

Will Saudi Arabia get older? Will its pension system be sustainable?

Spectral answers

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answers

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Received 17 December 2017
Revised 17 January 2018
Accepted 19 February 2018

Abstract

Purpose – The purpose of this paper is to answer the following two questions: Will Saudi Arabia get older? Will its pension system be sustainable?

Design/methodology/approach – The methodology/approach is to forecast KSA's population with wavelet analysis combined with the Burg model which fits a p th order autoregressive model to the input signal by minimizing (least squares) the forward and backward prediction errors while constraining the autoregressive parameters to satisfy the Levinson-Durbin recursion, then relies on an infinite impulse response prediction error filter.

Findings – Spectral analysis projections of Saudi age groups are more optimistic than the Bayesian probabilistic model sponsored by the United Nations Population Division: Saudi Arabia will not get older as fast as projected by the United Nations model. The KSA's pension system will stay sustainable based on spectral analysis, whereas it will not based on the U.N. model.

Originality/value – Spectral analysis will provide better insight and understanding of population dynamics for Saudi government policymakers, as well as economic, health and pension planners.

Keywords Spectral analysis, Wavelet analysis, Burg model, Kingdom of Saudi Arabia's population, Pension system, Population projection

Paper type Research paper

Introduction

The proportion of people aged 60 or more will represent 25 per cent of the total Saudi population of 40 million by the end of 2050. The number of people aged 80 or more is expected to reach 1.6 million, or 4 per cent of the total population in the same period (Abusaaq, 2015). The number of pension benefits claimants will rise and the working class will have to pay extra pension costs in that period. Health expenditures are expected to increase between 2015 and 2050 due to an increase in the elderly rate from 5.4 per cent (1.6 million) to 25 per cent (10 million), but health expenditures will be offset by a decrease of the public expenses related to education in the same period as the youth dependency ratio will decrease to 24 per cent from 41 per cent. The objective of our paper is to add a spectral perspective to the Saudi population projections of the Population Division of the United

JEL classification – C53, J11, J26

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Nations (U.N.) used in Abusaaq's paper. We forecast 80 years of Saudi Arabia population by age groups with spectral analysis presented in [Rostan and Rostan \(2017a\)](#). The [Burg \(1975\)](#) model combined with wavelet analysis is a versatile and robust model that may help the Saudi government policymakers, planners and pension managers to gain insight into the future pyramid of ages. Spectral analysis represents a broad range of applications, algorithms and implementations of processing signals. The foundation of spectral analysis is retraced in the classical numerical analysis techniques of the seventeenth century ([Oppenheim and Schaffer, 1975](#)). Its applications encompass many fields from electrical signals to audio signal processing, wireless communication, waveform generations, demodulation, filtering, equalization or seismology whenever the forecast of waveform time series is involved. In recent years, spectral analysis has been applied to time series outside the field of Physics. A first attempt was the appraisal of the financial sustainability of the Spanish pension system using Spanish population forecasts ([Rostan et al., 2015](#)), then an application to Spanish GDPs forecasts ([Rostan and Rostan, 2018](#)). Spectral analysis was also applied to yield curve forecasting ([Rostan et al., 2017](#)) with a robust outcome. With a refined methodology using multiscale principal component analysis to take into account the co-dynamics of age groups, [Rostan and Rostan \(2017a\)](#) forecasted European and Asian populations which lead to original outcomes when compared to more conformist population projections of the U.N. Finally, the versatility of spectral analysis applied to the forecast of financial times series with distinctive properties was illustrated with market data in [Rostan and Rostan \(2017b\)](#). Population estimates follow mean-reverting processes overtime and are conspicuous candidates of waveform time series forecasting with spectral analysis.

Our benchmark, promoted by the Population Division of the U.N. Secretariat, follows the methodology presented in [Raftery et al. \(2012\)](#): it is based on Bayesian probabilistic population projections where “the total fertility rate and female and male life expectancies at birth are projected probabilistically using Bayesian hierarchical models estimated via Markov chain Monte Carlo using U.N. population data for all countries. These are then converted to age-specific rates and combined with a cohort component projection model. This yields probabilistic projections of any population quantity of interest”.

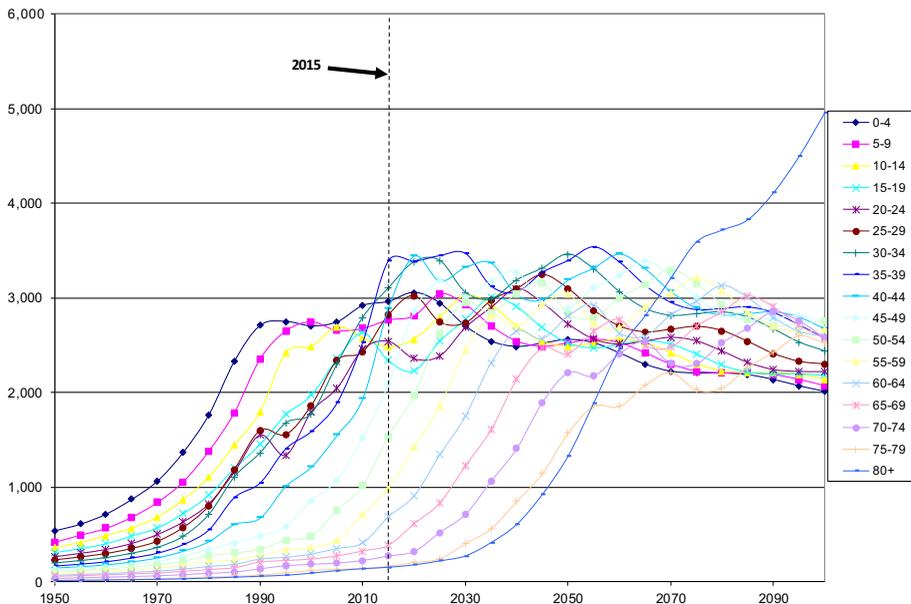
In the next section, we present the rationale of selecting spectral analysis for population projections and a four-step methodology based on spectral analysis that belongs to deterministic methods as opposed to probabilistic methods, such as our benchmark which is a Bayesian probabilistic model.

