Optimal re-allocation of mooring areas for yachts
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Abstract

Purpose – The purpose of this study is to propose a mathematical optimization model to solve the yacht mooring area re-allocation problem (YMARP). The objective of the problem is to allow the maximum number of yachts to be moored at their ideal mooring areas.

Design/methodology/approach – In this paper, the YMARP is introduced, formally defined and discussed. The authors develop a 0-1 integer programming optimization model for the problem, which can be solved efficiently using off-shelf solvers. The performance of the model is tested on extensive numerical experiments.

Findings – The results of the numerical experiments demonstrate that the proposed model can solve the YMARP very efficiently using off-shelf solvers like CPLEX. In particular, problems with up to 30,000 yachts and 400 mooring areas can be solved to optimum within 30 s.

Originality/value – This study is one of the preliminary studies to consider problems arising in yacht management in a quantitative manner. The proposed model has three main merits. First, it enables the government to better manage yachts and mooring areas. Second, with more yacht owners assigned with ideal mooring areas, the model helps reduce the traveling time of the yacht owners to yacht mooring areas. Third, by reducing the traveling time of the yacht owners, the model contributes to lessening the traffic burden in cities.

Keywords Optimization model, Mooring area re-allocation, Yacht management

Paper type Research paper

1. Introduction

Yacht tourism is an emerging form of leisure tourism in the coastal regions. It not only contributes significantly to the development of the economy (Chen et al., 2016) but also creates new job opportunities (Stone, 2000). A “yacht”, which is defined as a pleasure vessel, can be any of various recreational watercraft, such as sailboats used for racing or large crafts used for pleasure cruising (Yang and Liu, 2016). Thanks to the increasing success of yachting tourism in recent years, the number of yachts in the world is rising steadily. For example, in Hong Kong, the number of pleasure vessels increased by 23 per cent in four years from 2012 to 2016 (Chan, 2017). In 2005, the total number of mega-yachts over 30 meters long was 2,243 worldwide. In 2010, there were 6,000 mega-yachts in total throughout the world and the number of mega-yachts more than 30 meters long was nearly doubled to 4,419 (Luković, 2012).

Yachts are required to be moored in mooring spaces when they are not sailing at sea. Mooring spaces protect vessels from tide and strong wind and can also help to maintain the traffic order in the sea, avoiding too many yachts staying in the open sea (Dulebenets, 2016;

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Dadashi et al., 2017). Generally speaking, one yacht is supposed to be moored at one assigned mooring space. Each mooring space belongs to a designated mooring area and one mooring area can consist of a number of mooring spaces. In recent years, an increasing number of coastal cities around the world suffering from an urgent shortage of yacht moorings because of the rapid increase in vessels and limited mooring spaces (Dulebenets, 2017). One example is Hong Kong, which has 46 private mooring areas (government’s mooring areas for berthing private yachts) consisting of a total of 2,123 mooring spaces. In August 2018, only 24 mooring spaces in two mooring areas were available for accommodating new yachts (Marine Department, 2018). Another example is the lack of moorings of London. Given London’s growing population and record-high house prices and rents, an increasing number of people are choosing to live on boats as a more affordable housing option. From March 2012 to March 2018, the total number of boats in London nearly doubled from 2,326 to 4,098, but the increase in mooring spaces is much slower. As the sailing vessels are supposed to be moored in the mooring spaces, the berths for yachts are also in an urgent shortage (Canal and River Trust, 2018).

For specific reasons, a yacht owner may prefer some mooring areas than others to moor his/her yacht, and these mooring areas are regarded as the ideal mooring areas for this yacht. Generally, ideal mooring areas for one yacht should be easy to reach from the owner’s living or working place, have easy access to fuel stations and maintenance centers and protects the yacht from severe weather conditions. It can be seen that different yachts may have different ideal mooring areas. Moreover, the ideal mooring areas of a yacht may change with time because of the changes in the owner’s working or living places, the chosen yacht maintenance centers, the city transportation system, etc. However, because of the shortage of mooring spaces, it will take a long time for a yacht owner to apply for the ideal mooring space. As a result, yachts are not always able to be moored at their ideal mooring places. Under this condition, mutual personal transferring and subletting mooring spaces occur. Thus, it is recommended that the marine department should rationalize the administrative measure for regulating the transfer of private moorings (Audit Commission of HKSAR, 2012).

