The implication of the stochastic gross-profit-per-day objective on the cargo ship profitability, capacity, and speed

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Abstract

Purpose – This paper aims to study the implication of the stochastic gross-profit-per-day objective on the ship profitability and the ship capacity and speed.


Findings – The paper finds that if the ship owner follows the rate concept and the cargo demand forecast, he can improve the profitability of his company and be able to select the proper capacities and speeds for the ships used.

Research limitations/implications – The findings are not only useful for the shipping or other cargo transport companies but also for businesses like gas reservoir development, car assembly lines in the industry, cooperative farming and crop harvesting in agriculture, port cargo handling in trade and road paving in construction.

Originality/value – The contribution of this paper lies in notifying the ship owners of the possible profitability improvement and the consequences of building ships of larger capacities and slower speeds.

Keywords Stochastic optimization, Gross profit-per-day objective, Ship capacity, Ship profitability, Ship speed

Paper type Research paper

Introduction

This paper opens the discussion widely about applying the concept of the stochastic gross-profit-per-day objective to all systems which have a time-variable operational cycle (El Noshokaty, 2018a, 2019) or systems which can employ different mixes of factors of production (El Noshokaty, 2018b, 2019). The former systems are by definition spend different times in each operational cycle while the latter systems spend different times while using each mix of factors of production. Either type of systems may be found in single, decoupled and coupled entities. Examples are found in all means of cargo transport, oil and gas reservoir development, car assembly lines in the industry, cooperative farming and crop harvesting in agriculture, port cargo handling in trade and road paving in construction.

Applying the concept of the stochastic gross-profit-per-day objective to these systems seems to influence the profitability and investments worth trillions of dollars.

The gross-profit-per-day objective is one form of the rate concept which is described thoroughly in the third section of this paper. In shipping, the gross-profit-per-day objective implies the fact that the ship voyage is variable in time, the case which occurs when the ship voyage is...
is following the tramp mode of operation, or, when the ship capacity and speed are strategically subject to selection either in the tramp or liner mode of operation. El Noshokaty, 2017a, 2017b has developed the mathematical models to optimize a stochastic gross profit-per-day objective for the time-variable ship voyage. It also applies sensitivity and what-if analysis if there are any possible changes that might happen to cargo quantity and freight, cargo handling rate and charges, and ship speed and fuel consumption. The same analysis is used when the stochastic cargo transport demand can’t be anticipated.

The above-mentioned research papers of El Noshokaty call for an important extension which the Author is trying to address in this paper. El Noshokaty demonstrates the case where it is a worse situation for the profitability if the rate concept is not followed. Regrettably, management of the cargo ship transport operations is used to maximize a voyage gross-profit objective, rather than a gross-profit-per-day objective, assuming a deterministic cargo transport demand. What is worse for the profitability is that the management has started to apply what is called in Economics as the ‘economies of scale’. These economies of scale act in time-variable operational cycles against the rate indicators the way described in the third section of this paper. World ship owners and governmental bodies are not aware of this situation. The Organization for Economic Co-operation and Development (OECD) is an international organization established as a strategic think tank with the objective of helping shape the transport policy agenda on a global level and ensuring that it contributes to economic growth, environmental protection, social inclusion and the preservation of human life and well-being. Fifty-six countries have attended the International Transport Forum at the OECD in 2015 to discuss the economies of mega-ships (OECD, 2015). The findings are:

- cost savings from bigger container ships are decreasing;
- the transport costs due to larger ships could be substantial;
- supply chain risks related to mega-container ships are rising;
- public policies need to better take account of this and act accordingly; and
- further increase of maximum container ship size would raise transport costs.

Nothing of the above-mentioned findings has to do with the efficiency of the mega-ships and whether these ships are following the efficiency rates as described in the third section.

The paper uses the same mathematical model and the solution methodology developed by El Noshokaty, 2017a, 2017b. A case study is given to demonstrate a possible magnitude of the implication of the stochastic gross-profit-per-day objective in the tramp shipping. The case study is not intended to establish a governing rule towards the building of ships of a certain capacity or a certain speed. It just demonstrates a possible case where the building of a large ship capacity with a lower speed is not profitable despite the economies of scale the ship owner thought he would gain.

The following section brings about a review of the literature on the possible effects on the ship profitability, capacity, and speed, and whether there is any research work on the possible effects caused by the stochastic gross-profit-per-day objective. The third section describes the implication of the stochastic gross-profit-per-day objective on the ship capacity and speed. The one next introduces the mathematical model and the solution methodology used to study the implication of the gross-profit-per-day objective on the ship profitability, capacity, and speed. The following section demonstrates a case study in the tramp shipping to find out if there is such implication and to size its magnitude if any. The last section gives a concluding statement of the paper.
Review of the literature

To review the possible effects of the gross profit and the gross profit-per-day objectives on the ship profitability, the reader is to refer to El Noshokaty, 2013, 2017b for the liner shipping and El Noshokaty, 2014, 2017a, 2017b, 2018c for the tramp shipping. The former papers recommend the optimization of a stochastic gross profit objective while the latter papers recommend the optimization of a stochastic gross profit-per-day objective, rather than a deterministic gross profit one. Both groups of references also recommend the use of sensitivity and what-if analysis if the stochastic cargo transport demand can’t be anticipated.

