Auction models with resource pooling in modern supply chain management

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Abstract
Purpose – In this paper, the main focus is on supply and demand auction systems with resource pooling in modern supply chain from a theoretical modeling perspective. The supply and demand auction systems in modern supply chains among manufacturers and suppliers serve as information sharing mechanisms. The purpose of this paper is to match the supply and demand such that a modern supply chain can achieve incentive compatibility and economic efficiency. The authors design such a supply and demand auction system that can integrate resources to efficiently match the supply and demand.

Design/methodology/approach – The authors propose three theoretic models of modern supply chain auctions with resource pooling according to the Vickrey auction principle. They are supply auction model with demand resource pooling, demand auction model with supply resource pooling, and double auction model with demand and supply resource pooling. For the proposed auction models, the authors present three corresponding algorithms to allocate resources in the auction process by linear programming, and study the incentive compatibility and define the Walrasian equilibriums for the proposed auction models. The authors show that the solutions of the proposed algorithms are Walrasian equilibriums.

Findings – By introducing the auction mechanism, the authors aim to realize the following three functions. First is price mining: auction is an open mechanism with multiple participants. Everyone has his own utility and purchasing ability. So, the final price reflects the market value of the auction. Second is dynamic modern supply chain construction: through auction, firm can find appropriate partner efficiently. Third is resources integration: in business practices, especially in modern supply chain auctions, auctioneers can integrate resources and ally buyers or sellers to gain more efficiency in auctions.

Originality/value – In the paper, the authors propose three theoretic models and corresponding algorithms of modern supply chain auctions with resource pooling according using the Vickrey auction principle, which achieves three functions: price mining, dynamic modern supply chain construction and resources integrating. Besides, these proposed models are much closer to practical settings and may have potential applications in modern supply chain management.

Keywords Modern SCM, Resource pooling, Vickrey auction mechanism, Walrasian equilibriums

Paper type Research paper

1. Introduction

Supply chain management (SCM) aims at improving the individual performance of all the enterprises in the supply chain as well as the whole supply chain. It integrates inside and outside enterprise resources to meet the quick change of customers’ needs and improves the
competitive edge of the supply chain. Supply chain considers all business processes from point of origin to point of consumption, involving product R&D, material procurement, manufacturing, marketing, logistics and after-sale services, etc. (Lambert et al., 1998; Sebastian and Douglas, 2003). To reduce operational cost and to respond quickly in modern SCM, three key factors should be considered: the supply chain network construction, the information sharing mechanism and the cooperation mechanism. However, the implementation of the three factors in modern supply chain invariably meet many problems in practices, such as what to cooperate for, who to cooperate with and how to cooperate, etc. Thus, it is necessary to introduce an efficient coordination mechanism in modern SCM to solve the problems above.

In highly competitive business environment, we can naturally expect participants in the modern supply chain to be motivated solely by self-interests, which often results in inefficiencies for the whole supply chain. Therefore, we could exploit some auction mechanisms to make the operation process more efficient and profitable, improving the well-being of all participants therein. Auction is a market mechanism where the buyers and seller agree on the items of interest and on the payment and delivery conditions (Feigenbaum and Shenker, 2004; Klemperer, 1999; Nisan and Ronen, 2001). There are three participants in an auction: the buyer, the seller, and the auctioneer. Usually, the auctioneer only plays the role as an agency to organize the auction. There are four basic types of auctions widely used and analyzed: the ascending-bid auction (also called the open, oral, or English auction), the descending-bid auction (used in the sale of flowers in the Netherlands and also called the Dutch auction by economists), the first-price sealed-bid auction and the second-price sealed-bid auction (the latter is also known as the Vickrey (1961) auction by economists). Besides, game theoretic and combinatorial ideas (Mas-Collel et al., 1995; Osborne and Rubinstein, 1994; Papadimitriou, 2001) have been widely applied in the design of algorithms and mechanisms for the various auction models, such as VCG mechanism (Clarke, 1971; Groves, 1973; Vickrey, 1961), single-minded auction (Archer et al., 2004; Chen et al., 2003; Chen et al.; Lehmann et al., 2002; Mu'alem and Nisan, 2008), combinatorial auction (Sven de Vries 2003; Lavi et al., 2003; Lehmann et al., 2001; Parkes and Ungar, 2000; Sandholm et al., 2002), online auction (Bar-Yossef et al., 2002; Blum et al., 2003; Lavi and Nisan, 2000), computing grid auction (Chen et al., 2005), digital goods auction (Bar-Yossef et al., 2002; Fiat et al., 2002; Goldberg et al., 2001; Goldberg and Hartline, 2003), market equilibrium (Conen and Sandholm, 2002; Xiaotie Deng and Papadimitriou, 2002; Devanur et al., 2002) and Walrasian equilibrium (Bikhchandani and Mamer, 1997; Chen et al., 2005; Gul and Stacchetti, 1999; Kelso and Crawford, 1982).

While several researchers have pursued auction models in SCM separately, less attention has been paid to auction mechanism with modern supply chain issues. In this paper, by introducing the auction mechanism, we aim to realize the following three functions. First is price mining: auction is an open mechanism with multiple participants. Every bid is based on its assessment of targeted products or services. Everyone has his own utility and purchasing ability. So, the final price reflects the market value of the auction. Second is dynamic modern supply chain construction: through auction, the firm can find appropriate partners efficiently. Most of the time auctions are one-time transactions. But if two firms, matched through auction, cooperate well in supply chain, they can continue this relationship. If one of parties is dissatisfactory, then a new auction can be held to find a new partner. Third is resources integration: in business practices, especially in modern supply chain auctions, auctioneers can integrate resources and ally buyers or sellers to gain more efficiency in auctions. In this paper, our focus is on supply and demand auction systems with resource pooling in modern supply chain from a theoretical modeling perspective. The supply and demand auction systems in modern supply chain among manufacturers and suppliers serve as information sharing mechanisms. Its main purpose is to match the supply and demand such that a modern supply chain can achieve incentive compatibility and economic efficiency. A key feature of an ideal auction system is that it provides access to a
rich set of information services and resource integration. We design such a supply and demand auction system that can integrate resources to match the supply and demand efficiently. The resources to be auctioned may be distributed in different locations or spots with connected paths, which we call a resource pooling network. Participating auctioneer desires to integrate the various resources to discover auction price efficiently although they also need to take into consideration some extra fees such as logistic costs besides the price.