One possible way is conducting the re-allocation of yacht mooring areas under the regulation of the government to allow more yachts to be moored at their ideal mooring areas to reduce the illegal private transferring (Lui, 2015). As the mooring space applications are processed in a first-come-first-served order by the government, one scenario is that person A and person B may submit their applications at different time with different mooring areas available. It is possible that person A lives near mooring area M, and person B lives near mooring area N, but A’s yacht is assigned to be moored in N, while B’s yacht is assigned to be moored in M. In another scenario, person C owns a yacht in mooring area O for a long time, but he/she is now going to live or work in a new place near mooring area P, which is quite far away from area O, while a mooring space in area P was issued to a yacht owner who lives near area O. In both situations, optimal re-allocation can be conducted by exchanging yacht mooring spaces between the two owners or even among more owners, resulting in more yacht owners provided with mooring spaces in ideal areas.

This paper aims to address the yacht mooring area re-allocation problem (YMARP). The contribution of this paper is as follows. This paper puts forward an optimal yacht mooring area re-allocation scheme to allow the maximum number of yachts to be berthed at their ideal moorings. A mathematical model is proposed and the optimal solution is found to solve the model. It is formulated as a 0-1 integer programing model and can be solved by the optimization solver called CPLEX optimizer. Computation result of a numerical example shows that the proposed model can solve randomly generated problems with up to 30,000 yachts distributed in 420 mooring areas within 30s while allowing 28,625 yachts to be
re-allocated to their ideal mooring areas. The purpose of this paper is to propose one of the first few models that address problems arising in yacht management in a quantitative fashion.

This paper is organized as follows. In Section 2, we review the literature on yacht management. In Section 3, we give a description of the problem and formulate a mathematical model for the proposed problem. Section 4 illustrates a small-scale numerical example and the computational results of large-scale experiments. Section 5 concludes.

2. Literature review

2.1 Literature on yacht management

Topics related to yacht management have been studied for several decades. One stream of related studies focuses on yacht tourism. Sariisik et al. (2011) used strength, weakness, opportunities and threats analysis to study the current situation of yachting and analyzed managerial conditions for yacht tourism in Turkey. Their paper focused on the resources for yacht tourism such as natural and social environment elements and adequacy of trained work-force. Alcover et al. (2011) evaluated the impact of yacht tourism on the economy of the Balearic Islands, a group of islands of Spain located in the western Mediterranean Sea, and came to the conclusion that the average daily expenditure for a yachting tourist is €203.90, which was much higher than the daily expenditure of a traditional tourist (€105.20 per day). Cho (2012) quantitatively examined 300 visitors’ preference to yacht tourism and perceptions to the importance of yacht tourism industry’s activation strategies by using survey methodology. The conclusion was to adopt strategies including developing yacht tourism products, and establishing yacht tourism and marketing strategies. These three papers mainly focus on the development process and the current situation of yacht tourism.

Another stream of studies focuses on yacht moorings; Bosman et al. (1991) considered problems in the pre-design investigation, design and construction of pleasure craft harbor. They used a harbor in South African West Coast as a case study. Adams et al. (1992) did the first extensive survey on recreational craft traffic during 1989-90 and proposed that boat mooring management policy should be formulated, but they did not put forward specific measures. George (1994) proposed an approach called the Soufriere Marine Management Area (SMMA) to zone the coastal district of Soufriere, a town on the West Coast of Saint Lucia and in the eastern Caribbean Sea, into several parts for different uses to resolve the conflicts between local fishermen and the growing yachting tourism sector. In a later review of SMMA by Nichols et al. (2004), they commented that the approach was much more difficult to be implemented than expected because of the ignorance of the interest of fishermen and the failure to deal with key issues such as stakeholder and public awareness and security needs of the approach. Piccinno and Zanini (2010) reviewed the development process of boating and yachting and the promoted demand for berth places. Their paper mainly emphasized the necessity and urgency of the construction of yacht harbors but did not present any concrete solutions. Park and Joo (2011) proposed a 0-1 integer programing mathematical model to determine the optimal yacht mooring location from four candidates in the Mokpo sea area in South Korea using 21 important factors to obtain the maximized efficiency. The objective function contained the 21 factors, which were classified into four weighted grades. The constraints consisted natural conditions such as waves and the depth of water, and the current situations including maritime traffic congestion, initial development cost and sea zone utilization. Numerical experiment results showed that the Square Sea area was the optimal marina’s establishment location among the candidates in the Mokpo sea area.
In the above-mentioned five papers, the researchers all did qualitative analysis and primarily focused on physical factors including environmental conditions and local government regulations.