From the optimization of the ship voyage perspective, a general review is given by Christiansen et al. (2004), Christiansen et al. (2013) and Christiansen and Fagerholt (2014). The problem of these research papers is to assign an optimal loading sequence of cargoes to each ship during a given time. Each cargo has a loading time window, size, type, port of loading, port of discharging and cargo handling time in these ports. Each ship has its operational characteristics of the initial position, and expected daily marginal revenue of optional cargoes which may become available during the planning period. All contracted cargoes must be loaded, whereas optional cargoes may be accepted or rejected. A ship may carry only one cargo at a time. The objective is to maximize the revenue of optional cargoes minus cargo handling and fuel cost.

Fagerholt (2001) has developed an optimization model for tramp shipping, where cargo time window (lay can) may be violated to a certain extent with a penalty cost in return. That is why cargo time window was given the name soft time window, and penalty cost was given the name inconvenience cost. The model designs a predetermined set of schedules for each ship to follow. In each schedule, there is a predetermined route with cargo pick-up and delivery nodes along with soft time window for each node and a predetermined ship speed on each sailing leg. The model objective is to find the schedule for each ship which minimizes the total operating and penalty cost. Fagerholt (2004) has also developed a computer-based decision support system for fleet scheduling based on heuristic algorithms. Fagerholt et al. (2010) have presented a decision support methodology for strategic planning in tramp and industrial shipping. The proposed methodology combines simulation and optimization, where a Monte Carlo simulation framework is built around an optimization-based decision support system for short-term routing and scheduling. Although these research papers have developed algorithms which are flexible, allow interactive user interface, and save time, their exact optimal solution is not guaranteed.

Lin and Liu (2011) have considered the ship routing problem of tramp shipping and proposed a combined mathematical model that simultaneously takes into account the ship allocation, freight assignment and ship routing problems. To solve this problem, they have developed an innovative genetic algorithm.

Laake and Zhang (2013) have developed a model to determine the best mix of long-term and spot cargo contracts for a given fleet. The model finds the optimal fleet size and a mix for a set of cargo contracts or a mix of both. The model assumes that transport demand is sufficiently large on each route. Each ship takes full loads and does not mix cargoes from different cargo contracts, which is a standard practice in the coal/iron ore trade.

It was found that the OR model of Osman et al. (1993) and Christiansen et al. (2007) holds characteristics close to the tramp shipping characteristics. The model of either research paper is based on a network of multiple cargo flows. Each network node either represents a load or a discharge event for each cargo. Ships compete in carrying cargoes by following selected arcs in the network, beginning with a start node and ending with an end node. If a network arc is used by a ship, this arc is restricted for use by other ships. An arc is used by a
ship if lay can of each arc node can be met and load available in each arc node is within remaining ship capacity. The model assigns network arcs to ships in an attempt to maximize total voyage gross profit for all ships. Both models are nonlinear. Hemmati et al. (2014) have presented better tramp shipping characteristics. They have used a linear objective but used heuristic algorithms to solve their problem. Laake and Zhang (2013), Vilhelmsen et al. (2015) have developed a linear model to handle the case where multiple cargoes can be carried simultaneously on board each ship.

Bakkehaug et al. (2016) and Vilhelmsen et al. (2017) have developed a model to schedule the voyages of a fleet of ships considering a minimum time spread between some voyages. The former has used the adaptive large neighborhood search (ALNS) heuristic to solve the problem, while the latter has used a decomposition approach with dynamic programming algorithm for column generation. Their model focuses on the time spread between voyages in response to a charter party clause which requires the voyages to be ‘fairly evenly spread’. This requires the voyage to become the model decision variable with a predetermined route and full-load cargo to be transported in each voyage.

As for the possible effects on the ship capacity and speed from the engineering design perspective, there are only several research papers. Examples are given by Papanikolaou, (2009), Michalski, (2016); and Szelangiewicz and Zelazny, (2016).

As for the possible effects on the ship capacity and speed from the economics perspective, Jansson and Shneerson (1982) have determined the optimal ship size by minimizing the costs per ton at sea and in port. While costs per ton at sea decreases with size, it is argued that costs per ton in port increases with size, particularly the cost per ship’s time. Cuncev (1984) has computed the speed which minimizes fuel consumption and maximizes revenue as great as possible. By Ozen and Guler, (2002), the optimum ship capacity is determined by minimizing the transportation cost objective function. This function depends on the ship capacity variable and has the following parameters: the total amount of cargo transported in unit time between two ports; the distance between two ports; accumulating time; loading and unloading time per ship; investment cost; holding cost per item per unit of time. Psaraftis and Kontovas, (2014) have incorporated those fundamental parameters and other considerations that weigh the most in a ship owner’s or charterer’s speed decision at the operational level. These are the fuel price, the state of the market (freight rate), the inventory cost of the cargo and the dependency of fuel consumption on the payload.

As for the economics of the mega-ships, Imai et al. (2006) analyze the container mega-ship viability by considering competitive circumstances. For the Asia–Europe trade, the mega-ship is found to be competitive in all scenarios, while it is viable for the Asia–North America only when the freight rate and feeder costs are low. The OECD forum, 2015 has raised some concerns about cost savings, transport cost, supply chain risks and public policy as reported earlier. Kimp (2015) has given his comments on fuel cost, slower steaming, and what he called unintended costs. The Global Shippers (GS) forum, 2016 has discussed issues related to inconvenience to shippers, higher barriers for ship owners to entry, and the vertical integration between the shippers and the shipping companies. Gcaptain, 2016 has made a study where it found that the economies of scale may be running out as vessel size increases up to and beyond 18,000 TEU. Helmy and Shrabia (2016) have stated the disadvantages related to the port problems, the imbalance of trade and uncertainty of the global economy, and the larger risks and environmental impact. Malchow (2017) showed that further effects of lower slot costs lessen while ships get larger. Hence, a further increase in ship sizes would not significantly reduce transport costs anymore.
No research papers, other than that of El Noshokaty, were found which study the implication of the gross profit-per-day objective as such on the ship profitability, capacity, and speed combined. This includes both liner and tramp shipping. The reader may wonder why a paper may study the implication of the gross profit-per-day objective for liner shipping, where the ship voyage is not variable in time. The reason is that it does use the gross profit-per-day objective when the ship capacity and speed are subject to change the way which causes the liner voyage time to change as well. The absence of the research papers in this concern is mainly due to the unawareness of the rate concept on one hand and on the other hand, the complications of the gross-profit-per-day models as being NP-hard and difficult to solve.