The paper is organized as follows. In Section 2, we propose three theoretic models of modern supply chain auctions with resource pooling according to the Vickrey auction principle. They are supply auction model with demand resource pooling, demand auction model with supply resource pooling, and double auction model with demand and supply resource pooling. In Section 3, for the proposed auction models, we present three corresponding algorithms to allocate resources in the auction process by linear programming. In Section 4, we study the incentive compatibility and define the Walrasian equilibriums for the proposed auction models. We show that the solutions of the proposed algorithms are Walrasian equilibriums. Finally, we conclude our paper with remarks and future research directions in Section 5.

2. Auction models with resource pooling

Resource pooling refers to the resources which can be auctioned and are distributed from different companies in supply chain. We consider three auction models with resource pooling: a demand auction model, a supply auction model and a double auction model. In these models, we introduce an auctioneer, who shall organize the auction and implement the mechanism in place. The proposed auction models can be utilized in different situations, but they share one common objective that economic efficiency can be improved under the condition of incentive compatibility, and from information collection and resource integration. The demand auction model is utilized for the market where the supply is much larger than demand. In such a market, the auctioneer collects all the demand from the manufacturers and conducts a reverse auction with the suppliers. Usually the more demand the auctioneer collects, the more commission from manufacturers and more discount from suppliers. Conversely, the supply auction model suits for the situation where the supply is short and there is serious information asymmetry between sellers and buyers. Finally, the double auction model fits for the market where both supply and demand situations are unknown. To facilitate further discussion, we first make some assumptions, give some definitions and explain some general notations:

Assumption 1. There is information asymmetry between manufactures and suppliers, which means that manufactures do not know any information about the suppliers and the suppliers also can not estimate how big the production’s market is.

Assumption 2. Supply and demand bidding processes are not open between manufactures and suppliers.

Assumption 3. The auctioneer gets its profit not only from auction service commission but also from resource integration and information collection, demands collection and supply resource control etc.

Definition 1. (Demand resource pooling network) Suppose that there exist \( m' \) manufacturers. Each of them has commercial alliance relationship with the auctioneer. Let the set of the \( m' \) manufacturers be expressed by \( M' = \{M'_1, \ldots, M'_{m'}\} \). We define the manufacturers network \( G_M = (M', E'_1, w'_1) \) as the demand resource pooling network, where \( E'_1 \) is the set of the edge \( e \) with two end nodes \( M'_i \in M' \) and \( M'_j \in M' \). \( (M'_i, M'_j) \in E'_1 \) means that it is transportable between the two end nodes \( M'_i \) and \( M'_j \); \( w'_1 \) is weights defined on every edge \( e \in E'_1 \) as the logistic fee per unit goods between the two ends of \( e \). If \( (M'_i, M'_j) \notin E'_1 \),
then define $w'_1(M'_i, M'_j) = \infty$. Denote $w'_1(M'_i, M'_j)$ as the lowest logistic cost between $M'_i$ and $M'_j$, which can be calculated by:

$$w'_1(M'_i, M'_j) = \min \left\{ \sum_{e \in E} w'_1(e) \left| \Gamma \text{ is a path between } M'_i \text{ and } M'_j \right. \right\}. \quad (1)$$

**Remark 1.** The commercial alliance relationship between the auctioneer and each manufacturer $M'_i \in M'$ means that the auctioneer can collect demand from every manufacturer in $M'$, make some kind of pricing recommendation for them, and integrate the demands for auction as a whole on behalf of them. In return, each manufacturer in $M'$ will pay a commission fee to the auctioneer in proportion to the purchase amount.

Similarly, we define the supply resource pooling network as follows:

**Definition 2.** (Supply resource pooling network) Suppose that there exist $n'$ suppliers. Each of them has commercial alliance relationship with the auctioneer. Let the set of the $n'$ suppliers be expressed by $S' = \{S'_1, \ldots, S'_{n'}\}$. We define the suppliers network $G_S = (S', E'_2, w'_2)$ as the supply resource pooling network, where $E'_2$ is the set of the edge $e$ with two end nodes $S'_i \in S'$ and $S'_j \in S'$. $(S'_i, S'_j) \in E'_2$ means that it is transportable between the two end nodes $S'_i$ and $S'_j$. $w'_2$ is weights defined on every edge $e \in E'_2$ as the logistic fee per unit goods between the two ends of $e$. If $(S'_i, S'_j) \notin E'_2$, then define $w'_2(S'_i, S'_j) = \infty$. Denote $w'_2(S'_i, S'_j)$ as the lowest logistic cost between $S'_i$ and $S'_j$, which can be calculated by:

$$w'_2(S'_i, S'_j) = \min \left\{ \sum_{e \in \Gamma} w'_2(e) \left| \Gamma \text{ is a path between } S'_i \text{ and } S'_j \right. \right\}. \quad (2)$$

**Remark 2.** The commercial alliance relationship between the auctioneer and each supplier $S'_j \in S'$ means that the auctioneer is also as a retailer of every supplier in $S'$. Besides, the auctioneer can also make some kinds of selling recommendation for them and can integrate the supply resource for them. On the other hand, each supplier in $S'$ will give the auctioneer some kind price discount and auction commission.