Methodology

Our methodology is the fusion of three sources: a book ([Rostan and Rostan, 2017a](#)), a seminal paper ([Rostan et al., 2015](#)) on population projections and a paper on financial time series forecasting ([Rostan and Rostan, 2017b](#)). The assumption of the model relies on the fact that population estimates follow waveform patterns like several physical phenomena such as electrical, audio or seismic signals which propagate through space in waveforms. Signal processing (SP) proposes sparse representations of signals for the purposes of analysis or enhancement. “Signals carry overwhelming amounts of data in which relevant information is often more difficult to find than a needle in a haystack. Processing is faster and simpler in a sparse representation where few coefficients reveal the information we are looking for” ([Mallat, 2009](#)). SP includes spectral analysis and enhancing acquired data using digital filtering. Spectral analysis is the technical process of decomposing a complex signal into a simpler one. Spectral analysis uses different techniques divided into two classes, non-parametric and parametric methods. Non-parametric methods include periodogram, Bartlett's method or non-uniform discrete Fourier transform. Parametric methods include

autoregressive model (AR), moving-average model (MA), autoregressive moving average (ARMA) and maximum entropy spectral estimation. In this paper, the Burg (1975) model combined with wavelet analysis and applied to population projection is a parametric AR model. The Burg model is able to capture the amplitude of the wave, the slope of the trend, the decaying or increasing amplitude of the wave overtime and may qualify for population projections.

Our database is represented by the Saudi Arabia pyramid of ages divided by the Population Division of the U.N. Secretariat by five-year age group into 17 classes: 0-4 years, 5-9, 10-14, 15-19, . . . , 80+. We project the population values of the 17 age groups from 2020 to 2100. We benchmark our model to population forecasts of the Population Division of the U.N. Secretariat. Figure 1 provides an insight of adapting SP to Saudi population projections: age groups represented by times series propagate through time like signals. We observe this behavior in other fields such as communication systems, SP and electrical engineering where a signal refers to “a function that conveys information about the behavior or attributes of some phenomenon” (Priemer, 1991). More specifically, in electrical engineering, the embodiment of a signal in electrical form is made by a transducer that converts the signal from its original form to a waveform expressed as a current or a voltage, or an electromagnetic waveform, for example, an optical signal or radio transmission. Figure 1 captures the oscillation of population time series in waveforms. Based on the U.N. population projections between 2020 and 2100, most of these waves have an uptrend until 2050-2060 and then experience a downtrend until 2100.



Notes: 1950-2015 data obtained by census; after 2015, population projections (medium variant) of the Population Division of the U.N. Secretariat

Source: U.N.: <http://esa.un.org/unpd/wpp/DVD/> (in thousands)

Figure 1.
The 17 age groups
constituting the Saudi
pyramid of ages for
the 1950-2100 periods

The alarming trends belong to the young population: age group 0-4 starts its downtrend in 2020, age group 5-9 in 2025, age group 10-14 in 2030, age group 15-19 in 2035, age group 20-24 in 2040 and age group 25-29 in 2045. All age groups except the 80+ decay in amplitude overtime to converge inside a 2 million to 2.7 million range in 2100.

Wave propagation is defined in SP as how waves travel. Once we acknowledge the wave propagation of the time series and the analogy with optical or radio transmission signals, we may apply SP to population estimates: the assumption of the paper is, thus, that population estimates propagate through time like signals through space. As observed in Figure 1, historical population data between 1950 and 2015 clearly behave in waveforms.

Figure 1 illustrates age groups population estimates in thousands (both sexes combined) by five-year age group propagating in waveforms. Historical estimates are obtained by census between 1950 and 2015. After 2015, Figure 1 illustrates probabilistic population projections of the U.N (medium variant) from 2020 until 2100, which converge toward a 2 million to 2.7 million range in 2100. The probabilistic U.N. model makes the 80+ age group an outlier group which increases steadily overtime, reaching a 4.9 million level in 2100.

We apply a four-step methodology to project population estimates with spectral analysis, illustrated with KSA's 0-4 age group population projection Figure 2.

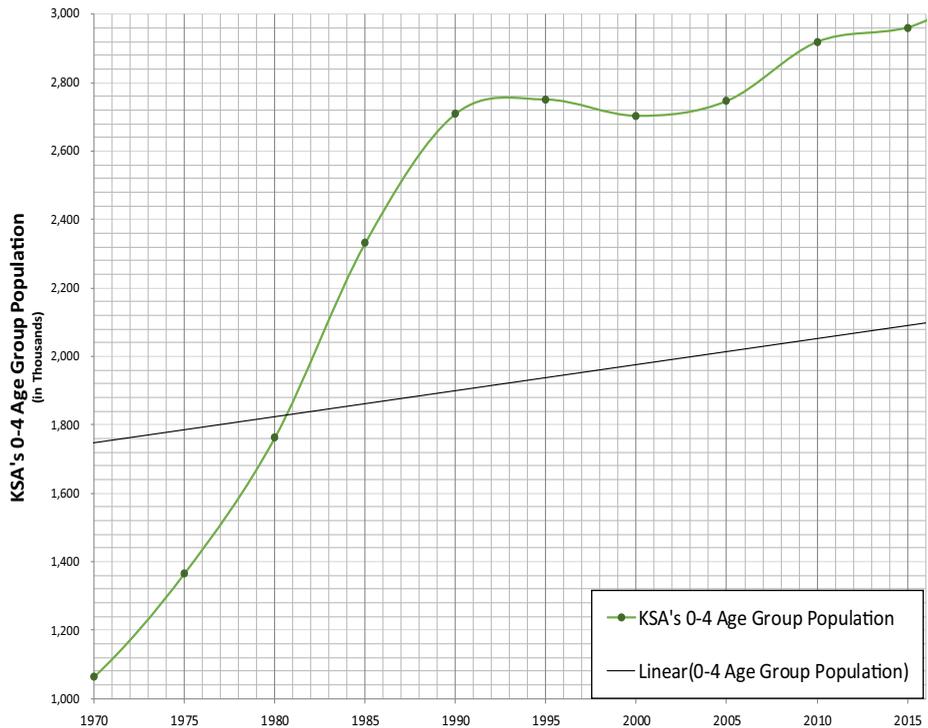


Figure 2.
Kingdom of Saudi Arabia's 0-4 age group population from 1950 to 2015 (14 data)