The implication of the stochastic gross-profit-per-day objective on the ship capacity and speed

The word ‘rate’ is used to proportionate one measurement to another. It sometimes uses the word ‘per’ between the two measurements, and it sometimes uses a fraction or a ratio; with a nominator dedicated for one measurement and a denominator for the other. The rates that have a non-time denominator include exchange rates, literacy rates and electric fields (in volts/meter). The rates that have a time denominator include heartbeat rate, speed and flux. In business, the term ‘rate of output’ is highly significant. It is used in capacity planning to specify the upper limit of the output expressed in product quantity per the production time; say per day or hour. In this case, the capacity will turn to a fraction, where its nominator is the output and its denominator is the time. Maintaining such a rate will lead to a maximum capacity per year (Stevenson, 1999). The same concept applies to other plans. The planning of the gross profit should be related to the time taken to generate such profit. Maintaining such profit will ensure maximum profit per year. To explain, assume for short that there are two cargoes and one must choose only one: ‘cargo A’ which yields a gross profit per voyage equals $2m in 200 days ($10,000 per day), and ‘cargo B’ which yields a gross profit per voyage equals $1.5m in 100 days ($15,000 per day). Although ‘cargo B’ generates less gross profit per voyage, it causes the ship owner to get $3m in 200 days instead of $2m. To conclude, according to the rate concept of the operational management, the time-variable operational cycle should maintain a maximum possible rate of output and should target a maximum possible gross profit per the operational cycle time. Unfortunately, this concept is not always followed where the operational management in some cases is still targeting the maximum possible amount of output and the maximum possible gross profit, regardless of how long the operational cycle might take to complete. These cases may be found more in businesses where future customer demand is poor and unknown. In such a situation, the operational management prefers to produce and sell now the maximum quantity the operational system can afford. The situation becomes worse when the customer demand does not amen to any forecasting pattern. When this situation persists, targeting the maximum possible gross profit becomes a normal practice. Among other industries, the shipping industry is experiencing such a situation. What is wondering is that Operations Research, which is now being ignored by most practitioners, can resolve most of these situations. It can build a stochastic model based on stochastic customer demand. The model objective is to maximize the stochastic gross-profit-per-cycle-time subject to the constraints put on the operational capacity and the stochastic customer demand. Based on the above-mentioned example of choosing ‘cargo A’ or ‘cargo B’, choosing ‘cargo B’ would be emphasized as being the right decision if the stochastic cargo demand shows probabilities supporting the availability of B-like cargoes. Even in situations where the stochastic
customer demand can’t be anticipated, sensitivity and what-if analysis can introduce different decision scenarios based on future customer demand.

In the production system, on the other hand, the economies of scale are cost advantages reaped by production systems when production becomes efficient. Systems can achieve economies of scale by increasing production and lowering costs. This happens because costs are spread over a larger number of products. Costs can be both fixed and variable. The optimal production quantity may be achieved in the short run by maintaining the right proportions among the factors of production employed in the production system, and in the long run by employing new production technologies and more skilled labor force. In fact, the average production cost which signals how much economies of scale are gained is also another form of the rate concept. The nominator is the total production cost and the denominator is the production quantity.

This paper has a contributing point in finding the possible impacts of the time-dependent rates over the resources causing the economies of scale. This is a research point which has never been discussed before. The paper is discussing the implication of the time-dependent rate, expressed by the stochastic gross-profit-per-day objective, on the ship capacity and speed which causes the economies of scale to collect. To explain, according to the economies of scale, the gross profit objective is assumed to yield a grosser profit if the ship transport average cost is minimized, which may be achieved by building ships of larger capacities. As a result, the voyage time of the ship tends to prolong because of the more cargo handling operations of larger loads, longer distance due to route restriction or more port calls, possible lightening of loads before passing canals, possible transshipment operations, and most likely a slower ship speed. From the time-independent rate perspective, this is a good situation where the average cost per unit load will decrease. But according to the time-dependent rate, this is a bad situation where the voyage time should not extend too long otherwise the gross profit-per-day will decrease and the larger capacity and the slower speed will then act against profitability. If the larger capacity from the time-dependent rate perspective is tending to be less profitable in carrying large loads, it will definitely be costly enough in carrying small loads due to the excessive port and canal dues.

