Mental time travel and the valuation of financial investments: analysing five biases that cause pricing anomalies

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Abstract

Purpose – The purpose of this paper is to analyse five biases in the valuation of financial investments using a mental time travel framework involving thought investments – with no objective time passing.

Design/methodology/approach – An investment’s initial value, together with any periodic funding cash-flows, are mentally projected forward (at an expected rate of return) to give the value at the investment horizon; and this projected value is mentally discounted back to the present. If there is a difference between the initial and present values, then this can imply a bias in valuation.

Findings – The study identifies (and gives examples of) five real-world valuation biases: biased funding cash-flow estimates (e.g., mega infrastructure projects); biased rate of return projections (e.g., market crises, tech stock carve-outs); biased discount rate estimates (e.g., dual-listed shares, dual-class shares, short-termism, time-risk misperception, and long-termism); time-duration misestimation or perception bias when projecting (e.g., time-contracted projections which lead to short-termism); and time-duration misestimation or perception bias when discounting (e.g., time-extended discounting which also leads to short-termism). More than one bias can be operating at the same time and we give an example of low levels of retirement savings being the result of the biased discounting of biased projections. Finally, we consider the effects of the different biases of different agents operating simultaneously.

Originality/value – The paper examines key systematic misestimation and psychological biases underlying financial investment valuation pricing anomalies.

Keywords Financial investment valuation, Thought investment, Mental time travel, Rational and irrational expectations, Projecting, Discounting, Biases, Heterogeneous agents

Paper type Research paper

1. Introduction

Valuing financial investments can be viewed as mental time travel (“MTT”). Before making an investment decision, an investor might conduct the following “thought investment”. An investment’s initial value is projected forward (at an expected rate of return) to some future date (the investor’s planned investment horizon) and then discounted back to the present, giving a present value. This is then compared with the initial value, allowing the investor to decide whether to invest for real. Clearly, when the projection path matches the discount path, an initial investment of say $1 has a present value of $1, making the investment marginally attractive. However, if projection and discount paths differ, present value could either exceed or fall below initial value, making the investment either positively attractive or positively unattractive, respectively.
The present value that results from this thought investment is this investor’s “personal value” and will reflect their biases. We define a bias as a systematic deviation from rational valuation; it will be the result of either a systematic misestimation or a psychological trait or a combination of these. An investment’s personal value might differ from its “market value” which is its current price traded in a financial market and results from the aggregation of the personal values of all market participants, weighted by the size of their transactions. This implies that market value can also be subject to bias. Personal value and market value can differ from “intrinsic (or fair) value” which is “a measure of what an [investment] is worth [and] is arrived at by means of a [rational] objective calculation or complex financial model” [1] and is therefore free of biases [2].

Blake and Pickles (2022) established the MTT valuation framework and used it to explain how a closed-end investment company might trade at a market price other than its net asset value [3] – which is an anomaly in a rational world. In Section 2 of the present paper, we generalise the framework to accommodate investments with periodic funding cash-flows and to introduce five parameters that measure the degree of rationality of (or, alternatively, the degree of bias in) valuation. Section 3 then considers a range of real-world examples of biased financial investment valuations. Section 4 concludes.

2. The mental time travel valuation framework

2.1 The basic framework
In the basic MMT framework, an initial value (IV) is projected forwards in time at an annual expected rate of return \( R \) over an investment horizon of \( T \) years and the projected future value (FV) is discounted back at an annual rate \( r \) over \( T \) years to give a present value (PV). Projection and discounting are typically exponential with value changing over time at a constant rate [4].

The basic projection equation is:

\[
FV = IV \times (1 + R)^T \tag{1}
\]

The basic discounting (or valuation) equation is:

\[
PV = FV / (1 + r)^T = IV \times (1 + R)^T / (1 + r)^T \tag{2}
\]

If the return (or projection or growth) rate \( R \) matches the discount rate \( r \) over the investment duration \( T \), IV equals PV. If \( R \) exceeds \( r \), PV will exceed IV; if \( r \) exceeds \( R \), PV will be less than IV.

Valuations reflect investor perceptions and preferences [5]. For example, different perceptions of the riskiness of an investment will lead to different values of \( R \). Similarly, preferences (e.g., in respect of risk or loss attitudes) can influence discount rates.

2.2 A general mental time travel valuation model
We now generalise the basic equations both to allow for periodic funding cash-flows and to introduce bias parameters. This will allow us to explain some key examples of pricing anomalies observed in the real world [6].

The general projection equation is:

\[
FV = D(1) \times \alpha \times (1 + R \times B)^{T \times 4^{(1)}} + D(2) \times \alpha \times (1 + R \times B)^{(T-1) \times 4^{(2)}} + \ldots + D(T) \times \alpha \times (1 + R \times B)^{4^{(T)}} \tag{3}
\]
Here, the future value of an investment with an investment horizon of $T$ years is the sum of the compounded projected net cash-flows, where:

1. $D(t)$ is the net funding cash-flow at time $t$ that is rationally expected, assumed to occur at the beginning of the relevant period. It will be positive where the sum of cash in-flows (from the issuance of new equity or debt) exceeds that of cash out-flows at time $t$ and it will be negative otherwise;

2. $\alpha$ is a measure of investor rationality concerning the expected net funding cash-flows, with $\alpha = 1$ implying rational expectations, $\alpha < 1$ downward-biased expectations, and $\alpha > 1$ upward-biased expectations;

3. $R$ is now defined as the rationally expected rate of return on the investment. It includes all changes in value (arising from, e.g., the revenues and costs generated by the investment) other than funding cash-flows;

4. $B$ is a measure of investor rationality concerning the expected return, with $B = 1$ implying rational expectations, $B < 1$ downward-biased expectations, and $B > 1$ upward-biased expectations;

5. $T$ is the objective investment duration in years;

6. $\Gamma(t)$ is a measure of investor rationality concerning either the estimated or the perceived investment duration for the $t$th funding cash-flow, where $\Gamma(t) < 1$ implies either an underestimation or perceived time-contraction compared with objective time and where $\Gamma(t) > 1$ implies either an overestimation or perceived time-extension.