Source: U.N.: <http://esa.un.org/unpd/wpp/DVD/> (in thousands)

Step 1: De-noising and compression of the first-order difference of Saudi Arabia's total population time series

We compute the first-order difference of the KSA's 0-4 age group population time series to transform non-stationary series into stationary series. We apply the augmented Dickey-Fuller test to the time series before and after differentiation: before differentiation, the time series are non-stationary (i.e. existence of a unit root), and after differentiation, the time series is stationary (rejection of the existence of a unit root). The choice of this transformation relies on the fact that wavelet analysis presents a more accurate forecasting ability with stationary time series rather than non-stationary time series. Refer to [Rostan and Rostan \(2017b\)](#) for a demonstration.

We then de-noise the series using a one-dimensional de-noising and compression-oriented function using wavelets. The function is called “wdencmp” in Matlab ([Misiti et al., 2015](#)). The underlying model for the noisy signal is of the form:

$$s(n) = f(n) + \sigma e(n) \quad (1)$$

where time n is equally spaced, $e(n)$ is a Gaussian white noise $N(0,1)$ and the noise level σ is supposed to be equal to 1. The de-noising objective is to suppress the noise part of the signal s and to recover f . The de-noising procedure proceeds in three steps:

- (1) *Decomposition*: We choose the wavelet *sym4*, and choose the level 2-decomposition. *Sym4* is a symlets wavelet of order 4 used as the mother wavelet for decomposition and reconstruction. It is a nearly symmetrical wavelet belonging to the family of *Symlets* proposed by [Daubechies \(1992\)](#). We compute the wavelet decomposition of the signal s at level 2.
- (2) *Detail coefficients thresholding*: For each level from 1 to 2, we select a threshold and apply soft thresholding to the detail coefficients.
- (3) *Reconstruction*: We compute wavelet reconstruction based on the original approximation coefficients of level 2 and the modified detail coefficients of levels from 1 to 2.

Like de-noising, the compression procedure contains three steps:

1) Decomposition. 2) Detail coefficient thresholding. For each level from 1 to 2, a threshold is selected and hard thresholding is applied to the detail coefficients. 3) Reconstruction. The difference with the de-noising procedure is found in step 2. The notion behind compression is based on the concept that the regular signal component can be accurately approximated using a small number of approximation coefficients (at a suitably selected level) and some of the detail coefficients.

We illustrate in [Figure 3](#) KSA's 0-4 age group population (14 years) before differentiation (top figure), after differentiation (middle) and after de-noising and compression (bottom).

Step 2: Wavelet decomposition

We decompose the signal after being differentiated, de-noised and compressed. The signal, i.e. the 14-year time series of KSA's 0-4 age group population transformed at step 1, is decomposed into decomposed signals cAs named approximations and cDs named details. The discrete wavelet transform is a kind of decomposition scheme evaluated by passing the signal through low-pass and high-pass filters ([Corinthios, 2009](#)), dividing it into a lower frequency band and an upper band, respectively. Each band is subsequently divided into a second-level lower and upper bands. The process is repeated, taking the form of a binary, or

“dyadic” tree. The lower band is referred to as the approximation cA and the upper band as the detail cD . The two sequences cA and cD are downsampled. The downsampling is costly in terms of data: with multilevel decomposition, at each one-level of decomposition, the sample size is reduced by half (in fact, slightly more than half the length of the original signal, as the filtering process is implemented by convolving the signal with a filter. The convolution “smears” the signal, introducing several extra samples into the result). Therefore, the decomposition can proceed only until the individual details consist of a single sample. Thus, the number of levels of decomposition will be limited by the initial number of data of the signal. The level of decomposition of the signal is left to the appreciation of the user. In this paper, we apply a third-level decomposition. The choice of the third level is explained at the end of the methodology section. Figure 4 illustrates the third-level

Figure 3. Observed KSA’s 0-4 age group population from 1950 to 2015, 14 annual data (top), first-order difference of KSA’s 0-4 age group population (middle), de-noising and compression of the first-order difference of KSA’s 0-4 age group population (bottom)

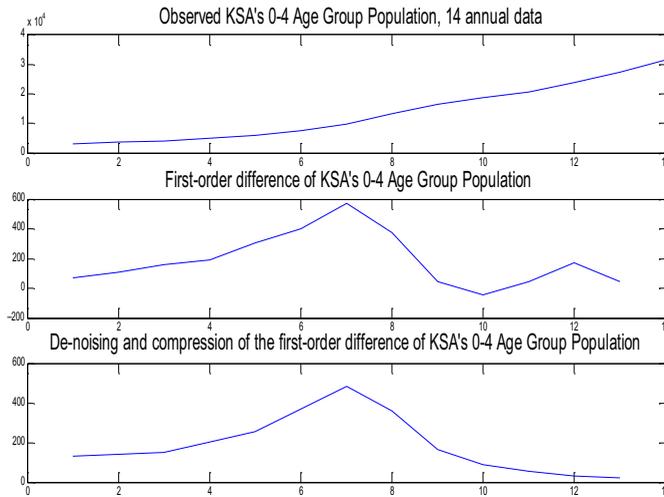
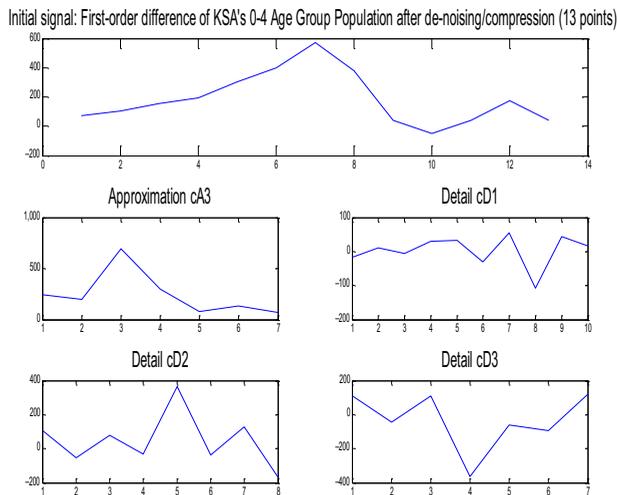