To conclude the contributing point from the Economics perspective, the word ‘efficient’, which is used to describe the production systems having the economies of scale, should be interpreted as adopting the right proportions not only between the factors of production but also between the factors of production and the time. In shipping, the right proportions with respect to time are ignored, the way that permits the building of economically unjustifiable ships of large capacities and lower speeds. To conclude it from the Mathematics perspective, let q denote the production quantity, c denote its production cost, p denote its gross profit, and t denote its production time. The economies of scale are reaped by increasing q and decreasing c, which causes p to increase as well. The production system is described as efficient if the economies of scale is able not only to decrease the rate $z_1 = c/q$ to its possible minimum level but also to increase the rate $z_2 = p/t$ to its possible maximum level. In fact, the rate $z_2$ overrides the rate $z_1$ since it includes revenue in addition to cost. The economies of scale are reaped at the maximum possible level of $z_2$. In shipping, the rate $z_2$ is ignored which permits the building of sluggish and profitless ships of large capacities and lower speeds. The rate $z_2$ addressed in this paper is the stochastic gross-profit-per-day where it is required to find its implication on the cargo ship profitability, capacity, and speed. It is rather simple; as the daily profit of $100/10$ days = $10$ outperforms the profit of $1000$, which is far greater than the $100$, IF the latter produces a corresponding daily profit less than the former such as $1000/200$ days = $5$. The shipping economies of scale that may be included in the $1000$ are mostly accompanied with slower operations expressed by the
200 days. So, the ship owner should not be happy with $1000 profit collected by a larger ship capacity and mega-ships, he should also think of the time taken to collect this amount.

As described in the next section, the rate concept is expressed in the model objective by a stochastic gross-profit-per-day function and as a result, the case-study section shows that the economies of scale are not reaped by increasing the ship capacity as believed. The paper recommends taking the stochastic gross-profit-per-day as an alternative objective in shipping rather than the traditional total-gross-profit oriented operation.

The mathematical model and the solution methodology

The mathematical model and the solution methodology for the tramp shipping are included in El Noshokaty, 2017a, 2017b; and SOS, 2019; and for the liner shipping in El Noshokaty, 2013 and SOS, 2019. Shipping Optimization Systems (SOS) is a suite of decision support systems which comprises a database management system incorporated with the aforementioned mathematical models. It is developed to support the ship owner optimizing the cargo mix selection of each ship voyage (SOS Voyager), the optimal allocation of ships’ voyages to lines and trade areas (SOS Allocator), and the appraisal of new ships to be built, purchased, or chartered (SOS Appraiser).

SOS can here be applied to a set of liner ships of different classes of capacities and speeds and to a set of designs of the same shipping line. Each design is tailored to suit each class of ships and a certain cargo transport demand, and describing the line ports and the port arrival dates. The ship which gives the maximum gross-profit-per-day is proposed to work on the shipping line.

Likewise, SOS can here be applied to a set of tramp ships of different classes of capacities and speeds and to a certain cargo transport demand available in a certain trade area. Cargo pick-up dates and other shipping elements and rules are assigned to all cargoes. The ships which give the maximum total gross profit-per-day are proposed to work on this trade area.

The ship-speed sensitivity and what-if analysis discussed in the above-mentioned references can be used here to study whether or not increasing the ship speed will improve the profitability. In applying these references, differentiation is made to whether the ship to be employed is to be chartered-in for a short period or purchased/built. In the latter case, a comparison between the profitability of the ships should be calculated for the ship lifetime in a present-value formulation.

As the following case study is using SOS Voyager for the tramp shipping, its mathematical model and the solution methodology are listed in Appendix A (adapted with permission).

A case study in tramp shipping

This case, which is an extension to the case given by El Noshokaty (2017a) demonstrates the situation where using a gross-profit-per-day objective with a stochastic transport demand is considerably more profitable than using a gross profit objective with deterministic transport demand. It also demonstrates that the former objective favors the ships of smaller capacities and higher speeds. It applies the model and methodology included in SOS (2019) and shown in Appendix A. A shipping company is planning to charter-in one oil tanker as to compete in carrying part of the cargo transport demand. Tanker I, Tanker II, and Tanker III are three proposed types of oil tankers of different capacities and speeds. In the last quarter of the year 2019, these tanker types can compete in carrying ten crude oil cargoes. Three of these cargoes are to be transported from Kuwait to the USA, another three from Ukraine to China, and four from Venezuela to Latvia. Data on tankers, ports and cargoes can be extracted and displayed using SOS (2019). Relevant data on ships are shown in Table I. For Tanker type I
and Tanker type III, their starting port is Alexandria, Egypt. For Tanker type II, its open port is Odessa, Ukraine. For all tankers, the close port is the last port of call, where the open date is 1/10 (dd/mm is the date format), the close date is 31/12, the voyage fixed cost is $1000, and the fixed time is 0.3 days. Relevant data on the port are shown in Table II. Ten crude oil cargoes represent the transport demand, of which eight cargoes have offered (confirmed) quantity and freight and two are not-yet-offered cargoes (unconfirmed). Relevant data on cargoes are shown in Table III. For the two unconfirmed cargoes, the company anticipates probabilities for five classes of quantity and freight for each cargo. The company also stipulates, by a least probability, to be able to transport a quantity of each cargo within its transport demand. Additional data on unconfirmed cargo are shown in Table IV. The company needs to know what type of tanker is most profitable. As the company is considering the use of a gross profit-per-day objective when selecting the optimal (best) cargo mix, it needs to know whether this new objective influences the tanker-type selection expressed in tanker capacity and speed, compared to the old gross profit objective. Also, it