The general discounting (or valuation) equation is:

$$PV = D(1) \ast \alpha \ast (1 + R \ast B)^{T \ast \Gamma(1)} / (1 + r \ast \beta(1))^{T \ast \gamma(1)} + D(2) \ast \alpha \ast (1 + R \ast B)^{(T-1) \ast \Gamma(2)} / (1 + r \ast \beta(2))^{T \ast \gamma(2)} + \ldots + D(T) \ast \alpha \ast (1 + R \ast B)^{T \ast \Gamma(T)} / (1 + r \ast \beta(T))^{T \ast \gamma(T)}$$

where:

1. $r$ is now the rationally expected discount rate;

2. $\beta(t)$ is a measure of investor rationality concerning the discount rate, with $\beta(t) = 1$ implying rational expectations, $\beta(t) < 1$ downward-biased expectations, and $\beta(t) > 1$ upward-biased expectations;

3. $\gamma(t)$ is a measure of investor rationality concerning either the estimated or the perceived duration of the discounting period for the $t$th funding cash-flow, where $\gamma(t) < 1$ implies either an underestimation or perceived time-contraction compared with objective time and where $\gamma(t) > 1$ implies either an overestimation or perceived time-extension.

Bias-free valuation requires that $\alpha = B = \beta(t) = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$). A value for any of these parameters differing from unity implies some form of bias. As noted earlier, a bias is a systematic deviation from rational valuation and can be due to either systematic misestimation or a psychological trait. In the first case, the investor seeks the best estimates for the parameters of Equations (3) and (4), but uses a model that systematically misestimates...
one or more of them. In the second case, some form of psychological bias prevents the use of the rational or objective value of a parameter. We will show below that some systematic misestimations may also be psychological [16].

Valuation anomalies can therefore occur for the following reasons: biased cash-flow estimates ($\alpha \neq 1$); biased expected return estimates ($B \neq 1$); biased discount rate estimates ($\beta(t) \neq 1$); time-duration misestimation or perception bias when projecting ($\Gamma(t) \neq 1$); and time-duration misestimation or perception bias when discounting ($\gamma(t) \neq 1$).

We can illustrate the effect of these biases using a simplified example of Equations (3) and (4) in which $D(1) = IV$, and $D(2) = \ldots = D(T) = 0$. When valuation is bias-free, an investment project with a required investment (IV) of $1 and an $r$ of 10% will have a FV of $2.59 after 10 years which, if discounted at an $r$ of 10%, will have a PV of $1 – as shown in the upper curve in Figure 1. However, as Figure 1 also shows, such an investment project will have a PV of $0.60 in each of the following circumstances:

1. underestimating the funding required to complete the project ($\alpha = 0.60$, which leads to a downward-biased FV = $1.55$);
2. underestimating the expected return on the project ($B = 0.45$, which leads to a downward-biased FV = $1.55$);
3. excessive discounting ($\beta = 1.58$) of an unbiased FV = $2.59$;
4. an underestimated or time-contracted investment project duration ($\Gamma = 0.46$, which leads to a downward-biased FV = $1.55$); or
5. an overestimated or time-extended discounting duration ($\gamma = 1.53$) of an unbiased FV = $2.59$.

In Section 3, we discuss real-world examples of each of the above causes of biased valuation.

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**Figure 1.** The effect of the bias parameters on valuation

**Source(s):** Authors’ own work
It is important to note that, in the MTT valuation framework, projection and discounting are instantaneous, so no objective time passes. This precludes the possibility that an investor observes deviations between the projected and actual future values – which might highlight irrationalities and enable revisions of the projection. Although important in real-world investing, this is not relevant in thought investments. Further, we argue that most investors must implicitly conduct such thought investments when making investment decisions.

We also need to take account of the following. First, it can be difficult to separate out the different effects of the bias parameters. So Equations (3) and (4) have “identification” issues to resolve which might require independent evidence on investor perceptions, preferences and estimation models. In the real-world examples we now discuss, we impose restrictions on the free parameters in Equations (3) and (4) to ensure the equations are identified: these are summarised in Table 1 in Section 4. Second, as already noted, preferences will influence the size of some parameters, such as $\beta$, which will reflect investors’ risk or loss attitudes and the shape of their utility or loss functions [17]. These factors are hard to quantify, but it is clear how there can be a de-coupling of discounting from projecting.

3. Examples of biased valuation
3.1 Biased funding cash-flow estimates ($\alpha \neq 1$)
3.1.1 Mega infrastructure projects 1. There are two common problems with mega infrastructure projects ("MIPs"): costs are underestimated and benefits are overestimated (Department for Transport, n.d.). In this section, we consider the first problem: downward-biased funding projections ($\alpha < 1$).

MIPs often feature a cocktail of psychological biases. First, “optimism bias” which can be defined as “a cognitive bias leading people to think they are more likely to succeed, or are less at risk of failure or of experiencing a negative event, than they are” (Behavioural Insights Team, 2017).