3. Problem description and model

3.1 Problem description

This section defines the YMARP formally. To begin with, in this paper, we classify the yachts into four types according to their length, that is, small, medium, large, and extra-large. In a certain mooring area, yachts of the same type are supposed to be moored in a sub-area, and one sub-area can only hold at most one type of yacht. In our model, we view each sub-area as an individual mooring area for simplicity, i.e., each mooring area in the proposed model contains only one type of yacht. One can make it clearer by an analogy between the definitions related to a mooring area and a parking lot. In a parking lot, there are separated sub-areas to park different types of vehicles, which may include a sub-area for motorcycles, a sub-area for cars, a sub-area for buses, and a sub-area for trucks. The sub-areas for parking different vehicles are like the sub-areas for mooring different types of yachts. Just like each vehicle should be parked at a parking space in a specific sub-area of a parking lot, the yachts are required to be moored at a mooring space in a specific sub-area of a mooring area. In reality, unless with specially permitted, yachts of a specific type are not allowed to be moored in sub-areas that are reserved for yachts of different types. One typical yacht marina in Hong Kong is the Royal Hong Kong Yacht Club (RHKYC), which has three marinas in Hong Kong and is the first Hong Kong yacht club. The RHKYC provides 132 moorings for approximately 100 class sailing yachts with sizes of 40 to 85 feet. The marinas also offer various boating services, including but not limited to freshwater, power, security, and convenience access to the best cruising waters in Hong Kong.

Applications for mooring spaces made by yacht owners are first delivered to the marine department and will be processed in a first-in-first-served order. In this stage, every successful applicant will be assigned a mooring space in a mooring area that suits exactly the size of the yacht. If the indicated ideal mooring areas are full, the application will be put in the waiting list. The yacht owners may apply to abolish the right to use their current mooring spaces when their yachts are abandoned or sold to other people or companies, or they move to other living places and thus apply for new mooring spaces. In these situations, the abolished mooring spaces are available and can be assigned to other applicants. In the current allocation mechanism, exchange is not allowed and re-allocation of mooring space can be very complicated. Suppose a yacht owner wants to change to a new mooring space, he/she has to make a new application and wait in the queue, which can be time-consuming and laborious. Also, it cannot be guaranteed that the largest number of yachts can be moved to their ideal mooring areas in the re-allocation stage as the applications will be processed again like the initial applications in a first-come-first-served sequence.

To allow more yachts to be moored at their ideal mooring areas and simplify processing steps, a mooring area re-allocation optimization model is developed in this paper. The information regarding the needs of yacht owners for the re-allocation of mooring areas can be obtained by the government through a survey via phone calls, emails, and text messages. There may be two situations. In the first situation, where the yachts will remain unchanged, the owners may provide wrong contact information and cannot be reached, or they just ignore the messages and do not reply. It is also possible that the owners will reply but their yachts have already been moored at ideal positions so they do not want a re-allocation. In the other situation, where reallocation will be conducted, the yacht owners are not quite satisfied with the current mooring areas, so
they want a change and provide their ideal mooring areas to the government. If any one of the indicated new mooring areas of a yacht owner has vacancies, the marine department can give permission directly to the owner to allow re-allocation of his/her yacht. If all the ideal mooring areas indicated by a yacht owner are all full, then a mooring area re-allocation optimization model can be applied to allow exchanges of mooring spaces among the owners.