<table>
<thead>
<tr>
<th>Data item*</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deadweight in mt</td>
<td>40,000</td>
<td>50,000</td>
<td>170,000</td>
<td></td>
</tr>
<tr>
<td>Low (economic), medium, and high speed in knots</td>
<td>15; 17; 19</td>
<td>14; 16; 18</td>
<td>13; 15; 17</td>
<td></td>
</tr>
<tr>
<td>Main engine laden fuel consumption in mt/day, each speed level</td>
<td>37; 54; 75</td>
<td>35; 52; 74</td>
<td>62; 96; 139</td>
<td></td>
</tr>
<tr>
<td>Main engine ballast fuel consumption in mt/day, each speed level</td>
<td>22; 32; 44</td>
<td>22; 33; 47</td>
<td>39; 60; 88</td>
<td></td>
</tr>
<tr>
<td>Auxiliary engine fuel consumption in mt/day</td>
<td>0.5</td>
<td>0.6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Heating fuel consumption in mt of main engine fuel/day/100 mt of cargo</td>
<td></td>
<td>0.11</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Sues Canal dues, laden and ballast in US$</td>
<td>158,960; 135,180</td>
<td>172,310; 146,560</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td>Panama Canal dues, laden and ballast in US$</td>
<td>79,000; 62,900</td>
<td>98,250; 78,150</td>
<td>na</td>
<td></td>
</tr>
<tr>
<td>Bosporus and Dardanelles dues in US$</td>
<td>9,640</td>
<td>12,150</td>
<td>21,673</td>
<td></td>
</tr>
<tr>
<td>Running cost in US$/day</td>
<td>5,000</td>
<td>7,000</td>
<td>7,700</td>
<td></td>
</tr>
</tbody>
</table>

* mt = metric ton. Fuel cost for main engine is 450 US$/mt. Fuel cost for auxiliary engine is 675 US$/mt ** na = not applicable since the tanker cannot pass the Suez Canal and the Panama Canal

<table>
<thead>
<tr>
<th>Port name</th>
<th>Data item</th>
<th>Cost/call in US$ (Lights, towage)</th>
<th>Cost/day in US$ (Quay services)</th>
<th>Waiting days (Anchor, idle)*</th>
<th>Cargo handling mt/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alexandria (Egypt)</td>
<td>1,500</td>
<td>150</td>
<td>0</td>
<td>34,000</td>
<td></td>
</tr>
<tr>
<td>Baltimore</td>
<td>12,000</td>
<td>1,200</td>
<td>0.3</td>
<td>40,000</td>
<td></td>
</tr>
<tr>
<td>Shuaiba (Kuwait)</td>
<td>8,000</td>
<td>800</td>
<td>0.5</td>
<td>37,000</td>
<td></td>
</tr>
<tr>
<td>Maracaibo</td>
<td>10,700</td>
<td>1,070</td>
<td>0.5</td>
<td>37,000</td>
<td></td>
</tr>
<tr>
<td>Odessa</td>
<td>10,000</td>
<td>1,000</td>
<td>0.5</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>Riga (Latvia)</td>
<td>11,000</td>
<td>1,100</td>
<td>0.3</td>
<td>35,000</td>
<td></td>
</tr>
<tr>
<td>Shanghai</td>
<td>9,000</td>
<td>900</td>
<td>0.4</td>
<td>35,000</td>
<td></td>
</tr>
</tbody>
</table>

* Port waiting days are classified as ‘force majeure’ and hence are not part of any demurrage or dispatch time counts
needs to know whether considering the unconfirmed cargoes has an additional impact on the selection of the tanker type.

In the beginning, SOS Voyager optimization model is used to find the optimal (best) cargo mix for each tanker type, where data in Table IV are turned to deterministic-equivalent quantities as shown in Table V (see Appendix A for details).

Applying the stochastic gross profit-per-day model to the tanker types gives the result reported in Table VI. The table displays the cargo mix, route, and the stochastic gross-profit-per-day classified by tanker type and speed level.

To hold a side-by-side comparison between the stochastic gross-profit–per-day model and the stochastic gross-profit, suppose that the stochastic gross-profit-per-day criterion is discarded and the stochastic gross profit criterion is used instead (which can also be handled by SOS Voyager). Table VII displays the results of this case.

On the other hand, to hold a side-by-side comparison between the stochastic gross-profit model and the non-stochastic (deterministic) gross-profit, suppose that the unconfirmed cargoes: ‘Crude Oil 8’ and ‘Crude Oil 10’ are discarded, the stochastic gross profit criterion is also discarded, and the gross profit criterion is used instead. The comparison is made to show the effect on the stochastic gross-profit model when the two cargoes, which happen to be not-yet-offered cargoes, are ignored, i.e. when the model is becoming deterministic gross-profit as what the ship owners are now following. The ship owners are targeting the maximum gross profit without taking into account any not-yet-offered cargoes even when probability of transporting these cargoes is quite high. Table VIII displays the results of this case, assuming all the tanker types are at low speed.

Table VIII is broken down into the voyage details displayed in Table IX. The following is some analysis based on the findings displayed in Tables VI to IX:

From the profitability perspective, Table VI shows a stochastic gross profit-per-day for Tanker type III at all speed levels greater than that given by Table VII (percentage increase is 8 per cent, 21 per cent, and 12 per cent for the low, medium, and high speed, respectively.)
Table VI is produced by a model of stochastic gross profit-per-day objective, while Table VII is produced by a model of stochastic gross profit objective. Table VII also displays a stochastic gross profit-per-day equivalent to the stochastic gross profit. Likewise, Table VI shows a stochastic gross profit-per-day for all tanker types at the low speed greater than that given by Table IX (percentage increase is 26 per cent.) Table IX gives a gross profit-per-day equivalent to the gross profits shown in Table VIII. Table VIII is produced by a model of a gross profit objective, where the gross profit is not proportionate to voyage time and no stochastic cargo transport demand is considered. It is here to highlight that ignoring Crude Oil 10 now, despite of its high probability of being offered within its lay-can dates, has a negative impact on the current and future schedules if less profitable cargoes were taken instead.