Optimism bias is self-deception. Its intentionally more deceitful partner is “strategic misrepresentation” – deliberately underestimating funding requirements (Flyvbjerg, 2006). Indeed, where a project is part-funded by the State, there is a perverse incentive for the State to award the contract to the most optimistic (or misrepresented) bid to (mistakenly) minimise State support (Goldsmith and Boeuf, 2016). This is the “winner’s curse” – the tendency for the successful auction bid to be that which most exceeds intrinsic value (Locatelli, 2018).

We take, as our example, High Speed Rail 2 (HS2). An “ambitious, controversial project for a high-speed rail line” connecting some of the UK’s largest cities (London, Manchester, Birmingham and East Midlands), HS2 is a State-funded project. At the heart of its strategic case is the desire to address capacity constraints on the north-south rail links in England. Proponents of the project said it would improve transport times, create jobs and help the country’s economy. In 2013, HS2 was estimated to cost £37.5bn (in 2009 prices). By 2020, the funding estimate had increased to £72bn–£98bn (in 2019 prices). In 2022, this estimate was further revised to £67bn–£86bn, following the decision to curtail the building of the eastern leg to East Midlands instead of Leeds to save costs (Pickett and Hirst, 2022). In October 2023, it was announced that the HS2 link between Birmingham and Manchester would be cancelled, due to further cost escalation. Instead, £36bn of the funding saved from the cancellation would be redirected to alternative transport projects (Bovenizer, 2023). HS2 is a clear example of underestimating the capital required to complete the project, so that $\alpha \ll 1$.

3.2 Biased expected return projections ($B \neq 1$)
3.2.1 Mega infrastructure projects 2. We now turn to biased rate of return assumptions in MIPs (e.g., overestimated project benefits).
MIPs typically have a three-stage business model: a planning phase; a construction phase in which projected costs exceed revenues; and an operating stage with positive returns (Park and Chang, 2013). Optimism bias and strategic misrepresentation occur during stage 1 and so are commonly termed the “planning fallacy” (Behavioural Insights Team, 2017).

A common stage 2 feature is the “sunk cost fallacy” – the decision to continue with a project, “throwing good money after bad”, because of already incurred costs, even when future costs outweigh future revenues (Behavioural Insights Team, 2017). The sunk cost fallacy is related to the “status-quo bias” – the tendency to stick to a current course of action – which is itself a consequence of “loss aversion” – the desire to avoid crystallising losses because we feel them more strongly than equivalent gains (Behavioural Insights Team, 2017). Loss aversion can, in turn, be explained by “Prospect Theory” which hypothesises that decisions are based not on absolute values, but values relative to a reference value. Once losses (relative to the reference value) have been incurred, continued losses are undervalued, whereas future gains are overvalued (Kahneman and Tversky, 1979; Behavioural Insights Team, 2017). In terms of project projections, this implies $B < 1$ for the cost component of the expected net return and $B > 1$ for the revenue component. Furthermore, the potential volatility of a MIP’s costs and revenues means that the rationally expected return may well be time-varying.

Finally, in stage 3 of the project, we have the “hiding hand principle” (Hirschman, 2011) – the proposition that, initially, ignorance of obstacles enables the commencement of a project and, subsequently, when the obstacles are recognised, creative solutions are found to complete it, built around the argument that the sunk costs already incurred render the project “too big to fail” (Goldsmith and Boeuf, 2019). Just as we underestimate the challenges of a MIP, so we underestimate our capacity to respond to those challenges with creative solutions. This can have a positive effect on the outcome.

We take, as our example of a MIP with biased return projections, Groupe Eurotunnel, the builder and operator of the Channel Tunnel (or Chunnel) which connects the UK and France by rail beneath the English Channel.

Funded entirely by private finance, in 1987, Groupe Eurotunnel’s IPO raised £770 million with listings in London and Paris partly on the back of a “worst case” promised 18.8% annual return (Goldsmith and Boeuf, 2019, citing Comfort, 2006). Shares were issued at an offer price of £3.50 and rose to over £11 in June 1989 (suggesting matching and increasing investor optimism) before eventually falling to 71p at the end of 1999. Construction delays, an overall cost overrun of 122%, demand for its passenger and freight transport services way below forecast and a massive debt burden led to rights issues in 1990 and 1994, followed by debt and other financial restructuring in 1997 and 2007. In 2017, Groupe Eurotunnel changed its name to Getlink and became a European public company.

It is apparent that the project’s initial rate of return projections were irrationally over-optimistic (or, as a cynic might say, “rationally” over-optimistic, otherwise the project would never have occurred). In terms of Equation (4), there were significantly biased expectations. Expected revenues were vastly overestimated from the outset and construction costs were more than double initial predictions (Global Infrastructure Hub, 2020).

Expert conclusions as to the Channel Tunnel’s place in history differ. According to Flyvbjerg (2014), it was a technological success but a financial failure. He concludes that the project proved to be non-viable with an internal rate of return on investment of minus 14.5% and with a total loss to the British economy of $17.8 billion. By contrast, perhaps with the benefit of greater hindsight and a longer assessment period, Goldsmith and Boeuf (2019) take a more positive view, calculating that, with a public-sector investment subsidy of 50% of the capital costs, the project could have generated an economic rate of return of between 3 and 6% over the life of its operating concession (now extended to 2086). At the time Goldsmith and Boeuf were writing (2019), Getlink was in robust financial health, and they conclude that if one takes a “holistic, multi-disciplinary, multi-stakeholder long run perspective”, the Channel...
Tunnel is a victory of optimism over adversity, a great engineering feat which stimulated the development of northern European high-speed rail networks and a demonstration of visionary political and corporate leadership. Shareholders in these other MIPs therefore benefitted from the Channel Tunnel’s completion, even if the original shareholders did not.