3.2 Mathematical model

A model for the YMARP is formulated in this section. We only take the yachts that need to be re-allocated and whose ideal mooring areas are all fully occupied into consideration (if any one of the ideal mooring areas still has vacancies, then the yacht can be moved there directly without exchange). Note that exchanges of mooring areas can only be conducted among the yachts of the same type. As an example, we take the yachts and mooring areas of medium type into consideration. Let $\mathcal{P}$ denote the set of medium type mooring areas, and $p \in \mathcal{P}$ denotes each separate medium type mooring area, in which all the medium type mooring spaces are identical. A set of medium type yachts $V$ is moored in different mooring spaces belonging to mooring area set $\mathcal{P}$ and the index for a medium type yacht is $v \in V$. Yacht $v$ has an ideal medium mooring area set $\mathcal{P}_v$ and is initially moored at $p^0_v \in \mathcal{P}\setminus\mathcal{P}_v$. As each yacht has an ideal mooring area set, $\mathcal{P}_v \neq \emptyset$ and ideally, $\mathcal{P}_v$ can contain all medium mooring areas except $p^0_v$. We further denote $V^0_\mathcal{P} \subset V$ as the set of yachts that are initially located in $p$, and therefore $V^0_\mathcal{P} = \{ v \in V | p^0_v = p \}$. Let $V^0_\mathcal{P} \subset V$ denote the set of yachts whose initial mooring area is not $p$ but one of its ideal mooring areas is $p$, i.e. $V^0_\mathcal{P} = \{ v \in V | p \in \mathcal{P}_v \}$. Let $n_p$ denote the size of mooring area $p$ which is defined as the maximum number of mooring spaces in $p$ to accommodate yachts re-allocated from other mooring areas. It is easy to infer that $n_p$ is equal to the number of yachts that need to be re-allocated and initially moored in $p$, i.e. $n_p = |V^0_\mathcal{P}|, p \in \mathcal{P}$ and $\sum_{p \in \mathcal{P}} n_p = |V|$.

For the ease of reading, the notation is listed in Notation.

Notation used in the model

**Indices:**
- $p$ Index of a mooring area
- $v$ Index of a yacht that needs to be re-allocated

**Sets:**
- $\mathcal{P}$ Set of all mooring areas. All spaces in a mooring area are identical
- $\mathcal{P}_v$ Set of ideal mooring areas of a yacht $v \in V$; $\mathcal{P}_v \neq \emptyset$ and $\mathcal{P}_v \subset \mathcal{P}$, i.e. $\mathcal{P}_v$ is non-empty and is a strict subset of $\mathcal{P}$ because the initial mooring area of yacht $v$ is not in $\mathcal{P}_v$
- $V$ Set of yachts that need to be re-allocated
- $V^0_\mathcal{P}$ Set of yachts whose initial mooring area is $p \in \mathcal{P}$, i.e. $V^0_\mathcal{P} = \{ v \in V | p^0_v = p \}$ and $|V^0_\mathcal{P}| = n_p$
- $V^0_\mathcal{P}$ Set of yachts whose initial mooring area is not $p \in \mathcal{P}$ and one of its ideal mooring areas is $p$, i.e. $V^0_\mathcal{P} = \{ v \in V | p \in \mathcal{P}_v \}$

**Parameters:**
- $p^0_v$ Initial mooring area of yacht $v \in V$; $p^0_v \in \mathcal{P}\setminus\mathcal{P}_v$
- $n_p$ The size of mooring area $p \in \mathcal{P}$ (number of mooring spaces); $n_p$ is equal to the number of yachts whose initial mooring area is $p$; therefore, $\sum_{p \in \mathcal{P}} n_p = |V|$

**Decision Variables:**
- $\theta_{vp}$ Binary, set to 1, if yacht $v \in V$ is re-allocated to mooring area $p \in \mathcal{P}_v$ and 0, otherwise.
Let $\theta_{vp}$ be a binary decision variable to indicate if yacht $v$ is re-allocated or not. The YMARP is formulated in M1 as follows.

$$\text{[M1]} \quad \max \sum_{v \in V} \sum_{p \in P} \theta_{vp}$$

(1)

Subject to

$$\sum_{p \in P} \theta_{vp} \leq 1, \quad \forall v \in V$$

(2)

$$\sum_{v \in V_p} \left(1 - \sum_{p' \in P_v} \theta_{vp'}\right) + \sum_{v \in V_p} \theta_{vp} = n_p, \quad \forall p \in P$$

(3)

$$\theta_{vp} \in \{0, 1\}, \quad \forall v \in V, \forall p \in P_v.$$  

(4)

Objective (1) maximizes the total number of yachts that are re-allocated to one of their ideal mooring areas. Constraint (2) ensures that each yacht can be moved to at most one of their ideal mooring areas. Constraint (3) guarantees the total number of used mooring spaces in each mooring area remains the same before and after the re-allocation. The left-hand side of Constraint (3) contains two terms which calculate the number of yachts that are initially located in a mooring area and remains unchanged and successfully re-allocated to mooring area $p$ from other mooring areas, respectively. Constraint (4) defines the domain of $\theta_{vp}$.

Model [M1] is an integer programming optimization model with a bounded feasible area, and hence there always exists an optimal solution for a specific instance, although the optimal solution may not be unique (Luenberger and Ye, 1984).