Figure 4. Third-level decomposition of the transformed KSA’s 0-4 age group population at current prices (after differentiation and de-noising/compression) using one-dimensional discrete wavelet analysis



decomposition of the transformed KSA's 0-4 age group population (after differentiation and de-noising/compression, 13 points). We observe in [Figure 4](#) that details cD s are small and look like high-frequency noise, whereas the approximation cA contains much less noise than does the initial signal. In addition, the higher the level of decomposition, the lower the noise generated by details. For a better understanding of signal decomposition using discrete wavelet transform, refer to the methodology section of [Rostan and Rostan \(2017b\)](#).

Step 3: Burg extension of approximations and details.

We apply the Burg extension to cA and cD . To run the Burg extension, we apply an autoregressive p th order from historical data, in this paper we choose a p th order equal to the longest available order when forecasting. For instance, in 2016, when forecasting KSA's 0-4 age group population for the subsequent years, the longest p th order available is 12 out of 13 data. Given x the decomposed signal (which is cA or cD), we generate a vector a of all-pole filter coefficients that model an input data sequence using the Levinson-Durbin algorithm ([Levinson, 1946](#); [Durbin, 1960](#)). We use the [Burg \(1975\)](#) model to fit a p^{th} order autoregressive (AR) model to the input signal, x , by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise.

Vector a contains the normalized estimate of the AR system parameters, $A(z)$, in descending powers of z :

$$H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_1z^{-1} + \dots + a_{(p+1)}z^{-p}} \quad (2)$$

As the method characterizes the input data using an all-pole model, the correct choice of the model order p is important. In [Figure 5](#), the prediction error, $e(n)$, can be viewed as the output of the prediction error filter $A(z)$, where $H(z)$ is the optimal linear predictor, $x(n)$ is the input signal and $\hat{x}(n)$ is the predicted signal.

In a last step, the infinite impulse response (IIR) filter extrapolates the index values for each forecast horizon. IIR filters are digital filters with infinite impulse response. Unlike a finite impulse response (FIR) filter, an IIR filter has the feedback (a recursive part of a filter) and is also known as recursive digital filter.

Step 4: Wavelet reconstruction

We recombine the forecasted signals after the Burg extension using the methodology illustrated in [Figure 6](#) for the third-level decomposition/reconstruction diagram. After reconstruction, we retransform the time series of the first-order difference of the KSA's 0-4 age group population into KSA's 0-4 age group population.

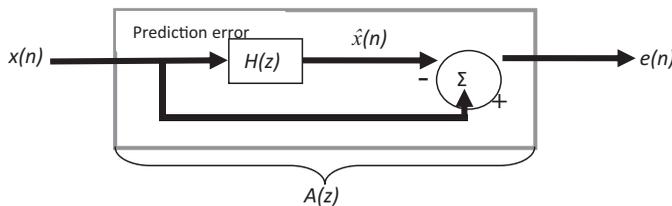


Figure 5. Prediction error filter to run the Burg extension

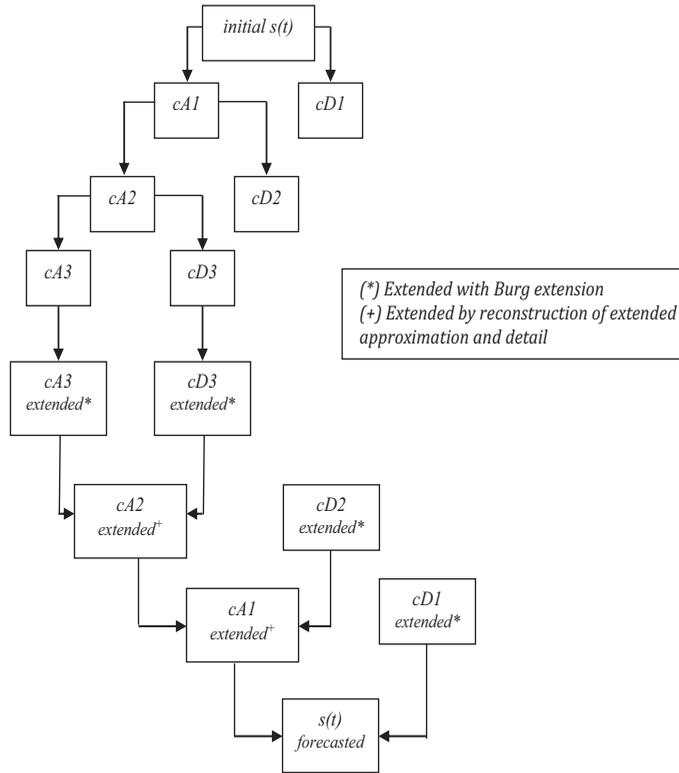


Figure 6.
Diagram of a third-level wavelet decomposition/reconstruction tree to forecast the initial signal $s(t)$

Identifying the optimal level of decomposition

Finally, we focus on the optimal level of decomposition of our forecasting model. We make the level varying from 1 to 7.

0-, 1- and 2-levels of decomposition return an error message. Figure 7 illustrates the average RMSE computed on the last five in-sample years of our database (forecasts versus observed data) from 1995 to 2015 of KSA's population for the 17 age groups.

Whichever the level of decomposition, RMSE is constant (425.47). For simplification, we use the lowest level of decomposition, the third level, to generate the forecasts in the results section.