**Definition 3.** (Auction supply and demand network) Suppose that there are $m$ manufacturers $M = \{M_1, \ldots, M_m\}$ and $n$ suppliers $S = \{S_1, \ldots, S_n\}$. We define the network $G = (M \cup S, E, w, Q)$ as auction supply and demand network, where $E$ is the edge set. The edge $e \in E$ means that both $S_j$ and $M_i$, the two end nodes of $e$, are transportable and $S_j$ is one of suppliers of $M_i$. $w$ is weights on every edge $e \in E$, which is defined as the logistic fee per unit goods between the two end nodes of $e \in E$, and $Q$ is another weights on every edge $e \in E$, which is defined as the amount that the supplier $S_j$ purchases to manufacturer $M_i$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n$.

Below, we suppose that there are $m$ manufacturers which are expressed by $M = \{M_1, \ldots, M_m\}$ and $n$ suppliers expressed by $S = \{S_1, \ldots, S_n\}$.
The demand auction model with demand resource pooling (DAMDRP)
The demand auction model with demand resource pooling is for the market where the 
supply is much larger than the demand. So, each supplier can meet the demand pooled 
across all manufacturers. In this situation, the auctioneer would collect the demands from 
the manufacturers and conduct a reverse auction with the suppliers to help procure the 
products for the manufacturers. Usually, the more demand the auctioneer collects, the 
more commission from the manufacturers and more discount from the suppliers. So, 
the auctioneer should not only have some information about the manufacturer’s willingness 
to pay from industry practices but also collect demand from its demand resource pooling network consisting of many manufacturers. In the demand auction model, the buyers consist of the all suppliers, the sellers are the manufacturers who hold demands and auction 
agent of both is the auctioneer.

Let \( G_{M'} = (M', E_1', w_1') \) be the demand resource pooling network of the auctioneer, where 
\( M' \subseteq M \). Without the loss of generality, suppose the auctioneer allocates the same spots as 
that of \( M'_1 \). Denote that \((P_{M'_i}, D_{M_i})\), \( i = 1, \ldots, m' \) are the demand of manufacturer \( i \), \( i = 1, \ldots, m' \) submitted to its auctioneer. Let \( P_{M'_i} \) satisfy:

\[
P_{M'_i} = P_{M'_i} + w_1'(M'_1, M'_i).
\]

As an alliance with auctioneer, the manufacturer \( M'_i \), \( i = 1, \ldots, m' \) can get \( \alpha \) discount from 
the auctioneer which usually satisfies \( \alpha < 5 \) percent. Let \( P_{M'_i}' = (1-\alpha)P_{M'_i} \) for \( i = 1, \ldots, m' \). 
Let \((P_{M'_i}, D_{M_i})\) denote the demand of the other \( m-m' \) manufacturers submitted to its 
auctioneer. Suppose the bidding price \( P_{M'_i} \) for all \( i = m'+1, \ldots, m \) includes the logistic fee 
per unit between the spot of manufacturer \( M_i \) and location of the auctioneer, i.e. the spot 
where \( M'_i \) is located. We denote \((P_{M'_i}, D_{M_i})\) as the actual demands of \( m \) manufacturers and 
expressed as follows:

\[
(P_{M'_i}, D_{M_i}) = \begin{cases} 
(P_{M'_i}, D_{M_i}), & i = 1, \ldots, m', \\
(P_{M'_i}, D_{M_i}), & i = m'+1, \ldots, m. 
\end{cases}
\] (3)

So, the total demand is \( D = \sum_{i=1}^{m} D_{M_i} \), and the average price is \( \overline{P}_{M} = \sum_{i=1}^{m} (D_{M_i}/D)P_{M_i}. \)

Note that \((P_{S_j}, D)\) is the bidding price of supplier \( j \), \( j = 1, \ldots, n \) for the total demand \( D \). 
The bidding price \( P_{S_j} \) for any \( j \) includes the logistic fee per unit between the spot of supplier \( S_j \) and location of the auctioneer, i.e. the spot where \( M'_i \) or \( M'_1 \) is located.

The auctioneer auctions the total demand \( D \) with the average price as the highest 
purchasing price, which serves as a reservation price in the reverse auction:

\[
\overline{P}_{M} = \sum_{i=1}^{m} \left( \frac{D_{M_i}}{D} \right)P_{M_i}. \] (4)

Remark 3. In the model (DAMDRP), the demand resource pooling network has the following 
characteristics: to guarantee the auctioneer’s lowest bidding demand; to be more 
efficient for the inside manufacturers; and the network will grow bigger because 
of the information asymmetry between manufacturers and suppliers.

2.2 Supply auction model with supply resource pooling (SAMSHP)
The supply auction model suits for the situation that the supply resource is short and there 
is serious information asymmetry between the sellers and buyers. Thus, the auctioneer
should not only have some information of supplier’s bidding from auction market but also integrate the supply resources from its supply resource pooling network consisting of the suppliers. Like the demand auction model, the buyers consist of the all manufacturers, and the sellers are the suppliers who hold resources. The auctioneer conducts the auction on the suppliers’ behalf, and collects commission from them.

Let \( G_S = (S', E'_2, w'_2) \) be the supply resource pooling network of the auctioneer, where \( S' \subseteq S \). Without the loss of generality, we assume that the auctioneer, \( M'_1 \) and \( S'_1 \) are located at the same spot. Denote that \((P'_{S'_j}, Q_{S'_j})\), \( j = 1, \ldots, n' \) are the supply of supplier \( j \), \( j = 1, \ldots, n' \) submitted to its auctioneer. Let \( P'_{S_j} \) satisfy:

\[
P'_{S_j} = P'_{S'_1} + w'_2(S'_1, S'_j).
\]