We demonstrate the model using a small-scale example. Consider there are four mooring areas, namely, A, B, C and D, containing six yachts that need to be re-allocated, namely, a, b, c, d, e and f. Thus, $P = \{A, B, C, D\}$, and $V = \{a, b, c, d, e, f\}$. The relationship between the yachts and the mooring areas is illustrated in Figure 1. The links in the figure represent the ideal mooring areas of the yachts, with the tail if a link is the yacht, and the head of the link

![Figure 1. An illustration of a mooring area in Hong Kong](image-url)
is its ideal mooring area. The specific information of the yachts and the mooring areas are provided in Tables I and II.

The objective of the model is to design a re-allocation scheme to allow as many yachts to move to their ideal mooring areas as possible.

### 3.3 Practical extension to re-allocating different types of yachts together

In the proposed re-allocation optimization model, only yachts of the same type can exchange their mooring spaces. As a practical extension of the system, we can consider different types of yachts together when re-allocating. Take a simple situation as an example. In mooring area A, two owners of large type yachts in front-rear-conjunct mooring spaces make applications to change to new mooring spaces in mooring area B during the same application period. Meanwhile, there is no large type mooring space available in area B but there is a yacht owner with an extra-large type yacht that wishes to change its berth to A.

In this situation, we may think about combining the two large type mooring spaces in A to form an extra-large type mooring space and dividing the extra-large type mooring space into two large type mooring spaces. If the combination and division are applicable, the exchange of these three yachts can be conducted. In the mooring area re-allocation optimization system, the two large yachts in A should be viewed as one yacht in quantity and the extra-large yacht in B should be viewed as two yachts in quantity to satisfy

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<table>
<thead>
<tr>
<th>Yacht no.</th>
<th>Initial mooring area</th>
<th>Ideal mooring area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$p_a^0 = A$</td>
<td>$P_a = {B, C}$</td>
</tr>
<tr>
<td>b</td>
<td>$p_b^0 = A$</td>
<td>$P_b = {C}$</td>
</tr>
<tr>
<td>c</td>
<td>$p_c^0 = B$</td>
<td>$P_c = {A}$</td>
</tr>
<tr>
<td>d</td>
<td>$p_d^0 = C$</td>
<td>$P_d = {A, D}$</td>
</tr>
<tr>
<td>e</td>
<td>$p_e^0 = D$</td>
<td>$P_e = {B}$</td>
</tr>
<tr>
<td>f</td>
<td>$p_f^0 = D$</td>
<td>$P_f = {C}$</td>
</tr>
</tbody>
</table>

**Table I. Information of the yachts**

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<table>
<thead>
<tr>
<th>Mooring area no.</th>
<th>Size</th>
<th>Yachts initially moor in</th>
<th>Yachts ideally moor in</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$n_A = 2$</td>
<td>$V_A^0 = {a, b}$</td>
<td>$V_A = {c, d}$</td>
</tr>
<tr>
<td>B</td>
<td>$n_B = 1$</td>
<td>$V_B^0 = {c}$</td>
<td>$V_B = {a, e}$</td>
</tr>
<tr>
<td>C</td>
<td>$n_C = 1$</td>
<td>$V_C^0 = {d}$</td>
<td>$V_C = {a, b, f}$</td>
</tr>
<tr>
<td>D</td>
<td>$n_D = 2$</td>
<td>$V_D^0 = {e, f}$</td>
<td>$V_D = {d}$</td>
</tr>
</tbody>
</table>

**Table II. Information of the mooring areas**
Constraint (3) in the model. This kind of inter-type re-allocation can be included in a larger mooring area exchange cycle and thus allow more yachts to be moored at their ideal areas.

Another extension of the model is to take the cost of re-allocation into consideration. In this model, we simply ignore the cost for the yacht owners to re-allocate their yachts as the satisfaction achieved by ex-changing yacht mooring areas is in the long run, even for several decades, while the cost for re-allocation in a short-term can ignored compared with the long-term satisfaction. If we take the cost for exchange into consideration, including the application fee, the fuel cost for yacht moving, and some associate fees, we can add other parameters in the model to represent the costs to make the model better reflect the true situation in the real world.