### 3.2.2 Market crises.

As noted earlier, it is usually hard to disentangle the valuation effects of projection and discount rates, given that neither is observable, but opportunities to do so arise occasionally at times of extreme price volatility, such as that seen during the Covid-19 pandemic. As Böni and Zimmermann (2021) observe, in the four weeks to 23 March 2020, the S&P 500 fell by almost 34% before recovering 59% of that loss in the succeeding period to 30 April 2020. This suggests that expectations were biased, in opposite directions, both immediately before and after the index fall. Böni and Zimmermann (2021, p. 1) confirm that, during the four-week period, there was a de-coupling of discounting from projecting: “stock returns were largely affected by a change in the long-run implied projection rate and only to a lesser extent by a change in discount rate”. In other words, at the height of the crisis, expectations about future growth rates were over-optimistic before being over-corrected, whilst discount rates remained broadly unchanged (i.e., $B$ moved from being $>> 1$ to being $<< 1$, and then recovered partially, but $\beta$ stayed more or less the same).

### 3.2.3 Tech stock carve-outs.

Over-optimistic expectations are a feature of research by Lamont and Thaler (2003) into US “tech stock carve-outs” – offerings of part of a subsidiary’s equity followed by the later spin-off to the parent company’s shareholders of the remaining equity. Lamont and Thaler’s prime example is Palm, a hand-held computer manufacturer, and its parent 3Com, a profitable IT company. In March 2000, 3Com sold 5% of Palm to the public and stated its intention to spin off the remaining equity. 3Com shareholders, being entitled to about 1.5 Palm shares per 3Com share, the 3Com price should have been about 1.5 times that of Palm plus the per share value of 3Com’s residual businesses. After its first day of trading, Palm closed at $95 per share, implying a 3Com price of about $145. Instead, 3Com traded at $81.

According to Lamont and Thaler (2003, p. 33), “Prior to the IPO, irrational optimists who desire to own Palm have to hold 3Com instead. 3Com trades in the optimistic segment of the market. Once the IPO occurs, these optimists buy Palm directly (ignoring the cheaper alternative of holding 3Com). 3Com now trades in the more rational segment of the market, and its price falls to the rational price”.

Using a Figure 1 analogy, it was as if, prior to the IPO, each intrinsic $1’s worth of 3Com’s shares was irrationally projected ($R = 0.038, B = 2.63 >> 1$) to grow to, say, $2.59 in 10 years’ time with this future value being rationally discounted ($r = 0.038, \beta = 1$) to a price of $1.79. Post-IPO, rational expectations ($R = 0.038, B = 1$) reduced the projected value to, say, $1.45 ($2.59*($81/$145)) which, when discounted rationally, returned the price to an intrinsic value of $1 (equal to $1.79*($81/$145)).

### 3.3 Biased discount rate estimates ($\beta(t) \neq 1$)

#### 3.3.1 Dual-listed shares.

Dual-listed shares provide an example of inconsistent discounting, which might itself be considered irrational. Eurotunnel PLC and its French twin Eurotunnel SA (jointly known as Groupe Eurotunnel) were “dual-listed” or “Siamese twin” companies – a pair of corporations each with its own share listing whose charters fix the division of common cash-flows. A modern-day example is Rio Tinto PLC and Rio Tinto Limited which are listed in London and Sydney, respectively. The twins’ share prices, adjusted for currency differences, should move in lockstep, in a ratio given by the proportional division of cash-flows. But this is not so in practice – rather, common expectations are discounted at different rates (implying that their $\beta$s differ). It is difficult to see how both valuations can be unbiased. Froot and Dabora (1999) show that the twins’ share prices are correlated with the stock-market indexes of the countries in which they are most actively traded. In a sample of 12 dual-listed companies during the period 1980–2002, de Jong et al. (2009) find mean deviations from
theoretical price parity ranging from 4 to 12%, with individual-year deviations of over 15% for all companies.

3.3.2 Dual-class shares. Similar considerations apply to “dual-class” shares – two classes of a company’s equity with identical economic interests (with the same earnings and dividends per share) but with differing voting rights. Typically, those owned by the company’s founders or controlling shareholders have greater voting powers than those held by outsiders.

According to Cremers et al. (2018), about 15% of US IPOs in the years 2011–2017 had a dual-class structure – and almost 30% in 2017–2019, according to Aggarwal et al. (2022). The theory here is that a controlling interest through voting shares gives management some protection from shareholder pressures. The argument continues that the entrepreneur-founders of businesses bring so much value to their companies that they deserve greater control than do ordinary shareholders: entrepreneurs should be free to fully develop their visions without, for example, having to worry about takeover threats (Allaire, 2019). On the other hand, those opposed to differentiated voting rights, such as the fund manager Blackrock, are sensitive to the risk of majority shareholders using their voting powers to extract private benefits from the companies they control.

Where the projected cash-flows attributable to different classes of share are identical, any differences in their prices may be due to differences in discount rates (their $\beta$s). A survey of investors indicates that superior voting shares generally trade at a premium to limited voting shares of up to 30% – implying that the $\beta$s for superior voting shares are lower than for limited voting shares (Blackrock, n.d.). While it may not be irrational for limited voting shares to trade at a discount to superior voting shares, does a 30% discount indicate some degree of bias?