As an alliance with the auctioneer, the supplier \( S'_j, j = 1, \ldots, n' \) will pay \( \beta \) percent as an appreciation to the auctioneer, usually with \( \beta \leq 5 \) percent. Let \( P'_{S'_1} = (1 + \beta)P'_{S'_1} \) for \( j = 1, \ldots, n' \). Let \((P'_{S'_j}, Q_{S'_j})\), \( j = n' + 1, \ldots, n \) denote the supply of other \( n-n' \) suppliers submitted to its auctioneer. Suppose the bidding price \( P'_{S'_j} \) for any \( j = n' + 1, \ldots, n \) includes the logistic fee per unit between the spot of supplier \( S'_j \) and location of the auctioneer, i.e. the spot for \( S'_1 \). Let \((P_{S_j}, Q_{S_j})\) denote the actual supply of the suppliers expressed as follows:

\[
(P_{S_j}, Q_{S_j}) = \begin{cases} 
(P'_{S_j}, Q_{S_j}) & j = 1, \ldots, n'; \\
(P'_{S'_j}, Q_{S'_j}) & j = n' + 1, \ldots, n.
\end{cases}
\]

So, the total supply is \( Q = \sum_{j=1}^{n} Q_{S_j} \) with the average price of \( \bar{P}_S = \sum_{j=1}^{n} (Q_{S_j}/Q)P_{S_j} \).

Note that \( P_{M_j} \) is the bidding price of manufacturer \( i, i = 1, \ldots, m \) for the demand \( D_{M_j} \). The bidding price \( P_{M_i} \) for any \( i \) includes the logistic fee per unit between the spot of manufacturer \( M_i \) and the location of the auctioneer, i.e. the spot where supplier \( S'_1 \) or \( S'_1 \) is located.

The auctioneer auctions the total supply \( Q_{S_j}, j = 1, \ldots, n', n'+1, \ldots, n \) to the \( m \) manufacturers with bidding \( (P_{M_i}, D_{M_i}) \), where \( i = 1, \ldots, m \):

**Remark 4.** The supply resource pooling network has the following characteristics: to guarantee the auctioneer’s lowest bidding supply; and to control the short resource of supply resource pooling network by providing allied suppliers with higher bidding price and service priority, which can also make the network bigger.

### 2.3 Double auction model with resource pooling (DAMRP)

The double auction model deals with the situation where the supply and demand scenarios are unknown. In the double auction model, the buyers consist of all the manufacturers who submit their demands to the auctioneer, and the sellers are all the suppliers with supply-side resources who place bids with the same auctioneer. The auctioneer not only has some information of both manufacturer’s demand bidding and supplier’s supply bidding in the auction but also can integrate the demands resource from its demand resource pooling network and the supply resources from its supply resource pooling network.

Let \( G_S = (S', E'_2, w'_2) \) be the supply resource pooling network of the auctioneer, where \( S' \subseteq S \). Without the loss of generality, suppose the auctioneer occupies the same spot as that of \( S'_1 \). Let \((P'_{S'_j}, Q_{S'_j})\), \( j = 1, \ldots, n' \) be the supply of supplier \( j, j = 1, \ldots, n' \) for its supply \( Q_{S'_j} \).
submitted to its auctioneer. Let \( P'_{S_j} \) satisfy:

\[
P'_{S_j} = P'_{S_j} + w_2(S'_j, S'_j).
\]

As an alliance or membership with auctioneer, the supplier \( S'_j, j = 1, \ldots, n' \) can get \( \beta \) appreciation from the auctioneer which usually satisfies \( \beta \leq 5 \) percent. Let \( P'_{S_j} = (1 + \beta)P'_{S_j} \) for \( j = 1, \ldots, n' \). Let \((P'_{S_j}, Q_{S_j})\), \( j = n' + 1, \ldots, n \) denote the supply of other \( n-n' \) suppliers submitted to the auctioneer. Suppose the bidding price \( P'_{S_j} \) for any \( j = n' + 1, \ldots, n \) includes the logistic fee per unit between the spot of supplier \( S_j \) and that of the auctioneer, i.e. the spot where \( S'_j \) is located. Let \((P_{S_j}, Q_{S_j})\) denote the actual supply of \( n \) suppliers, which can be expressed as follows:

\[
(P_{S_j}, Q_{S_j}) = \begin{cases} (P'_{S_j}, Q_{S_j}) & j = 1, \ldots, n'; \\ (P_{S_j}, Q_{S_j}) & j = n' + 1, \ldots, n. \end{cases}
\]

Clearly, the total supply is \( Q = \sum_{j=1}^n Q_{S_j} \) and the average price is \( P_S = \frac{\sum_{j=1}^n (Q_{S_j}/Q)P'_{S_j}}{Q} \).

Let \( M_0 = (M', E_1, w'_1) \) be the demand resource pooling network of the auctioneer, where \( M' \subseteq M \). Without the loss of generality, we assume that the auctioneer, \( M'_1 \) and \( S_1 \) are located at same spot. Let \((P'_{M_i}, D_{M_i})\), \( i = 1, \ldots, m' \) denote the demand price of manufacturer \( i, i = 1, \ldots, m' \) for a quantity of demand \( D_{M_i} \) submitted to the auctioneer. Let \( P'_{M_i} \) be calculated as:

\[
P'_{M_i} = P'_{M_i} + w'_1(M'_1, M'_1).
\]

As an alliance or membership with auctioneer, the manufacturer \( M'_i, i = 1, \ldots, m' \) can get \( \alpha \) discount from the auctioneer, which usually satisfies \( \alpha \leq 5 \) percent. Let \( P'_{M_i} = \alpha P_{M_i} \) for \( i = 1, \ldots, m' \). Let \((P_{M_i}, D_{M_i})\) denote the demand of other \( m-m' \) manufacturers submitted to its auctioneer. Suppose the bidding price \( P_{M_i} \) for any \( i = m'+1, \ldots, m \) includes the logistic fee per unit between the spot of manufacturer \( M'_i \) and the location of the auctioneer, i.e. the spot where \( M'_1 \) is located. Let \((P_{M_i}, D_{M_i})\) denote the actual demands of \( m \) manufacturers, and we have:

\[
(P_{M_i}, D_{M_i}) = \begin{cases} (P'_{M_i}, D_{M_i}) & i = 1, \ldots, m', \\ (P_{M_i}, D_{M_i}) & i = m'+1, \ldots, m. \end{cases}
\]

Clearly, the total demand is \( D = \sum_{i=1}^m D_{M_i} \) and the average price is \( P_M = \frac{\sum_{i=1}^m (D_{M_i}/D)P_{M_i}}{D} \).