From the ship design perspective, namely, the ship capacity and speed, Table VI shows an inefficient use of the larger capacity and the slower speed. Tanker type III of 170,000 dwt is carrying Crude Oil 2 and 8 totaling 111,000 tons ignoring Crude Oil 9 of 170,000 tons because it causes inefficient utilization of the tanker, time-wise, as it cuts a long distance from Odessa in Ukraine to Shanghai in China via Cape of Good Hope. Whereas Table VII
shows full utilization of the larger capacity, capacity-wise, where Tanker type III picks Crude Oil 9 apart from how long it takes the ship to reach Shanghai. Careful analysis of Table VI shows an increase in the stochastic gross profit-per-day for all tanker types of about 22 per cent due to the increase in tanker speed from an average 14 knots to an average of 16, while the decrease is about 6 per cent due to the increase in tanker speed from an average 16 knots to an average 18. Another useful observation is that the decrease in the stochastic gross profit-per-day for all ship speeds is about 95 per cent due the chartering-in of Tanker type II compared to Tanker type I, while the increase is about 196 per cent due the chartering-in of Tanker type III compared to Tanker type I. For the cargo transport demand shown in Tables III to V, the above-mentioned analysis suggests the chartering-in of tanker Type III of an average capacity of about 120,000 dwt at an average speed around 16 knots.

From the economies of scale point of view, the shipping company may believe it can collect more economies by choosing a capacity of 170,000 dwt for Tanker type III to transport Crude Oil 9 at low speed and as a result it will collect $2,045,783 stochastic gross-profit, as displayed by Table VII. Advisably, profitability wise, the shipping company should consider a better alternative decision by choosing a capacity of 120,000 dwt at 16 knots for Tanker type III, as suggested by the above-mentioned analysis, to transport Crude Oil 2, 5, 6, 7 and 8. The stochastic gross profit of this alternative decision is $3,653,855. What is more important is that the stochastic gross-profit-per-day of this alternative decision is $43,292 while it is only $23,169 for the 170,000 dwt at 13 knots speed, which means that the company can surprisingly increase profitability by 87 per cent, according to the data given in Tables I to V. This analysis demonstrates the fallacy of what the company believed and leads the analysis to the conclusion given by the next paragraph.

### Table VI.

<table>
<thead>
<tr>
<th>Cargo mix</th>
<th>Ship name and speed level</th>
<th>Total stochastic gross profit per day in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo mix</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>Low Stoc. GPD in $</td>
<td>10,239 ($845,758/82.6 days)</td>
<td>25,014 ($1,463,376/58.5 days)</td>
</tr>
<tr>
<td>Route</td>
<td>Maracibo - Riga</td>
<td>Shuaiba-Baltimore (directly)</td>
</tr>
<tr>
<td>Cargo mix</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>Medium Stoc. GPD in $</td>
<td>11,652 ($882,786/58.6 days)</td>
<td>34,002 ($2,900,360/85.3 days)</td>
</tr>
<tr>
<td>Route</td>
<td>Shuaiba-Baltimore (directly)-Maracibo-Riga</td>
<td>Shuaiba-Baltimore (directly)-Maracibo-Riga</td>
</tr>
<tr>
<td>Cargo mix</td>
<td>High</td>
<td></td>
</tr>
<tr>
<td>High Stoc. GPD in $</td>
<td>9,626 ($513,079/53.3 days)</td>
<td>34,284 ($2,876,450/83.9 days)</td>
</tr>
<tr>
<td>Route</td>
<td>Shuaiba-Baltimore (directly)-Maracibo-Riga</td>
<td>Shuaiba-Baltimore (directly)-Maracibo-Riga</td>
</tr>
</tbody>
</table>

**Note:** *Stoc. GPD = Stochastic gross-profit-per-day*
More insights into the overall tables reveal two key management concepts behind better profitability and the optimal ship capacity and speed. They are the **per-day rate** and the **demand forecast**. The former concept cares for both the voyage gross profit and time the way which leads to a maximum gross profit at the end of the year. The latter concept works in synchronization with the former. It looks ahead in the future to guarantee that the rate concept will not work against better profitability by picking a future poor demand on the account of the current rich one if any. Moreover, the economies of scale that ship owners believe are reaped by increasing the ship capacity despite its slow operations is completely

<table>
<thead>
<tr>
<th>Speed level/Data item*</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total stochastic gross profit in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Cargo mix</td>
<td>Crude oil 7 and 10</td>
<td>Crude Oil 6</td>
<td>Crude oil 9</td>
<td>3,058,893</td>
</tr>
<tr>
<td></td>
<td>Stoc. GP in $</td>
<td>845,758</td>
<td>167,352</td>
<td>2,045,783</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Route</td>
<td>Maracibo - Riga</td>
<td>Maracibo - Riga</td>
<td>Odessa-Shanghai (directly)</td>
<td></td>
</tr>
<tr>
<td>Medium</td>
<td>Cargo mix</td>
<td>Crude oil 1, 7, and 10</td>
<td>Crude oil 6</td>
<td>Crude oil 9</td>
<td>3,578,084</td>
</tr>
<tr>
<td></td>
<td>Stoc. GP in $</td>
<td>1,180,521</td>
<td>77,572</td>
<td>2,319,991</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Route</td>
<td>Shuaiba - Baltimore (directly)-Maracibo-Riga</td>
<td>Maracibo - Riga</td>
<td>Odessa-Shanghai (directly)</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>Cargo mix</td>
<td>Crude oil 1, 7, and 10</td>
<td>Crude oil 6</td>
<td>Crude oil 9</td>
<td>3,315,387</td>
</tr>
<tr>
<td></td>
<td>Stoc. GP in $</td>
<td>984,368</td>
<td>-40,535</td>
<td>2,371,554</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Route</td>
<td>Shuaiba - Baltimore (directly)-Maracibo-Riga</td>
<td>Maracibo - Riga</td>
<td>Odessa-Shanghai (directly)</td>
<td></td>
</tr>
<tr>
<td>Total stochastic gross profit in US$</td>
<td>3,010,647</td>
<td>204,389</td>
<td>6,737,328</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** * Stoc. GP = Stochastic gross-profit