3.3.3 Short-termism I – undervaluation of listed securities due to excessive discounting. A body of empirical research which analyses the discount rates implicit in the prices of UK and US listed equities includes: Miles (1993); Haldane and Davies (2011) and Davies et al. (2014) (collectively Haldane et al.); and Sampson and Shi (2020) (S&S). In these studies, share price is compared with intrinsic value. Where, because of excessive discounting it is lower, valuation is deemed to be irrational and the phenomenon is known as “short-termism” [18]. Short-termism leads not only to companies being undervalued, but also to profitable long-term investment projects being rejected because they appear to have a negative net present value [19].

These researchers all find evidence of excessive discounting: Miles, in the UK equity market in the period 1980–88; [20] Haldane et al. in the UK and US equity markets in 1995–2004; [21] and S&S for securities listed on the New York Stock Exchange and the NASDAQ exchange over the period 1980 to 2013. This finding held across many industrial sectors over the period. However, we will show later (Section 3.7) that the recent tech revolution in the US has reduced short-termism in the US equity markets.

3.3.4 Short-termism II – undervaluation of listed securities due to time-risk misperception. Another possible cause of short-termism is investment “time-risk” misperception – the risk that an investment project might generate positive cash-flows for a shorter period than is rationally anticipated [22]. This causes investors to apply excessively heavy discount rates to more distant cash-flows due to the greater perceived risk that they will not materialise. In terms of Equation (4), $\beta(t) > \beta(t-1) ≥ 1$, indicating that investors perceive time-risk increasing with the discounting duration $t$ [23].

Miles (1993) finds evidence that the stock market discriminates between short-term and long-term cash flows with, for example, cash flows accruing more than five years in the future being discounted at twice the rate of shorter-term flows, implying that $\beta(t) = 1$ (for $t = 1,5) < \beta(t) = 2$ (for $t > 5$). Applying these parameters to the illustrative investment in Figure 1, and assuming all other parameters are equal to unity, an IV of $1$ has a PV of $0.65$.

Returning to the Channel Tunnel example, we noted that MIPs typically have a three-stage business model: planning, construction and operation. Viewed separately, the time-risks in stage 3 are likely to be lower (possibly considerably lower) than those in stage 2, implying that $\beta$(stage 3, conditional on stage 2 completion) $< \beta$(stage 2). However, taken
together, the risk that stage 2 is abandoned introduces the risk that stage 3 is never reached, implying that it is possible for the unconditional $\beta_{(\text{stage 3})} \rightarrow \infty >> \beta_{(\text{stage 2})}$.

3.3.5 Long-termism – overvaluation of listed securities due to insufficient discounting. At certain times and for certain stock market securities and sectors, there is evidence of long-termism – of the stock market overvaluing listed securities because the discount rate is too low, implying that with $\beta < 1$. Haldane et al., for example, report evidence of long-termism in the financials’ sector for the period 1985–1994 [24].

3.4 Time-duration misestimation or perception bias when projecting ($\Gamma(t) \neq 1$)
It is important to recall that, in the MTT valuation framework, projection and discounting are instantaneous, so no objective time passes [25]. Given this, there are two types of future time-duration bias, as noted above.

First, future event duration estimates can be erroneous. Investors can simply underestimate how long it will take to complete a project, as in the case of Eurotunnel (Roy et al., 2005). One view attributes this to optimism bias. Another explanation is that future duration projections are based on past duration memories, but these memories are systematic underestimates (Roy et al., 2005) [26].

Second, time perception bias may be psychological. Research by Zauberman et al. (2009) and Bradford et al. (2019) into consumer decision-making finds that, for many of us, subjective perceptions of future time are transformations of objective future time. For example, for some people, subjective time itself is contracted (Kim and Zauberman, 2009) [27].

Suppose that the length of all future time periods is equally contracted, such that subjective time is just under half the length of objective time ($\Gamma = 0.46$). $1$ invested at $R = 0.10$ in subjective time (4.6 years) will have a projected value of $1.55$ which, when, rationally discounted (at $r = 0.10$) over objective time (10 years), gives a PV of $0.60$.

So, the rational discounting of time-contracted projections provides another possible explanation of short-termism. But if investment durations are time-contracted, why is discounting not similarly affected? The answer may be asymmetrical time-duration perception, or a “temporal Doppler effect” (Caruso et al., 2013), whereby future events are psychologically closer than past events of equivalent objective temporal distance. If we think of projecting as mentally visiting the future and discounting as mentally returning to the present from the future, [28] then, perhaps, the future seems closer to the present (more contracted) when projecting than does the present to the future when discounting. This would imply that $\Gamma(t) < 1$, while $\gamma(t) = 1$. So even if $R = r$ and $B = \beta = 1$, price can be less than intrinsic value.

3.5 Time-duration misestimation or perception bias when discounting ($\gamma(t) \neq 1$)
The problem of time-duration misestimation or perception bias also applies to discounting. Here we will focus on perception bias.

The time-extended discounting ($\gamma(t) > 1$) of rational projections can produce the same result as the rational discounting of time-contracted ($\Gamma(t) < 1$) projections (PV = 0.60) if we project the $1$ investment at $R = 0.10$ over 10 years in objective time and discount the projected value at that rate over subjective time in which a calendar year equals just over one-and-a-half subjective years ($\gamma(t) = 1.53$) for all time periods. This implies that year-1 cash-flows are discounted at the rational rate appropriate to 18-month cash-flows; cash-flows in year-5 are discounted at a rate more appropriate to year-8, etc.