The auctioneer tries to make the auction most efficient because he/she can not only integrate resources from both sides, but also utilize the information asymmetry between manufacturers and suppliers. With these considerations, the auctioneer tries to construct the auction supply and demand network \( G = (M_0S, E, w, Q) \) to maximize the profit subject to the condition that both manufacturers and suppliers are incentive compatible and there is an improvement of economic efficiency.

### 3. Algorithms via Vickrey auction mechanism

From the perspective of the auctioneer, we propose three algorithms via Vickrey auction mechanism for the models (DAMDRP), (SAMSRP) and (DAMRP), respectively. We can employ linear programming to obtain the final allocation.

For the model (DAMDRP), we propose algorithm (DAMDRP) as follows.
3.1 Algorithm (DAMDRP) via Vickrey auction mechanism

- Event 0: demand collection: from manufacturers’ demands network, the auctioneer can collect the network’s demand bidding \((P_{M_i}, D_{M_i})\), \(i = 1, \ldots, m\); from auction market, the auctioneer can get the other manufacturers’ demand bidding \((P_{M_i}', D_{M_i}')\), where \(i = m' + 1, \ldots, m\); and let \(P_{M_i} = aP_{M_i}'\) for \(i = 1, \ldots, m', \) such as \(P_{M_i}\) can be calculated by (3) as the actual demand price of manufacturer \(i, i = 1, \ldots, m', m' + 1, \ldots, m\), for its demand \(D_{M_i}\) submitted to its auctioneer.

- Event 1: calculate the total demand \(D = \sum_{i=1}^{m} D_{M_i}\) and the highest purchasing price as follows:

\[
P_{M} = \sum_{i=1}^{m} \left( \frac{D_{M_i}}{D} \right) P_{M_i}.
\]

- Event 2: auction the demand \(D\) with the lowest price \(P_{M}\).

- Event 3: suppose that the final suppliers’ bidding is \((P_{S_j}, D)\), where \(j = 1, \ldots, n\). Without the loss of generality, let \(P_{S_1} \leq P_{S_2} \leq \cdots \leq P_{S_n}\).

- Event 4: make auction decision via Vickrey auction mechanism. If \(P_{S_1} \leq P_{S_2} \leq P_{M}\), supplier \(S_1\) wins and hedging price is \(P_{S_1}\). Go to Event 5; if \(P_{S_1} \leq P_{M} \leq P_{S_2}\), supplier \(S_1\) wins and hedging price is \(P_{M}\). Go to Event 5. Otherwise, remove the manufacturer with the lowest demand price and let \(m = m-1\). If \(m \geq 1\), go to Event 2; else, increase the bidding price of the demand resource pooling network, and repeat the whole process again, i.e. go to Event 0.

- Event 5: allocate the supply amount \(D\) to satisfy remained manufacturers. Assign the actual demand \((P_{M_i}, D_{M_i})\) to manufacturer \(M_i\), where \(i, i = 1, \ldots, m', m' + 1, \ldots, m\) for its demand \(D_{M_i}\) excepted those manufacturers who are removed during above auction process:

**Remark 5.** Note that some manufacturers are removed during the auction process because of their high bidding prices, which promotes them to joint auctioneer’s demand resource pooling network and get alliance discount. Thus, the auctioneer can accumulate large quantities of demand and can get lower hedging price.

**Remark 6.** The auctioneer’s profit can be calculated by:

\[
\eta \sum_{i=1}^{m} P_{M_i} D_{M_i} + \eta P_{hedging} D + \sum_{i=1}^{m} P_{M_i} D_{M_i} - P_{hedging} D - \sum_{i=2}^{m} D_{M_i} w(S_1, M_i),
\]

where \(\eta\) is the service fee for the auction process and \(P_{hedging}\) is the hedging price as expressed by:

\[
P_{hedging} = \begin{cases} 
P_{S_2}, & P_{S_1} \leq P_{S_2} \leq P_{M} \\
P_{M}, & P_{S_1} \leq P_{M} \leq P_{S_2}.
\end{cases}
\]

For the model (SAMSMP), we propose algorithm (SAMSMP) as follows.
3.2 Algorithm (SAMSRP) via Vickrey auction mechanism

- Event 0: resource collection: from the suppliers’ resource network, the auctioneer can collect the its price commitments $(P_{S_j}^i, Q_{S_j}^i), j = 1, \ldots, n’$; from the auction market, the auctioneer can get other supplier’s $(S_j, j = n’ + 1, \ldots, n)$ resource bidding $(P_{S_j}^i, Q_{S_j}^i)$; and let $P_{S_j}^i = (1 + \beta)P_{S_j}^i$ for $j = 1, \ldots, n’$, where $\beta$ (usually $\beta \leq 5$ percent) is the appreciation rate as the allied member of the supplier resource network from the auctioneer, such as $(P_{S_j}^i, Q_{S_j}^i)$ can be found by (5) as the actual supply bidding of supplier $S_j, j = n’ + 1, \ldots, n$. Calculate the total supply $Q = \sum_{j=1}^{n} Q_{S_j}$ and the average price $P_S = \sum_{j=1}^{n} (Q_{S_j} / Q) P_{S_j}$ as the reservation price.

- Event 1: auction with the resource limitation $Q = \sum_{j=1}^{n} Q_{S_j}$ with the average price $P_S = \sum_{j=1}^{n} (Q_{S_j} / Q) P_{S_j}$.