4. Numerical examples
In this section, we first generate a small-scale example to demonstrate the application of the model, and then we test the performance of the proposed model on randomly generated large-scale instances. Model M1 is solved using CPLEX for all the instances. CPLEX is a mathematical programing software tool developed by IBM. It provides flexible, high-performance mathematical programing solvers for linear programing, mixed integer programing, quadratic programing and quadratically constrained programing problems. It enables the researchers to find the optimal solution from among billions of alternatives for their models (CPLEX, 2009). A desktop computer (Intel Core i7, 3.40 GHz; Memory, 8G) is used to conduct these experiments with the programing language C++ (Visual Studio 2010).

4.1 A small-scale example
In the small-scale example, we assume that there are eight mooring areas (with ID from 1 to 8) that are fully occupied and thirty yachts of medium type (with ID from 1 to 30) within these areas need to be re-allocated. The initial mooring areas of the yachts are listed in Table III. Assume that the number of ideal mooring areas of one yacht is no more than two. The ideal mooring areas of the yachts are shown in Table IV.

After using the proposed model to describe the problem and CPLEX to solve the model, the results show that the optimal solution allows 23 yachts, which account for 76.67 per cent of the total, to be moved to one of their ideal mooring areas. A specific re-assigning scheme is provided in Table V.

From Table V, we can see that except for yacht whose ID numbers are 6, 7, 9, 11, 12, 15 and 19 that remain unchanged, all the other yachts have been removed to their ideal mooring areas.

<table>
<thead>
<tr>
<th>Yacht ID</th>
<th>Initial area ID</th>
<th>Yacht ID</th>
<th>Initial area ID</th>
<th>Yacht ID</th>
<th>Initial area ID</th>
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<td>5</td>
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</tr>
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</table>

Table III. Initial mooring areas of the yachts
4.2 Computational results for large-scale experiments

In this section, we generate large-scale instances to test the efficiency of the proposed model. The problem sizes of the instances are between 100 yachts in 10 mooring areas and 30,000 yachts in 400 mooring areas. In each instance, the initial mooring area of a yacht is randomly generated. Each yacht can have one ideal mooring area that is different from its original mooring area, and this ideal mooring area is also randomly chosen. For each combination of number of yachts and number of mooring areas, we randomly generate five instances. The results are presented in Table VI, including the average number of re-allocated yachts and the average processing time over the five instances.

Table VI shows that although a yacht has only one ideal mooring area, the model can provide an optimal solution that allows a significant number of yachts to be re-allocated to their ideal mooring areas. As the scale of the problems increases, the model can be solved in an acceptable time as it only takes 29.346 s to allow 28,126 yachts among all the 30,000 yachts mooring in 400 mooring areas to be re-allocated to their ideal mooring areas. Among all the re-allocation schemes, the best situation is that all the yachts can be re-allocated, and the worst situation is that no yacht can be moved to their ideal mooring areas. Moreover, the proportion of yachts that can be re-allocated to their ideal mooring areas increases in the problem size. This means the proposed model

<table>
<thead>
<tr>
<th>Yacht ID</th>
<th>Ideal area ID*</th>
<th>Yacht ID</th>
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Table IV.
Ideal mooring areas of the yachts

Note: *Different ideal mooring areas of one yacht are separated by a comma

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Table V.
Optimal re-allocation scheme for yachts

Note: *\ Denotes the mooring area of the yacht remains unchanged
is more effective for large coastal cities with a strong culture of yachting. Table VI further shows that instances up to the scale of 30,000 yachts in 400 mooring areas can be solved within computation time for 30 s. Therefore, the proposed model can be used to solve practical-size problems efficiently.

5. Conclusions
This paper has conducted a quantitative analysis of a problem about re-allocating yacht mooring areas. We formulated a 0-1 integer programing model for the proposed problem, which allows the maximum number of yachts to be moored at their ideal mooring areas while taking yachts owners’ preference for some mooring areas into consideration. We generated a small-scale numerical example with thirty medium type yachts in eight mooring areas to demonstrate the application of the model. The result showed that the optimal solution allows 23 yachts (76.67 per cent) to be re-assigned to their ideal mooring areas. Then, we conducted computational experiments and the computational results showed that random examples with up to 30,000 yachts and 400 mooring areas can be solved by the proposed model efficiently using CPLEX.

This paper proposed one of the first few models consisting of quantitative analysis for yacht management. The model can help the government to better manage yachts and mooring areas, bring convenience to yacht owners, help reduce their traveling time and distance and thereby reduce urban road travel congestion. For future studies, we consider taking all types of yachts into consideration simultaneously while redesigning mooring areas to allow more yachts to be moored at their ideal mooring areas.

References


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