Miles’ (1993) analysis of the UK stock market for the period 1980–88 revealed estimates of $\gamma$ “around 1.8”, the result of averaging values of $\gamma(t) = 1$ (for $t = 1.5$) and $\gamma(t) = 2$ (for $t > 5$). This implies that a $1$ invested at $R = 0.10$ over 10 years in objective time and discounted at that rate over subjective time in which $\gamma(t) = 1.8$ for all time periods has a PV of $0.47$. This is also broadly consistent with Haldane et al.’s finding that in “the UK and US, cash-flows 5 years’
ahead are discounted at rates more appropriate 8 or more years hence; 10-year ahead cash-flows are valued as if 16 or more years ahead; and cash-flows more than 30 years ahead are scarcely valued at all”. As Haldane et al. put it, the “long is short” (Haldane and Davies, 2011, page 1).

Now Zauber et al. (2009) find, for the subjects in their study, that subjective time is contracted, not extended, objective time. However, Damasio (2002) considers reasons why time perception might be extended. One possible reason is “subadditive discounting” (Read, 2001; Read and Roelofsma, 2003), whereby subjects’ discount rates increase when their time-durations are broken into subintervals, that is, they are less patient (per-time-unit) over shorter intervals. One implication of this is that quarterly, or even half-yearly, financial reporting requirements might extend investors’ perceived discounting time-durations and thereby have the perverse effect of reducing investor valuations of companies, other things being equal – a case of information overload (Hemp, 2009) and another potential contributor to short-termism. Interestingly, Zauber et al. (2009) themselves found that time-contraction reduced when time-durations were divided into sub-intervals.

The challenge, again, is to understand why the bias does not affect projecting and discounting equally, that is, why is the bias asymmetric? It might be possible to provide an explanation in the case of, say, bonds and preferred shares. Their projected values are immune to time-duration misperception because their expected cash flows are fixed, or anchored, in objective time; they pay coupons and dividends and repay principal (in the case of bonds) on pre-determined calendar dates. But these temporal anchors do not necessarily apply to discount rates.

3.6 Biased discounting of biased projections ($\beta(t) \neq 1; B \neq 1$)

It is, of course, possible for more than one bias to be operating at the same time.

Blake and Pickles (2021) apply the MTT valuation framework to the valuation of pension contributions and show how “exponential growth bias” (underestimating the effects of a compound projection rate, $B < 1$) might combine with “present bias” (overestimating the near-term discount rate, $\beta(t) > 1$ for low values of $t$) to undervalue pension investments. A body of independent research (Goda et al., 2019) identifies these two biases as significant predictors of (generally low) retirement savings.

3.7 The biases of heterogeneous agents

Finally, we consider the effects of the different biases of different agents operating simultaneously.

Barzuza and Talley (2020) portray financial investment valuation as a tussle between managerial and investor expectations, perceptions and preferences. Under the right circumstances, they suggest that one can counterbalance the other. For example, upward-biased managerial projections will be neutralised if they are discounted by investors at the same irrationally expected projection rate ($r = R$ and $B = \beta > 1$). In this case, price equals intrinsic value. At other times, investor short-sightedness (“myopia”) – a preference for current rather than future cash-flows (Hirshleifer and Chordia, 1992) – might prevail, leading to the sort of excessive discounting ($r./\beta > R.B$) we discussed earlier, resulting in market price being less than intrinsic value. Miles (1993) touched on this, referring to theoretical models based on the existence of asymmetric information between managers and shareholders which result in positive net present value MIPs being systematically rejected. Further, Haldane et al. show that if investors are myopic, it becomes rational for managers to commit relatively fewer resources to investment and relatively more to maintaining a constant dividend stream.

Bainbridge (2021) points out that the biases can differ even within the same group of agents, for example, amongst managers, shareholders and even stock exchanges. In recent years, US exchanges have attributed higher valuations to companies listed on their markets than have European exchanges to theirs. This is so across all industries and applies especially to high-growth tech companies (e.g., Apple and Google). In part, this is attributable
to the much larger pool of liquidity found in the US and partly to the much lower risk tolerance of many European equity investors. The implication is that US investors’ prospective time duration perceptions exceed those of their European counterparts ($\Gamma(\text{US}) > \Gamma(\text{Europe})$) and that they discount projected corporate earnings less steeply ($\beta(\text{US}) < \beta(\text{Europe})$). In short, US investors now suffer less from short-termism than European investors (and than they did previously) (Martin and Asgari, 2023). But if European investors have limited funds to invest and are risk averse, short-termism may not be irrational for them. In terms of biases then, it may be that projected earnings are discounted more lightly on US exchanges ($\beta(\text{US}) < 1$) and more heavily on European exchanges ($\beta(\text{Europe}) > 1$).

Furthermore, biases can pull in opposite directions. To illustrate, although, in the case of some MIPs, there is evidence of managerial long-sightedness ("hyperopia") – a preference for future over current cash-flows – there are also countervailing examples of corporate myopia, such as the cutting of capital investment and increasing debt to fund share buybacks, incentivised by career-length defining time-durations and by the perception that the market rewards short-term performance. Similarly, just as there are short-sighted activist-investors, so too are there shareholders (e.g., family offices, endowments and sovereign wealth funds) whose time-durations match the potentially infinite lives of corporations. Bainbridge also points out that, because shareholders can diversify away firm-specific risk, they may be less risk averse than are managers (i.e., their $r^*\beta$s may be lower than managerial $R^*B$s). The Kay Review of UK equity markets (2012, paragraph 5.3, p. 37) notes the distinction suggested by the Investment Management Association between traders and investors. Traders make decisions based on a framework of days, hours, minutes, and, even, seconds, whereas investors make decisions based on time-scales that span weeks, months, quarters, and years (The Econophysics Blog, 2006).