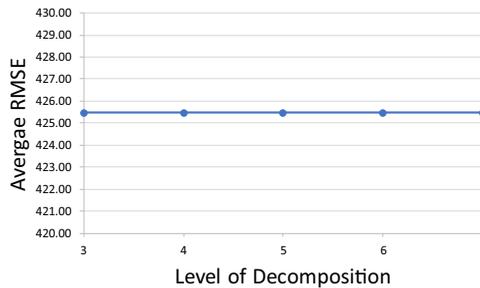


Figure 7.
Average RMSE versus level of decomposition

Results for KSA's population projections

Results for the 17 age groups

KSA's pyramid of ages is divided into 17 age groups: 0-4, 5-9, 10-14, 15-19, . . . , 80+. We forecast the population values of the 17 age groups from 2020 to 2100 using the Burg method combined with wavelet analysis. We benchmark our model to KSA's population projections of the Population Division of the Department of Economic and Social Affairs of the U.N. Secretariat. We review some examples of age group population projections obtained with spectral analysis. The first example is the 0-4 age group population represented by Figure 8.

Figure 8 shows that the 0-4 age group projection obtained with spectral analysis captures the uptrend depicted by population estimates observed before 2020. Spectral analysis adjusts the frequency and amplitude of the age group projection to the information provided by past data based on the SP theory. The uptrend is explained by the baby bust, which will last in Saudi Arabia, based on spectral analysis, until the end of the century, but which will end, based on the U.N. model, in 2020.

A second example is the 30-34 age group population represented by Figure 9. We observe again a divergence of the forecasts generated by the two models, spectral analysis being bullish with the 30-34 age group, reaching 9.3 million on 2100. In contrast, the U.N. model displays a steady decline of the value of the 30-34 age group after 2050, reaching 2.4 million in 2100.

A third example is the 80+ age group represented in Figure 10. This outlier 80+ age group displays a clear rupture between spectral analysis and the U.N. benchmark. Like all other age groups, spectral analysis identifies a trend whose amplitude and frequency are explained by past data. On the contrary, the U.N. 80+ age group points sharply upward for the remaining 80 years of the twenty-first century. The U.N. secretariat obviously assumes that medical progress and social protection of elderly people will make their age group number skyrocketing, but does not take into consideration:

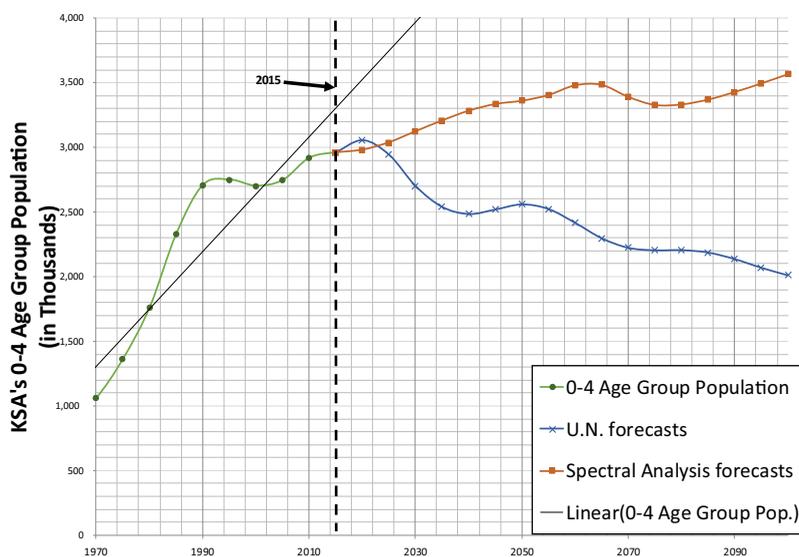


Figure 8.
0-4 age group
population;
population forecasted
with spectral analysis
versus population
projection (medium
variant) of the U.N.
Secretariat (in
thousands)

Figure 9.
Age group 30-34 years; population forecasted with spectral analysis versus population projection (medium variant) of the U.N. Secretariat (in thousands)

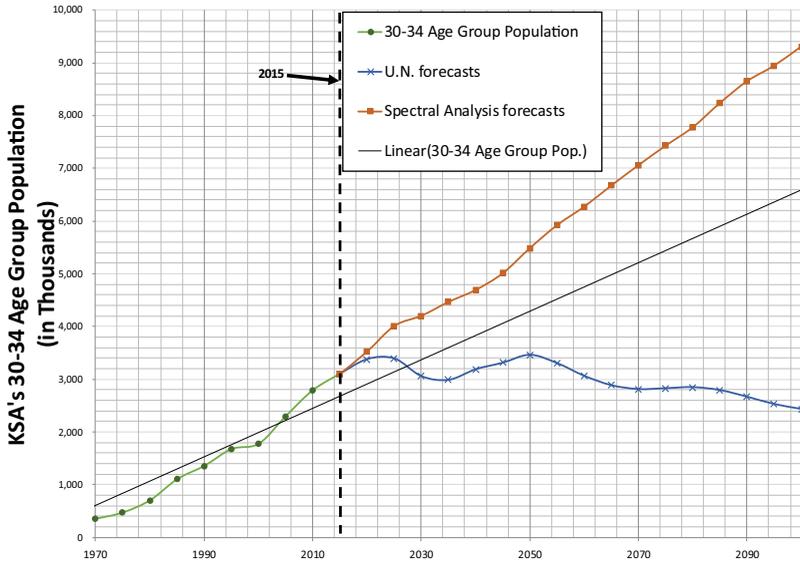
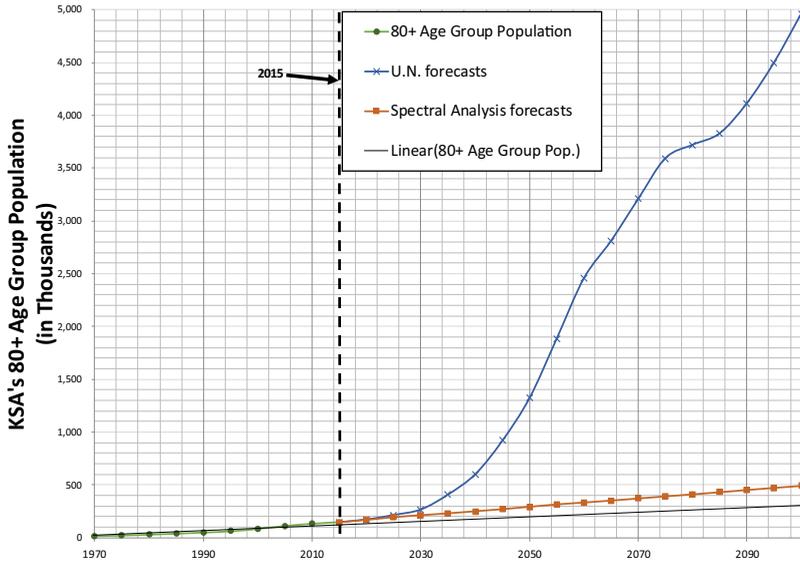


