S–N curves defined in the IIW, BS7608, Eurocode 3 and JIS 4207. It can be seen that the S/N curves given by different standards are subjected to huge deviations, which gives a big challenge to select the right S/N curve based on the design detail. Therefore, Dong et al. proposed the structural stress-based master S/N curve in terms of the structure stress rather than the nominal stress (Dong, 2021).

5. Research gaps and discussions
The above-mentioned investigations invariably suggest that the main driving force for the vibration fatigue of railway bogie is the short-pitch irregularities-induced high frequency vibration. Whereas the current design method is still based on the quasi-static method and neglects the high frequency vibration-induced mode resonance in the structure. The methodologies used to assess the fatigue damage of structure could be considered as a
postprocess procedure for dynamic stress and focus on improving the assessment accuracy of fatigue damage for a given loading condition. The system dynamics and the fatigue strength assessment are not coupled and are considered as separated fields. It is known that the loading of fatigue strength assessment is the dynamic response of system dynamics. Therefore, the core of improving the survival probability of vibration fatigue is to develop a systematic design method coupling the strength of structure and the system dynamics of the vehicle and track system, so as to achieve a reasonable matching relationship between the considered components and the service boundary conditions.

5.1 Systematic design methodologies for the vibration fatigue
A systematic design methodology is needed to account for the contribution of mode vibration of structure through coupling the system dynamics and the strength on the basis of tradition design and verification methods. Figure 22 illustrates a suggested design flowchart for the vibration fatigue. In the quasi-static design phase, the traditional static and fatigue strength analyses are conducted based on the loads given in the standard, such as EN 13749, UIC 515–4/615–4 and JIS 4207/4208. This phase is used to demonstrate the satisfaction of current design codes based on the simulation of the FE model of interested component, which neglects the real operating conditions. Therefore, a dynamic design phase is suggested to consider the typical vibration conditions, in which a vibration spectrum obtained through either field tests or numerical simulations is considered as an input for a vibration model. This vibration model could be a single component or the whole vehicle neglecting the wheel/rail contact based on the finite element method or rigid/flexible coupled theory. The identified week point is considered as the dynamic week point since the contribution of mode vibration is taken into consideration which could lead to different week points. The week points identified in both quasi-static and dynamic phases should be treated carefully for the design of vibration fatigue. Another important aspect of this phase is to evaluate the contribution of vibration mode based on the typical vibration spectrum, where the critical geometry parameters could be adjusted to achieve a reasonable modal matching from the vibration and fatigue damage point of view. In the third phase, a more comprehensive model of vehicle/track coupled dynamics based on the rigid/flexible coupling theory is needed to simulate all possible service conditions for rail vehicle, so as to obtain the characteristic load spectrum of

<table>
<thead>
<tr>
<th>Model</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basquin</td>
<td>( \log N = A - B \log \Delta \sigma )</td>
</tr>
<tr>
<td>Stromeyer</td>
<td>( \log N = A - B \log(\Delta \sigma - \Delta \sigma_0) )</td>
</tr>
<tr>
<td>Palmgren</td>
<td>( \log(N + D) = A - B \log(\Delta \sigma - \Delta \sigma_0) )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( \log(N + D) = A - B \log((\Delta \sigma - \Delta \sigma_0)/(\Delta \sigma_i - \Delta \sigma)) )</td>
</tr>
<tr>
<td>Stößible</td>
<td>( \log N = A - B \log((\Delta \sigma - \Delta \sigma_0)/(\Delta \sigma_i - \Delta \sigma)) )</td>
</tr>
<tr>
<td>Bastenaire</td>
<td>( \log \left( \frac{N}{N_0} \right) = A \log(\Delta \sigma/\Delta \sigma_0) - B \log(\Delta \sigma/\Delta \sigma_0) + )</td>
</tr>
<tr>
<td>Spindel–Haibach</td>
<td>( +B{(1/\alpha)\log[1 + (\Delta \sigma/\Delta \sigma_0)^{-2\alpha}]} )</td>
</tr>
<tr>
<td>Castillo–Cantelli</td>
<td>( \log\left( \frac{N}{N_0} \right) = \frac{\log(\Delta \sigma/\Delta \sigma_0)}{\log(\Delta \sigma/\Delta \sigma_0)} )</td>
</tr>
<tr>
<td>Kohout–Vechet</td>
<td>( \log\left( \frac{\Delta \sigma}{\Delta \sigma_0} \right) = \log \left( \frac{N+N_1}{N_0} \right)^{\delta} )</td>
</tr>
<tr>
<td>Pascual–Meeker</td>
<td>( \log N = A - B \log(\Delta \sigma - \Delta \sigma_0) )</td>
</tr>
</tbody>
</table>

Source(s): Castillo and Fernández-Canteli (2009)

| Table 7. Typical S–N curve models |
Figure 21. S–N curves defined in IIW, BS7608, Eurocode 3, and JIS 4207

Source(s): Authors own work
the considered region. This can further facilitate the compilation of the stress spectrum of interested points, thereby the fatigue damage estimation and fatigue life prediction. To obtain a reasonable estimation, the S/N curve and fatigue damage should be selected carefully and the degradation process of loading and material properties also should be taken into consideration. This framework aims to integrate the system dynamics into the strength design of interested components so as to consider the contribution of vibration mode, which is expected to improve the survival probability of vibration fatigue.