Traders have traditionally been subdivided into informed traders (who trade on the basis of fundamental news) and noise traders (who trade on the basis of irrelevant information such as recent price movements and so can end up buying high and selling low). While informed traders can attempt to arbitrage away the pricing anomalies created by noise traders, the increased price volatility generated by noise traders makes arbitrage risky and prices can deviate from fundamental values for some time (Shleifer and Summers, 1990). We conjecture that similar issues would arise if informed traders attempted to exploit the misestimation and psychological biases we have identified in this study. Furthermore, it may be that even informed traders are biased.

4. Conclusions
Our thesis is that valuing financial investments can be understood neatly within a framework of mental time travel: a series of funding cash-flows is projected forward in time at the investment’s expected rate of return; and the projected value is discounted back to the present. Valuation anomalies can arise from this mental journey because of one or more of five biases.

We provide examples of: biased funding cash-flow estimates (e.g., mega infrastructure projects); biased return rate projections (e.g., market crises, tech stock carve-outs); biased discount rate estimates (e.g., dual-listed shares, dual-class shares, short-termism, time-risk misperception, and long-termism); time-duration misestimation and perception bias when projecting (e.g., time-contracted projections which lead to short-termism); and time-duration perception bias when discounting (e.g., time-extended discounting which also leads to short-termism). More than one bias can be operating at the same time and we give an example of low levels of retirement savings being the result of the biased discounting of biased projections. Finally, we consider the effects of the interaction of the different biases of different agents.

We accept that it can be hard in practice to uniquely identify the impact of the different causes of irrational valuation, especially when more than one bias may be operating at the same time. Table 1 lists the identification requirements for the use of Equation (4) (and by implication Equation (3)) in the real-world examples examined in Section 3.
<table>
<thead>
<tr>
<th>Nature of bias</th>
<th>Example</th>
<th>Parameters of interest</th>
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<tr>
<td>Funding cash-flow estimates ($\alpha \neq 1$)</td>
<td>Mega infrastructure projects I</td>
<td>$\alpha &lt; 1$ (optimism bias, strategic misrepresentation) $B = \beta(t) = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Return rate projections ($B \neq 1$)</td>
<td>Mega infrastructure projects II</td>
<td>$B &gt; 1$ (optimism bias, strategic misrepresentation, hiding hand principle, too big to fail) $\alpha = \beta(t) = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Market crises</td>
<td>$B &gt;&gt; 1$ (over-optimism), followed by $B &lt;&lt; 1$ (over-pessimism)</td>
<td>$\alpha = \beta(t) = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Technology stock carve-outs</td>
<td>$B &gt;&gt; 1$ (irrational exuberance – price exceeds intrinsic value)</td>
<td>$\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Discount rate estimates ($\beta(t) \neq 1$)</td>
<td>Dual-listed shares</td>
<td>$\beta$s differ (resulting in different prices in different listing centres) $\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td></td>
<td>Dual-class shares</td>
<td>$\beta$s for superior voting shares are lower than for limited voting shares (although this may not be entirely irrational, unless the differences in $\beta$s is excessive) $\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Short-termism I – undervaluation of listed securities and investment projects due to excessive discounting</td>
<td>$\beta(t) &gt; \beta(t-1) \geq 1$ (excessive discounting which increases as $t$ – the discounting horizon – increases)</td>
<td>$\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Short-termism II – undervaluation of listed securities and investment projects due to time-risk misperception (the risk that an investment project might generate positive cash-flows for a shorter period than was initially expected)</td>
<td>$\beta(t) \rightarrow \infty$ as $t$ increases</td>
<td>$\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Long-termism (overvaluation of listed securities)</td>
<td>$\beta &lt; 1$</td>
<td>$\alpha = B = \Gamma(t) = \gamma(t) = 1$ (for all $t = 1, T$)</td>
</tr>
<tr>
<td>Time-duration misestimation or perception bias when projecting ($\Gamma(t) \neq 1$)</td>
<td>Rational discounting of underestimated time-duration or time-contracted projections (another explanation of short-termism)</td>
<td>$\Gamma(t) &lt; 1$</td>
</tr>
</tbody>
</table>

Table 1. Biases in the MTT valuation framework: Examples and bias parameter values
As our examples suggest, some of the biases can only really be identified retrospectively. For evidence of biased discounting, it means comparing the prices of securities which share common, or pre-determined, cash-flows (such as dual-listed shares). When the underlying causes of the bias reflect time or risk misestimations or misperceptions, it requires independent evidence collected through appropriately designed survey questionnaires. It is also clear that some biases are greater in some countries than others and can also change over time. To illustrate, short-termism used to be greater in countries with Anglo-Saxon-style financial markets, such as the UK and US, than in Continental European countries which rely on the banking system to raise investment funds (Kay, 2012; Aspen Institute, 2010); however, in recent years, US investors have become less short-termist than either their UK or Continental European counterparts (Martin and Asgari, 2023).

In conclusion, we believe that the mental time travel valuation framework is useful for drawing together in a single model the underlying causes of the biased valuation of financial investments and for understanding, and then potentially correcting, the biases.