Figure 10.
80+ Age group; population forecasted with spectral analysis versus population projection (medium variant) of the U.N. Secretariat (in thousands)



- That epidemiologic factors such as the emergence of new viruses (e.g. Zika virus), antibiotic-resistant bacteria (e.g. half a million cases of multidrug-resistant tuberculosis in 2013), depletion of the immune system by genetically modified food or overuse of pesticide, deterioration of the environment quality by global warming may slowdown the rise of octogenarians. Global warming responsible for many

epidemiologic occurrences was evidenced by year 2016, the warmest year since modern record-keeping began in 1880 (NASA's Goddard Institute for Space Studies, 2017) and by the fact that 16 of the 17 warmest years on record have occurred since 2001. This makes 2016 the third year in a row to set a new record for global average surface temperatures;

- That economic factors such as a deep and prolonged recession due to rampant oil prices, which have already taken a toll on the Saudi economy, may lead to significant budget deficits which may reduce transfer payments from the government, trimming the financial support of senior citizens.
- That political factors such as war against terror may impair the social protection of the 80+ vulnerable population with government budget cuts. Recently, by declaring war against terrorism at an Islamic Military Counter Terrorism Coalition meeting gathering its 41 members, Saudi Arabia has coordinated a collective response to fight terrorism through military actions and security measures (Naqvi, 2017). The leadership of Saudi Arabia in the war against terror will force the country to reassess the economic and social costs that these measures represent. In addition to the terrorism threat, risk of war has reached alarming levels in the Middle East. The risk is supported by an aggressive arms industry, which is lobbying superpowers. Out of the top 10 international arms producers, eight are American. The arms industry spends millions lobbying US Congress and state legislatures and defends its turf with efficiency and vigor (Hallinan and Wofsy, 2015). Tensions in sensitive areas are, therefore, created and maintained by the arms industries and their political advocates that exploit religious differences to support conflicts: the historical Shia-Sunni enmity has led to many conflicts in human history, the war between Iraq and Iran (1980-1988) is a recent example (Rostan and Rostan, 2017a). The death toll was estimated to 1.5 million casualties. Proxy war is another example of conflict created and encouraged by superpowers and their arms industries. As a result, Saudi Arabia has purchased a record US\$1.15bn arms to the USA in 2016 (Zengerle, 2016). Following the visit of US President Trump in Riyadh in 2017, a weapons' deal worth nearly US\$110bn was sealed immediately and an agreement of US\$350bn was signed over 10 years (David, 2017). The Syrian conflict, which has extended from 2011 until today, is an example of proxy war between Russia and its allies, who are supporting President Bashar al-Assad government representing the Shia minority and the USA, Saudi Arabia and their allies who are arming and supporting Sunni rebels. Another example of proxy war is the war in Yemen between Saudi Arabia and Iran. In this ailing economic and political context, the optimal allocation of resources for senior citizens in Saudi Arabia may not be a priority for the country.

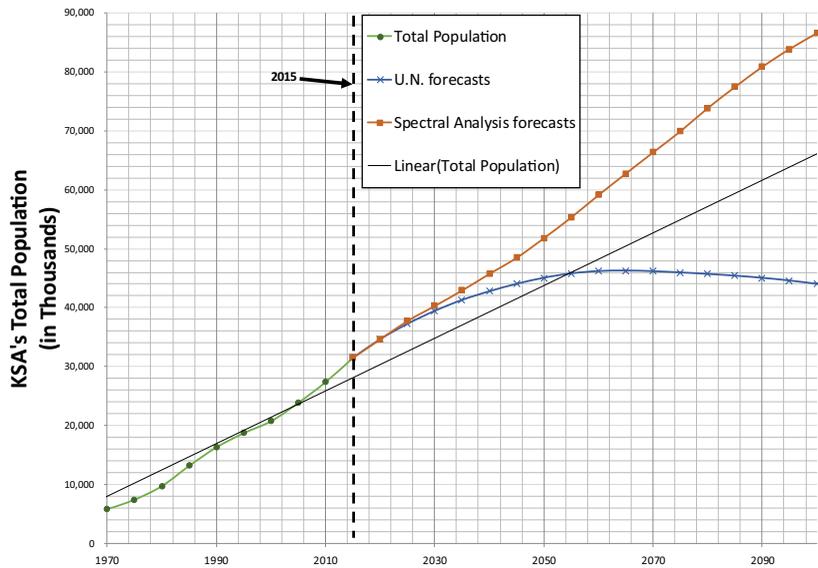
The U.N. foresees the 80+ age group skyrocketing by more than 3,276 per cent by the end of the century compare to 2015 with 4.9 million of senior citizens in 2100 versus 146,856 in 2015, which makes this figure rather unlikely. Spectral analysis forecasts a constant increase of the 80+ age group, reaching a 492,115 level in 2100 with a 235 per cent increase compare to 2015. The reality might lie somewhere inside this range.

Results for the total KSA's population

Figure 11 illustrates the total KSA's population forecasted with spectral analysis and by the U.N.

From 2020 to 2100, spectral analysis forecasts a steady increase of KSA's population estimates. U.N. estimates will experience an increase until 2065 toping a 46-million

Figure 11. Total population forecasted with spectral analysis versus population projection (medium variant) of the U.N. (in thousands)



high, then the population will deplete reaching a 44-million level in 2100. Spectral analysis estimates will hit the 86 million marks in 2100 about two times the U.N. estimate.

Reviewing the differences between KSA's population projections obtained with spectral analysis and the U.N. benchmark

Population projections forecasted with the two methods converge to one conclusion: KSA's population will be larger by the end of the century than today, 44 million in 2100 with U.N. projections, 86 million with spectral analysis versus 31.5 million in 2015. However, the drivers of KSA's population growth are different depending on methods.