<table>
<thead>
<tr>
<th>Data item</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total gross profit in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo mix</td>
<td>Crude oil 7</td>
<td>Crude Oil 6</td>
<td>Crude oil 9</td>
<td>2,565,682</td>
<td></td>
</tr>
<tr>
<td>Route</td>
<td>Maracibo-Riga</td>
<td>Maracibo- Riga</td>
<td>Odessa-Shanghai (directly)</td>
<td>2,565,682</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Voyage details</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross profit ($)</td>
<td>352,547</td>
<td>167,352</td>
<td>2,045,783</td>
<td>2,565,682</td>
<td></td>
</tr>
<tr>
<td>Days</td>
<td>81.7</td>
<td>71.9</td>
<td>88.3</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Gross profit/day ($)</td>
<td>4,315</td>
<td>2,328</td>
<td>23,169</td>
<td>29,812</td>
<td></td>
</tr>
</tbody>
</table>

**Table VII.** Cargo mix and route classified by ship name and speed level and total stochastic gross-profit

**Table VIII.** Cargo mix and route of each ship at low speed and total gross profit

**Table IX.** Voyage details reported by Table VIII for each ship at low speed
not true. The economies are reaped if they consider the stochastic gross-profit-per-day as an alternative objective to the total gross profit.

The above-mentioned analysis was developed to demonstrate the outperformance, profitability wise, of the stochastic gross-profit-per-day objective over the stochastic gross-profit and on the non-stochastic gross profit. The demonstration is based on different design factors, expressed by the ship capacity on one side and the speed and fuel consumption on the other side. The analysis can go further to demonstrate the outperformance under different levels of cargo transport demand, expressed by cargo quantity and freight rate, or, under different levels of cargo handling platform, expressed by cargo handling rate and charges. To give an example of the former type of analysis, suppose that the operation of the three tanker types is facing unrest in the middle-east where all cargoes shipped from port of Shuaiba, Kuwait are subject to some possible values of a war risk surcharge. One possible value is a 10 per cent increase in the freight rate and the other is 20 per cent. To account for these two values of the surcharge, Table X displays the results when applying the stochastic gross-profit-per-day objective, while Table XI displays the results when applying the stochastic gross-profit objective. Both tables assume the tanker types are at low ship speed.

From Tables X and XI, the freight analysis shows again the outperformance, profitability wise, of the stochastic gross-profit-per-day over the stochastic gross-profit where there are 5 and 46 per cent increase in profitability corresponding to the 10 and the 20 per cent surcharge percentage-increase for all cargoes transported from Shuaiba, Kuwait, respectively.

To give an example of the cargo-handling type of analysis, suppose that port of Maracaibo is planning to improve its cargo handling rate. There are two proposed plans; plan one suggests an increase of 5 per cent in the handling rate, while plan two suggests 10 per cent. To account for these two values of the cargo handling rate, Table XII displays the results when applying the stochastic gross-profit-per-day objective, while Table XIII displays the results when applying the stochastic gross-profit objective. Both tables assume the tanker types are running at low ship speed.

### Table X.
The Stochastic gross-profit-per-day classified by tanker type and surcharge percentage increase

<table>
<thead>
<tr>
<th>Surcharge percentage-increase</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total Stochastic gross-profit-per-day in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>10,239 (845,758/82.6 days)</td>
<td>2,328 (167,352/71.9 days)</td>
<td>24,246 (1,418,376/58.5 days)</td>
<td>36,813</td>
</tr>
<tr>
<td>20%</td>
<td>10,239 (845,758/82.6 days)</td>
<td>2,328 (167,352/71.9 days)</td>
<td>33,733 (1,973,376/58.5 days)</td>
<td>46,300</td>
</tr>
</tbody>
</table>

### Table XI.
The Stochastic gross-profit classified by tanker type and surcharge percentage increase

<table>
<thead>
<tr>
<th>Surcharge percentage-increase</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total Stochastic gross profit in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>845,758</td>
<td>167,352</td>
<td>2,045,783</td>
<td>3,058,893</td>
</tr>
<tr>
<td>($10,239*82.6 days)</td>
<td>($2,328*71.9 days)</td>
<td>($23,169*88.3 days)</td>
<td>3,058,893</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>845,758</td>
<td>167,352</td>
<td>2,045,783</td>
<td>3,058,893</td>
</tr>
<tr>
<td>($10,239*82.6 days)</td>
<td>($2,328*71.9 days)</td>
<td>($23,169*88.3 days)</td>
<td>3,058,893</td>
<td></td>
</tr>
</tbody>
</table>
From Tables XII and XIII, the cargo handling analysis shows for the third time the outperformance, profitability wise, of the stochastic gross-profit-per-day over the stochastic gross-profit where there is 8 per cent increase in profitability corresponding to the 5 and the 10 per cent percentage-increase in the cargo handling rate in Maracaibo.