**Notes**

1. The financial model used to calculate intrinsic value may also involve calculating a present value, but does so in a “rational” or “objective” (i.e., bias-free) way; [https://www.investopedia.com/terms/i/intrinsicvalue.asp](https://www.investopedia.com/terms/i/intrinsicvalue.asp)

2. We exclude from our definition of bias both random (unsystematic) misestimations and market frictions (or imperfections). While these can affect initial value and present value, we assume in this paper that they have no differential effects on a thought investment’s projection and discount paths. Market frictions include: asymmetric information, search frictions, participation costs,
non-competitive behaviour (e.g., collusive behaviour by market participants), transactions costs (market makers’ spreads, brokers’ commissions, order-processing costs, transaction taxes), short sales constraints and borrowing/leverage constraints (see, e.g., Vayanos and Wang, 2009, 2010, 2012; de Roon and Szymanowska, 2012).

3. The net value of an investment fund’s assets less its liabilities, divided by the number of shares outstanding. This should be close to intrinsic value, but may differ due to investors’ assessments of the fund manager’s investment expertise.

4. However, there are also other valuation methods, such as “hyperbolic” valuation in which value changes more steeply over the near-future than the far-future.

5. We distinguish between perception (the awareness of something through the senses, such as risk) and preference (where something gives higher utility (i.e., welfare) than something else, for example, the result of a utility ranking of risky financial investments).

6. To keep things simple, in the original MTT valuation framework, the return on a single initial cash-flow was assumed to take the form of capital growth only.

7. We adopt the following definition of rational expectations: investors form unbiased expectations based on the best available information in the market (see Muth, 1961). For investments in the form of, say, construction projects, the net funding cash-flows that are rationally expected are those that are required to complete the project.

8. In an even more general model, $\alpha$ could be made time-varying, i.e., $\alpha(t), t = 1,T$.

9. In an even more general model, $R$ could be made time-varying, i.e., $R(t), t = 1,T$. In this case, Equation (3) becomes:

\[
FV = \frac{D(1)\times B}{R(1)} \prod_{i=1}^{T-1} (1+R(t)\times B)^{i} + D(2)\times B \prod_{i=1}^{T-2} (1+R(t)\times B)^{i} + \ldots + D(T)\times B \prod_{i=1}^{T-1} (1+R(t)\times B)^{i}
\]

10. In an even more general model, $B$ could be made time-varying, i.e., $B(t), t = 1,T$.

11. Future perceived or subjective time is shorter than objective or calendar time, implying that the investment duration is perceived by the investor to be shorter than it is objectively (Zauberan et al., 2009).

12. Future perceived or subjective time is longer than objective or calendar time, implying that the investment duration is perceived by the investor to be longer than it is objectively (Zauberan et al., 2009).

13. In an even more general model, $r$ could be made time-varying, i.e., $r(t), t = 1,T$.

14. This is a sufficient condition, not a necessary one. It is possible that a different configuration of parameter values also gives an unbiased valuation, i.e., the biases neutralise or offset each another.

15. In addition, if $D(1) = IV$, and $D(2) = \ldots = D(T) = 0$, then Equation (4) reduces to Equation (2).

16. Some might go so far as to argue that all the biases have a psychological foundation.

17. It is conventional in economics to separate valuation, preference over alternatives, and choice (i.e., decision making amongst the alternatives) into three distinct processes, even if there is some joint dependence between them. We acknowledge this distinction even though our focus is on financial valuation and not decision-making.

18. Haldane and Davies (2011) define short-termism as the tendency to weight too heavily near-term factors (such as the upfront cost of investments) at the expense of longer-term opportunities. The key feature of short-termism is that $\beta(t)$ increases with $t$, the discounting horizon (i.e., $\beta(t) > \beta(t-1) \geq 1$).

19. It is the persistent underestimation of the value of companies by public markets that has caused increasing numbers of them to convert into private equity companies, particularly in the US (Mauboussin and Callahan, 2020).

20. Miles (1993, Equation (9)) uses a modification of Equation (4) and quantifies short-termism through the equivalent of the parameter $\alpha$ (or $\chi$ in Miles’ notation), which in his modification takes values $\alpha$ in period 1, $\alpha^2$ in period 2, etc. He estimates $\alpha = 0.90$ (and implicitly assumes $\beta = \Gamma = \gamma = 1$) which
implies that cash-flows accruing a year ahead are underestimated by 10% and those 5 years ahead are underestimated by almost 40%. Miles also recognises that he could get the same results by assuming $\alpha = 1$ and selecting an appropriate value of $\beta > 1$.

21. Haldane et al., following Miles’ approach, estimate $\alpha = 0.938$. Again, we could get the same results by assuming $\alpha = 1$ and selecting an appropriate value of $\beta > 1$.

22. While there are many examples in finance of misvaluations caused by misperceptions of risk, time-risk misperception is the most useful for illustrating our thesis in this paper.

23. This suggests that the undervaluation of listed securities due to time-risk misperception is a special case of excessive discounting.

24. The model of Haldane et al. estimates $\alpha = 1.09$ for the financials’ sector over the period 1985–1994 which is equivalent to a value of $\beta < 1$ (and $\alpha = 1$) in our model.

25. Our discussion of time perception bias here is therefore necessarily restricted to future time duration and therefore excludes time passage experience.

26. This supports the view noted earlier that all biases could have a psychological foundation.

27. Pigou (1920) suggested that some people had a “defective telescopic faculty”.

28. So the present is in the past when viewed from the future.

29. A trade body that represents investment managers and investment management firms in the UK.

30. As another example of pulling in opposite directions, Read (2001) and Read and Roelofsma (2003) find evidence that discount rates increase with sub-intervals, whereas Malkoc et al. (2015) find that a stream of cash-flows is discounted at lower rates than an equivalently valued lump-sum.

References


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