Figure 12 illustrates the 17 age groups forecasted with spectral analysis, which should be compared to the 17 age groups forecasted by the U.N. and illustrated in Figure 1. A general comment is that population estimates forecasted with spectral analysis are not converging toward the 2.5 million level as we observe in Figure 1 with U.N. projections. This convergence toward the 2.5 million level is a clear limitation of the U.N. Bayesian probabilistic model, which derives the age group projections from common denominators, the total fertility rate and female and male life expectancies at birth, projected probabilistically using Bayesian hierarchical models estimated via Markov chain Monte Carlo. With the U.N. model, all age groups projections (except the 80+) converge toward a specific level because the model volatility decreases overtime, making the range of individual age groups narrowing. Time series forecasted by spectral analysis and illustrated by Figure 12 are not converging toward a specific level; they are more diffuse and look more realistic. In 2100, spectral analysis series range between 12.4 million for the 35-39 age group and 492,115 for the 80+ age group. In addition, we observe that age groups of young people (0-4, 5-9, 10-14, 15-19) stagnate after years 2050-2065. All other age groups have a clear uptrend between 2020 and 2100.

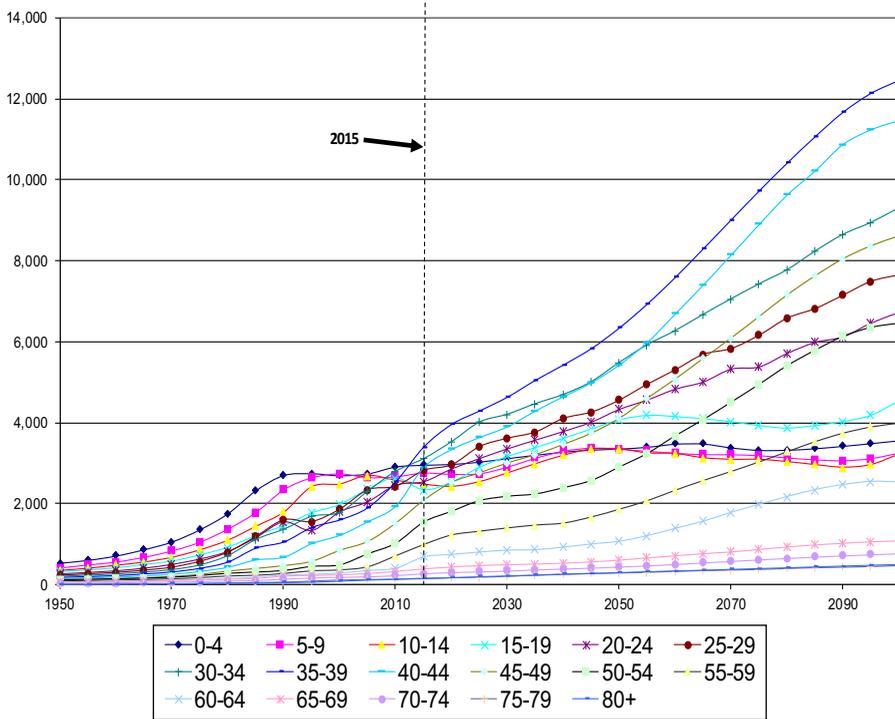


Figure 12.
The 17 Age groups constituting the Saudi pyramid of ages for the 1950-2100 period

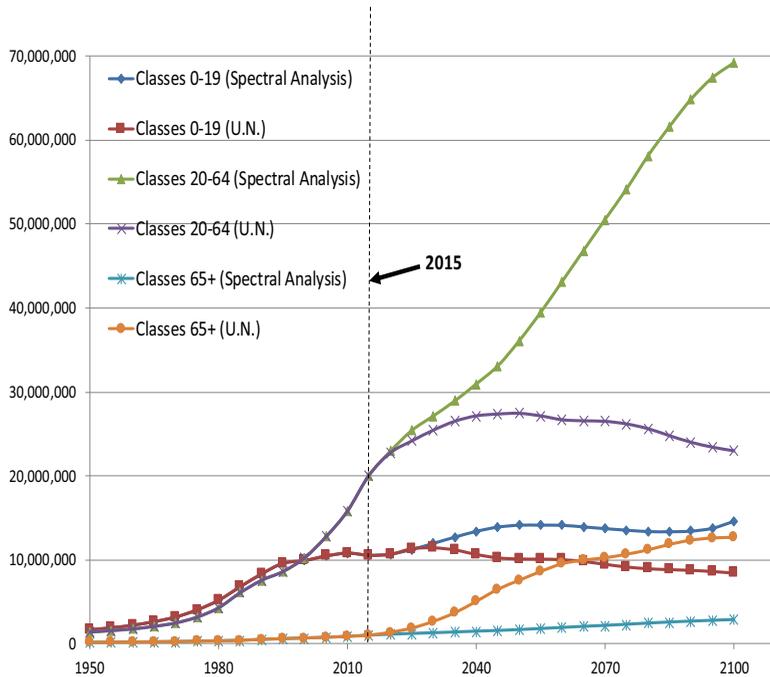
Notes: 1950-2015 data obtained by census; after 2015, population projections obtained with spectrum analysis
Source: U.N.: <http://esa.un.org/unpd/wpp/DVD/> (in thousands)

Appraising the financial sustainability of KSA’s pension system

As an application of population projections to the appraisal of the financial sustainability of KSA’s pension system, we regroup the 17 age groups in three major age groups, the 0-19, 20-64, 65+ illustrated by Figure 13.