5.2 Characterization of typical vibration spectrum of wheel/rail for railway bogie

The design of vibration fatigue should be based on the understanding of typical loads arising from coupling subsystems of railway bogie, including the wheel/rail interaction, traction and braking and aerodynamics, as well as loads from the car body. The wheel/rail interaction serves as the basic input for rail vehicles, it is thus desirable to determine the features of coupled vibration of wheel/rail considering different track properties. The dominating frequency bands, including P2 force and other dominating frequencies contributing to the formation of rail corrugation and wheel polygonal wear, should be treated carefully in the modal matching. Wu et al. summarized the typical wheel/rail coupled vibration modes of rail vehicles in the vertical direction (Wu et al., 2023), whereas the coupled vibration modes in the lateral direction are still needed to be further clarified. The axle box vibration spectrum could be taken as a representative of wheel/rail coupled vibration, and a database for characteristic vibration spectrums of axle box is thus suggested on the basis of field test measurements. This could be considered as an input for the random vibration model of railway bogie in the dynamic design phase. Regarding traction and braking-induced loads, the effects of mechanical–electrical coupled loads on railway bogie should be taken into consideration, especially for the high frequency component. For the thin-shell structure in rail vehicle, the fluid–solid coupling-induced dynamic loads should be taken into consideration through either simulation of a fluid–solid coupled dynamic model or field tests.
5.3 Systematic modal matching between the railway bogie and track system

Modal matching is the core of avoiding the vibration fatigue of railway bogie arising from the wheel/rail high frequency vibration. Theoretically, the natural frequencies of railway bogie should avoid typical dominating frequency bands of wheel/rail interaction. Whereas the principle for determination of the difference between the nature frequency and the excitation frequency still remains in question. In the mechanical vibration, the natural frequency of the system is suggested to stay away from the frequency range of bandwidth of dominating frequency. This could be still a big challenge for railway bogie design since the railway bogie is characterized by multi-modes. The natural frequencies of bogie frame cannot completely avoid all possible exciting frequencies, which mean that the railway bogie has to withstand the impacts due to the modal vibration arising from external excitations. A sophisticated modal matching criterion is thus needed to achieve the reasonable matching between the railway bogie and the coupling boundary.

5.4 Fatigue damage assessment for full life cycle of railway bogie

The fatigue damage assessment for railway bogie still shows deviations with respect to the real operation conditions, although huge efforts have been made. For vibration fatigue, the failure of interested components usually failed due to the structural resonance at a relatively high frequency range. Whereas the definition of S/N curve in the current standard neglects the influence of loading frequency and few of investigations address the frequency effects of S/N curve for railway bogie material. Although a number of fatigue damage assessment models have been developed based on different considerations, the linear Palmgren–Miner model is still a widely used model in the application due to the features of simplicity and convenient. For the frequency domain method, a well-validated model should be established to deal with the non-Gaussian and non-stationary signal in the railway. The load spectrum definition is essential for fatigue damage assessment. In the railway, the wheel/rail interaction is subjected to a degradation process due to the formation of uneven wear on the wheel and railhead surfaces, thereby the evolution of loading for railway bogie. For the full life cycle fatigue damage assessment, the degradation process in the load spectrum has to be addressed in the load definition, especially the evolution process arising from wheel OOR. In addition, it is suggested that the fatigue mechanism and damage mechanism under resonance conditions should be further studied and expounded, which could facilitate to improve the structural design so as to reduce the structural resonance-induced fatigue failure.

5.5 Mitigation of wheel/rail high frequency vibration

The vibration fatigue of railway bogie is mainly caused by the wheel/rail high frequency vibration. It is thus desirable to study methodologies to reduce the wheel/rail high frequency vibration-induced impacts, such as suppression of formation of wheel polygonal wear and rail corrugation, a reasonable limit value for the short-pitch irregularities and vibration reduction for high frequency vibration transmission.

6. Conclusions

The vibration fatigue of railway bogie has been considered the main concern of railway operators due to its highly adverse influences. This paper deals with the wheel/rail high frequency vibration-induced vibration fatigue in the railway bogie, mainly focusing on the characteristics of wheel/rail high frequency vibration, high frequency vibration-induced vibration fatigue failures in railway bogie, and the methodologies used to assess the vibration fatigue-induced fatigue damage, then the research gaps have been further discussed. This review is expected to explore the vibration fatigue of railway bogie and facilitate the development of the forward design methodology of vibration fatigue for railway bogie.
References


**Further reading**


EN 13749 (2011). Railway applications - wheelsets and bogies - method of specifying the structural requirements of bogie frames.

ERRI B12/RP 17 (1997). Programme of tests to be carried out on wagons with steel underframe and body structure.


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