- Event 2: for the total resource commitment $(S, Q)$, suppose that the final manufacturers’ bidding $(P_{M_i}, D_{M_i}), i = 1, \ldots, m$ with $P_{M_1} \geq P_{M_2} \geq \ldots \geq P_{M_m}$ and $P_{M_{m+1}} = P_{M_m}$.

- Event 3: let $Q_{ij}$ be supplied from supplier $j$ to manufacturer $i, i = 1, \ldots, n$ and $j = 1, \ldots, m$. By the Vickrey auction mechanism, suppose the manufacturer $M_i$ wins with the hedging price is $P_{M_{i+1}}$ for every $i = 1, \ldots, m$ and $P_{M_{m+1}} = P_{M_m}$. Solve the linear programming problem:

$$\text{maximize} \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{S_j} - w(M_i, S_j) \right) Q_{ij}$$

subject to

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{S_j} - w(M_i, S_j) \right) Q_{ij} \geq 0$$

$$\sum_{j=1}^{n} Q_{ij} \leq D_{M_i}, \ i = 1, \ldots, m$$

$$\sum_{i=1}^{m} Q_{ij} = Q_{S_j}, \ j = 1, \ldots, n$$

$$P_{M_i} \geq P_S, \ i = 1, \ldots, m$$

$Q_{ij} \geq 0, \ i = 1, \ldots, m, j = 1, \ldots, n.$ \hspace{1cm} (10)

- Event 4: make auction decision.
  If linear programming (1) has optimal solution $Q_{ij}, i = 1, \ldots, m$ and $j = 1, \ldots, n$. Manufacturer $M_i$ wins with the hedging price is $P_{M_{i+1}}$ and amount $D_{M_i}, i = 1, \ldots, m$. Go to Event 5; otherwise, remove the manufacturer with lowest bidding price and let $m = m-1$. If $m \geq 1$, then go to Event 3, else remove the supplier with highest price and let $n = n-1$. If $n \geq 1$, then go to Event 1; else, based on the auction information, decrease the bidding price of supply resource pooling network, and start all over again, i.e. go to Event 0.

- Event 5: allocate the $Q_{ij}$ from supplier $j$ to manufacturer $i, i = 1, \ldots, n, j = 1, \ldots, m.$
Remark 7. The auction profit can be calculated by above optimal value:

\[
\eta \sum_{i=1}^{m} P_{M_{i+1}} D_{M_{i}} + \eta \sum_{i=1}^{m} P_{S_{j}} Q_{S_{j}} + \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{S_{j}} - w(M_{i}, S_{j}) \right) Q_{ij}. \quad (11)
\]

For the model (DAMRP), we propose algorithm (DAMRP) as follows.

### 3.3 Algorithm (DAMRP) via Vickrey auction mechanism

- Event 0: collect bidding information: suppose the final actual manufacturer \( M_{i} \) bidding \( (P_{M_{i}}, D_{M_{i}}) \), \( i = 1, \ldots, m \) with \( P_{M_{1}} \geq P_{M_{2}} \geq \cdots \geq P_{M_{m}} \) and \( P_{M_{m+1}} = P_{M_{m}} \). \( (P_{S_{j}}, Q_{S_{j}}) \) is the final actual supply bidding of supplier \( S_{j}, j = 1, \ldots, n' \), \( n' + 1, \ldots, n \) with \( P_{S_{1}} \leq P_{S_{2}} \leq \cdots \leq P_{S_{n}} \) and \( P_{S_{n+1}} = P_{S_{n}} \), where the series \( (P_{M_{i}}, D_{M_{i}}) \) has been rearranged in decreasing order from (3) and the series \( (P_{S_{j}}, Q_{S_{j}}) \) has been rearranged in increasing order from (5). Without the loss of generality, we assume that the auctioneer, \( M_{1} \) and \( S_{1} \) are located at same spot.

- Event 1: let \( Q_{ij} \) be supplied from supplier \( j, j = 1, \ldots, n \) to manufacturer \( i, i = 1, \ldots, m \). By Vickrey auction mechanism, suppose the manufacturer \( M_{i} \) wins the amount \( D_{M_{i}} \) with hedging price \( P_{M_{i+1}} \) and \( P_{M_{m+1}} = P_{M_{m}} \) for every \( i = 1, \ldots, m \) and the supplier \( S_{j} \) wins the supply \( Q_{S_{j}} \) with hedging price \( P_{S_{j+1}} \) and \( P_{S_{n+1}} = P_{S_{n}} \) for every \( j = 1, \ldots, n \). Solve the linear programming problem as follows:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{S_{j+1}} - w(M_{i}, S_{j}) \right) Q_{ij} \\
\text{subject to} & \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{S_{j+1}} - w(M_{i}, S_{j}) \right) Q_{ij} \geq 0 \\
& \quad \sum_{j=1}^{n} Q_{ij} \leq D_{M_{i}}, i = 1, \ldots, m \\
& \quad \sum_{i=1}^{m} Q_{ij} = Q_{S_{j}}, j = 1, \ldots, n \\
& \quad P_{M_{i}} \geq \overline{P}_{S_{i}}, i = 1, \ldots, m \\
& \quad P_{S_{j}} \geq \overline{P}_{M_{j}}, j = 1, \ldots, n \\
& \quad Q_{ij} \geq 0, i = 1, \ldots, m, j = 1, \ldots, n, \quad (12)
\end{align*}
\]

where \( \overline{P}_{M} \) is the highest purchasing price of the \( m \) manufacturers and \( \overline{P}_{S} \) is the lowest selling price of \( n \) suppliers.

- Event 2: make auction decision.