From Figure 13, starting with the upper curves, we observe that the working class 20-64 who feeds the pension system is at lower levels between 2020 and 2100 with the U.N. projections (25.5 million on average) than spectral analysis projections (44.6 million on average). The 65+ age group who depletes the pension system is at significantly higher levels between 2020 and 2100 with the U.N. forecasts (8 million on average) than spectral analysis forecasts (1.9 million on average). Finally, the 0-19 age group that is considered the future generation to feed the pension system is at higher levels between 2020 and 2100 with spectral analysis (13.2 million on average) than the U.N. model (9.8 million on average). Overall, spectral analysis population projections are more optimistic than the U.N. model projections concerning financial sustainability of KSA’s pension system with 19.1 million more workers on average who feed the pension system over the period 2020-2100, 3.4 million more next-to-become-workers on average (the 0-19 age group) with spectral analysis than

Figure 13.
Three combined age groups, the 0-19, 20-64, 65+ constituting KSA's pyramid of ages for the 1950-2100 period



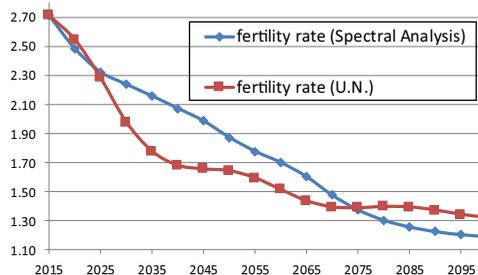
Notes: 1950-2015 data obtained by census; after 2015; population projections obtained with spectral analysis versus U.N. model (medium variant)

the U.N. projections and 6.1 million *less* retirees on average (who deplete the pension system) with spectral analysis than the U.N. model.

Forecasting the KSA's fertility rate

Finally, focusing on KSA's fertility rate defined as the ratio of live births to the population, expressed per 1,000 population per year, we choose the age group 0-4 as proxy of the live births that we adjust to the 2015 World Bank fertility rate estimate for KSA, i.e. 2.71 per 1,000. We obtain Figure 14. From Figure 14, the fertility rate forecasts

Figure 14.
Forecasting the fertility rate with spectral analysis versus U.N. model (medium variant) from 2020 to 2100



Source: U.N.: <http://esa.un.org/unpd/wpp/DVD/>

obtained by the two models are both bearish between 2020 and 2100, the decline being stronger with the U.N. model than spectral analysis at the beginning. However, by 2100, the spectral analysis estimate ends up lower with a value of 1.19 versus 1.32 with the U.N. model.

Conclusion on KSA's population projections

Our objective is to provide a robust model to government policymakers, planners or pension managers, to gain insight into the future KSA's pyramid of ages: spectral analysis assumes that population time series propagate through time like signals through space. KSA's pyramid of ages is divided into 17 age groups by the Population Division of the Department of Economic and Social Affairs of the U.N. Secretariat: 0-4, 5-9, 10-14, . . . , 80+. We forecast estimates of the 17 age groups from 2020 to 2100. We benchmark our model to the Bayesian probabilistic model promoted by the U.N. Secretariat.

We show that the overall population forecasts of the two models have positive trends, spectral analysis making the KSA's population increasing at a positive rate until 2100 when it reaches the 86-million marks, the U.N. model increasing at a negative rate and decreasing after 2065 to reach 44 million in 2100.

However, the drivers of KSA's population growth are different depending on the method. The 80+ age group of the U.N. follows a distinct pattern, a sharp uptrend for the last 80 years of the twenty-first century. The U.N. obviously assumes that medical progress and social protection of elderly people will make their age group number skyrocketing. The U.N. foresees the 80+ age group jumping by more than 3,276 per cent (4.9 million elderly people in 2100) by the end of the century compare to 2015, which makes this figure rather unlikely. Spectral analysis forecasts a smooth increase of the 80+ age group reaching 492,115 in 2100 (+235 per cent compare to 2015). The reality might lie somewhere in the middle. Concerning the 16 remaining age groups, the U.N. model makes the age groups following a waveform pattern. All groups converge inside a 2 million to 2.7 million range in 2100. This pattern is a clear limitation of the U.N. Bayesian probabilistic model, which derives the age group projections from common denominators, the total fertility rate and female and male life expectancies at birth, projected probabilistically using Bayesian hierarchical models estimated via Markov chain Monte Carlo. The projections converge toward a specific range because the model volatility decreases overtime, making the range of individual age groups narrowing and converging. Time series forecasted with spectral analysis are not converging toward the same level; they are more diffuse and look more realistic.

In addition, forecasts of age groups of young people (0-4, 5-9, 10-14, 15-19) obtained with spectral analysis stagnate after years 2050-2065. All other age groups have a clear uptrend between 2020 and 2100.

In an attempt to appraise the financial sustainability of KSA's pension system by analyzing the age groups feeding and depleting the pension system, we regroup the 17 age groups in three main age groups, the 0-19, 20-64, 65+. We show that over the period 2020-2100, the financial sustainability of KSA's pension system is more optimistic with spectral analysis than the U.N. with 19.1 million *more* workers on average who feed the pension system over the period 2020-2100, 3.4 million *more* next-to-become-workers on average (the 0-19 age group) with spectral analysis than the U.N. projections and 6.1 million *less* retirees on average (who deplete the pension system) with spectral analysis than the U.N. model.

Focusing on the fertility rate, we show that the fertility rate forecasts obtained by the two models are both bearish between 2020 and 2100, the decline being stronger with the U.N. model than spectral analysis at the beginning. However, by 2100, the spectral analysis estimate ends up lower with a value of 1.19 versus 1.32 with the U.N. model.

To recap, “Will Saudi Arabia Get Older by 2100?”. The U.N. model says that the 65+ age group will increase from 3 per cent of the total population in 2015 to 29 per cent in 2100 when spectral analysis says that the proportion of this age group will stay the same at 3 per cent. The 20-64 age group that represented 64 per cent of the population in 2015 will grow to 80 per cent with spectral analysis and slip to 52 per cent with the U.N. model. However, the young generations (0-19 age group) who represented 33 per cent of the total KSA’s population in 2015 will slip to 17 per cent with spectral analysis and 19 per cent with the U.N. model. Thus, the two models converge regarding the estimated proportion of the 0-19 age group in 2100. Overall, whichever the model, Saudi Arabia will get older.

Will KSA’s pension system be sustainable? The answer is no with the U.N. model, but yes with spectral analysis.

Compliance with ethical standards

Disclosure of potential conflicts of interest: All authors declare that no potential conflict of interest exists.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent: Informed consent was not necessary for this study.

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Further reading

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