If linear programming (2) has optimal solution \( Q_{ij}, i = 1, \ldots, n, j = 1, \ldots, m \), Manufacturer \( M_{i} \) wins \( D_{M_{i}} \) with the hedging price \( P_{M_{i+1}} \), \( i = 1, \ldots, m \) and supplier \( S_{j} \) wins the supply \( Q_{S_{j}} \) with hedging price \( P_{S_{j+1}} \), \( j = 1, \ldots, n \). Go to Event 3; otherwise, remove the manufacturer with the lowest bidding price and let \( m = m - 1 \). Meanwhile, remove the supplier with the highest bidding price and let \( n = n - 1 \). If both \( m \geq 1 \) and \( n \geq 1 \), go to Event 1, else recalculate \( D = \sum_{i=1}^{m} D_{M_{i}} \) and \( Q = \sum_{j=1}^{n} Q_{S_{j}} \). If \( D < Q \), then decrease the bidding price of supply resource pooling.
network; else increase the bidding price of demand resource pooling network, and make auction again, i.e. go to Event 0.

- Event 3: allocate the supply amount $Q_{ij}$ from supplier $j$ to manufacturer $i, i = 1, \ldots, n$ and $j = 1, \ldots, m$:

Remark 8. In Algorithm (DAMRP), $Q_{S_j}, j = 1, \ldots, n$ is also the resource limit of supplier $S_j$.

Remark 9. In the Event 3, auctioneer can utilize auction information to increase the bidding price of demand resource pooling network and decrease the bidding price of supply resource pooling network, which shows that the auctioneer can not only integrate resources but also coordinate market for the economic efficient and auction’s incentive compatibility by sharing information with its alliances.

Remark 10. The auction profit can be calculated by above optimal value:

$$\eta \sum_{i=1}^{m} P_{M_{i+1}} D_{M_i} + \eta \sum_{j=1}^{n} P_{S_{j+1}} Q_{S_j} + \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} Q_{S_j} - w(M_i, S_j) \right) Q_{ij}. \quad (13)$$

4. Incentive compatibility and Walrasian equilibrium

4.1 Incentive compatibility

In the general auction model, we know that Vickrey auction is incentive compatible (Vickrey, 1961). That is to say that the buyer is satisfied by the transaction that the buyer with the highest bid wins the resource at the price of the second highest bid. In our proposed three models, we apply the idea of Vickrey auction mechanism. We can prove that the proposed three auctions with resource pooling for modern SCM are also incentive compatible, i.e. all of the auction winners are satisfied by the transaction that manufacturer winners pay less than they expected and supplier winners get more than they submitted to the auctioneer:

Theorem 1. The three auction models (DAMDRP), (SAMSRP) and (DAMRP) with algorithms (DAMDRP), (SAMSRP) and (DAMRP), respectively, are incentive compatible.

Proof. The proof can directly be conducted from the incentive compatibility of Vickrey auction. Indeed, we employ the Vickrey auction mechanism in our proposed three auctions with resource pooling for modern SCM and we have the results that the hedging price is always less than the bidding price for every manufacturer winner, i.e. we have $P_{M_{i+1}} \leq P_{M_i}$, $i = 1, \ldots, m$ or we have the results that the hedging price is always greater that bidding price for every supply winner, i.e. we have $P_{S_{j+1}} \geq P_{S_j}$, $j = 1, \ldots, m$. Thus, the theorem is proved. ■

4.2 Walrasian equilibrium

Intuitively, Walrasian equilibrium is a vector of price and allocation matrix such that all auction winners are satisfied with the corresponding allocation, and the market clears or the price of non-allocated supply is zero. In the proposed auction models with resources pooling for modern SCM, we define Walrasian equilibria for our models respectively as follows:

Definition 4. (Walrasian equilibrium for auction model (DAMDRP)) For the proposed auction model (DAMDRP) with algorithm (DAMDRP) via Vickrey Auction Mechanism, we define a Walrasian equilibrium is a hedging supply series \( (P_{S_j}, Q_{S_j}), j = 1, \ldots, n \) and demand series \( (P_{M_i}, D_{M_i}), i = 1, \ldots, m \)
allocation matrix $Q_{ij}$ from supplier $S_j$ to manufacturer $M_i$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$ such that each supply winner is satisfied with the hedging price and each manufacturer winner is satisfied with the demand allocated; the auctioneer’s utility is maximized by the corresponding allocation; and the final sum of bidding demands can be produced by supplies and sold out to the manufacturer winners.

**Definition 5.** (Walrasian equilibrium for auction model (SAMSRP)) For the proposed auction model (SAMSRP) with algorithm (SAMSRP) via Vickrey Auction Mechanism, we define a Walrasian equilibrium is a hedging demand series $(P_{Mi}, D_{Mi})$, $i = 1, \ldots, m$, supply series $((P_{Sj}, Q_{Sj}), j = 1, \ldots, n)$ and allocation matrix $Q_{ij}$ from supplier $S_j$ to manufacturer $M_i$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$, such that each manufacturer winner is satisfied with the hedging price and amounts allocated; the auctioneer’s utility is maximized by the corresponding allocation; and the final sum of bidding supply can be sold out to the manufacturer winners.

**Definition 6.** (Walrasian equilibrium for auction model (DAMRP)) For the proposed auction model (DAMRP) with algorithm (DAMRP) via Vickrey Auction Mechanism, we define a Walrasian equilibrium is a hedging demand series $(P_{Mi}, D_{Mi})$, $i = 1, \ldots, m$, a hedging supply series $((P_{Sj}, Q_{Sj}), j = 1, \ldots, n)$ and allocation matrix allocation matrix $Q_{ij}$ from supplier $S_j$ to manufacturer $M_i$, $i = 1, \ldots, n$ and $j = 1, \ldots, m$, such that each manufacturer and supply winner is satisfied with the hedging price and amounts allocated; the auctioneer’s utility is maximized by the corresponding allocation; and each winner’s bidding supply can be sold out to the manufacturer winners and each manufacturer’s bidding demand can be supplied.

**Theorem 2.** The auction model (DAMDRP) with algorithm (DAMDRP) has Walrasian equilibrium.

Proof. According to Algorithm (DAMDRP), first, the hedging price of supply winner is calculated by:

$$P_{\text{hedging}} = \begin{cases} P_{S_1}, & P_{S_1} \leq P_{S_2} \leq P_M, \\ P_M, & P_{S_1} \leq P_M \leq P_{S_2}. \end{cases}$$

(14)

Because $P_{S_1} \leq P_{\text{hedging}}$, the supply winner $S_1$ is satisfied with the hedging price. For each manufacturer winner, its demand commitment can be met exactly and satisfied with the auction. Second, by Remark 6, in the auction model (DAMDRP) with algorithm (DAMDRP), the auctioneer’s utility can be calculated by:

$$\eta \sum_{i=1}^{m} P_{Mi} D_{Mi} + \eta P_{\text{hedging}} D + \sum_{i=1}^{m} P_{Mi} D_{Mi} - P_{\text{hedging}} D - \sum_{i=2}^{m} D_{Mi} w(S_1, M_i),$$

(15)

where $\eta$ is the service fee for the auction process and $P_{\text{hedging}}$ is the hedging price. Note that by (15), we have:

$$\sum_{i=1}^{m} P_{Mi} D_{Mi} - P_{\text{hedging}} D \geq \sum_{i=1}^{m} P_{Mi} D_{Mi} - P_M D = \sum_{i=1}^{m} P_{Mi} D_{Mi} - D \sum_{i=1}^{m} \left( \frac{D_{Mi}}{D} \right) P_{Mi} = 0,$$

which means that the auctioneer’s utility is maximized by the auction strategy. Third,
because \( D = \sum_{i=1}^{m} D_{Mi} \), the final sum of bidding demands can be produced by supplies and sold out to the manufacturer winners. By Definition 4, The auction model (DAMDRP) with algorithm (DAMDRP) has Walrasian equilibrium:

**Theorem 3.** The auction model (SAMSRP) with algorithm (SAMSRP) has Walrasian equilibrium.

Proof. According to algorithm (SAMSRP), first, the manufacturer \( M_i \) wins with hedging price \( P_M \) for every \( i = 1, \ldots, m \) and \( P_{M_{m+1}} = P_{M_n} \). For any \( i = 1, \ldots, m \), we have \( P_M \geq P_{M_{i+1}} \) and each manufacturer winner will be satisfied with the lower hedging price than expected. Second, by Remark 7, in the auction model (SAMSRP) with algorithm (SAMSRP), the auctioneer’s utility can be calculated by:

\[
\eta \sum_{i=1}^{m} P_{M_{i+1}} D_{Mi} + \eta \sum_{i=1}^{m} P_{S_{j+1}} Q_{Sj} + \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{Sj} - w(M_i, S_j) \right) Q_{ij}.
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{Sj} - w(M_i, S_j) \right) Q_{ij} \geq 0 \text{ by (10) which means that the auctioneer's utility is maximized by the allocation solution. Second, by (10), the allocation solution } Q_{ij}, i = 1, \ldots, m, j = 1, \ldots, n \text{ satisfies } \sum_{i=1}^{m} Q_{ij} = Q_{Sj}, j = 1, \ldots, n \text{ which means that the supply market is clear at last, i.e. final sum of bidding supply can be sold out to the manufacturer winners. By Definition 5, the auction model (SAMSRP) with algorithm (SAMSRP) has Walrasian equilibrium:}

**Theorem 4.** The auction model (DAMRP) with algorithm (DAMRP) has Walrasian equilibrium.

Proof. According to algorithm (DAMRP), first, the manufacturer \( M_i \) wins \( D_{Mi} \) with hedging price \( P_{M_{i+1}} \) for every \( i = 1, \ldots, m \) and \( P_{M_{m+1}} = P_{M_n} \). Besides, supplier \( S_j \) wins supply \( Q_{Sj} \) with the hedging price \( P_{S_{j+1}} \). For any \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \), we have \( P_M \geq P_{M_{i+1}} \). Thus, each manufacturer winner and each supplier winner will be satisfied with the hedging price than expected, respectively. Second, by Remark 9, in the auction model (DAMRP) with algorithm (DAMRP), the auctioneer’s utility can be calculated by:

\[
\eta \sum_{i=1}^{m} P_{M_{i+1}} D_{Mi} + \eta \sum_{i=1}^{m} P_{S_{j+1}} Q_{Sj} + \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{Sj} - w(M_i, S_j) \right) Q_{ij}.
\]

We have \( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_{M_{i+1}} - P_{Sj} - w(M_i, S_j) \right) Q_{ij} \geq 0 \) by (12), so the auctioneer’s utility is maximized by the allocation solution. Third, by (12), the allocation solution \( Q_{ij}, i = 1, \ldots, m, j = 1, \ldots, n \) satisfies \( \sum_{j=1}^{n} Q_{ij} \leq D_{Mi}, i = 1, \ldots, m \) and \( \sum_{i=1}^{m} Q_{ij} \leq Q_{Sj}, j = 1, \ldots, n \). By the algorithm (DAMRP), the supply market is clear when the total supply \( Q \) is less than the total demand \( D \). The demand market is clear when the total supply \( Q \) is greater than the total demand \( D \), and both supply and demand markets are clear when the total supply \( Q \) is equal to the total demand \( D \). Thus, each winner's bidding supply can be sold out to the manufacturer winners and each manufacturer's bidding demand can be supplied. By Definition 6, The auction model (DAMRP) with algorithm (DAMRP) has Walrasian equilibrium.

### 5. Conclusion and further research

In the paper, we propose three theoretic models and corresponding algorithms of modern supply chain auctions with resource pooling according the Vickrey auction principle, which achieves three functions: price mining, dynamic modern supply chain construction and
resources integrating. Besides, these proposed models are much closer to practical settings and may have potential applications in modern SCM.

There are many possible directions for future theoretical studies of these models, for example, the concept and existence about other kinds of equilibrium, the optimal allocations (corresponding to linear and duality approach), complexity analysis, etc.

References


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