To conclude what the case study is all about, the following statement will put everything into perspective. If the ship owner believes he can run the 170,000-dwt ship at 13 knots and reap the economies of scale leading to $2,045,783 gross profit in 88.3 days, then it is advisable, profitability wise, to run it adopting the rate concept and collect $1,463,376 in 58.5 days since the latter decision enables him getting $25,014 stochastic gross-profit-per-day rather than $23,169 equivalent gross-profit-per-day (8 per cent increase), or, replace it with a 120,000-dwt ship at 16 knots and collect $3,653,855 with $43,292 stochastic gross-profit-per-day (87 per cent increase).

**Concluding statement**

This paper takes the lead in revising the current practices of the management of cargo ship transport operations. The management is used to maximize a voyage gross-profit objective, assuming a deterministic cargo transport demand. According to the economies of scale, this objective may yield a grosser profit if the ship average fixed cost is minimized, which may be achieved by building ships of larger capacities. As a result, the voyage time of the ship tends to prolong due to more cargo handling operations of large loads, longer distance due to route restriction or more port calls, possible lightening of loads before passing canals, possible transshipment operations, and a most likely slower ship speed. Latest research papers recommend a voyage stochastic gross-profit-per-day objective to be used instead, assuming both deterministic and stochastic cargo transport demand. This new objective cares not only for the more voyage gross profit the ship is expected to earn but also for the fewer number of days the ship is expected to take to earn this gross profit. The voyage

<table>
<thead>
<tr>
<th>Handling charges percentage-increase</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total Stochastic gross-profit-per-day in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td></td>
<td>10,267 ($847,041/82.5 days)</td>
<td>2,334 ($167,827/71.9 days)</td>
<td>25,015 ($1,463,376/58.5 days)</td>
<td>37,616</td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>10,294 ($848,209/82.4 days)</td>
<td>2,340 ($168,260/71.9 days)</td>
<td>25,015 ($1,463,376/58.5 days)</td>
<td>37,649</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Handling charges percentage-increase</th>
<th>Ship</th>
<th>Tanker type I</th>
<th>Tanker type II</th>
<th>Tanker type III</th>
<th>Total Stochastic gross-profit in US$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td></td>
<td>847,041</td>
<td>167,827</td>
<td>2,045,783</td>
<td>3,060,651</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($10,267*82.5 days)</td>
<td>($2,334*71.9 days)</td>
<td>($23,169*88.3 days)</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td></td>
<td>848,209</td>
<td>168,260</td>
<td>2,045,783</td>
<td>3,062,252</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($10,294*82.4 days)</td>
<td>($2,340*71.9 days)</td>
<td>($23,169*88.3 days)</td>
<td></td>
</tr>
</tbody>
</table>
gross-profit-per-day objective permits the ship owner to maximize the yearly gross profit by repeating an expected higher daily voyage gross profit more number of times the year around. And, because the shipping management is not always sure whether the same magnitude of the gross profit can be maintained in the future of such ship voyages, maximization of the gross-profit-per-day objective needs to have a stochastic formulation based on a stochastic cargo transport demand.

This paper studies the implication on the ship profitability and design factors, namely, the ship capacity and speed, when maximization of the stochastic gross-profit-per-day objective is considered. The paper introduces the mathematical model and the solution methodology used in the study. A case study is given to demonstrate the possible magnitude of the implication. The case concluded that if the management of the ship operations follows the per-day rate concept and the cargo demand forecast, it can improve the profitability of the shipping company and be able to select the proper capacities and speeds for the ships the company uses. This conclusion is not only useful for the shipping or other cargo transport companies but also for businesses like gas reservoir development, car assembly lines in the industry, cooperative farming and crop harvesting in agriculture, port cargo handling in trade and road paving in construction. Moreover, the economies of scale that ship owners believe are reaped by increasing the ship capacity despite its slow operations is completely not true. The economies are reaped if they consider the stochastic gross-profit-per-day as an alternative objective to the total gross profit.

References


**Further reading**


**Appendix**

**SOS voyager mathematical model and solution methodology**

Appendix A contains the model objective function, flow constraints, capacity constraints, time constraints and non-negativity and integrality constraints. The objective function is expressed in total voyage gross profit per day for all ships. The flow constraints connect selected cargo transport links of each ship from voyage beginning to voyage end. They also ensure the flow of at most one transport link towards each cargo. The capacity constraints ensure the ship capacity; expressed in weight, volume, or units, is not violated by the cargo mix selected in each transport link. They also decide whether the ship has to be in a laden or a ballast position when sailing the transport link, and decide whether to pass or bypass the canals and straits. The time constraints ensure the time window allowed for loading or discharging of each cargo is not violated by the time spent in ports and sailing towards the cargo. They also calculate the ship waiting time spent before the opening time of each cargo time window. Also, they ensure the total voyage allowable time is not violated by the actual time. The non-negativity constraints ensure the model variables do not go negative. The integrality constraints turn the variables, dedicated for the transverse of transport links and chartering-in, to yes-or-no decisions. A chance-constrained (stochastic) version of the model is described at the end of the Appendix. The reason for formulating the model as chance-constrained is that it consumes a smaller number of variables if compared to dual-stage or multi-stage stochastic models, which are likely to be beyond practicality for most real linear programming applications.

In this model, it is assumed that each ship starts its voyage at home port (open event) and returns back to its home port (close event). In this model let:

- $S = \{1, 2, 3, \ldots, s_0\}$ be the set of ships;
- $P = \{1, 2, 3, \ldots, p_0\}$ be the set of ports of a working trade area;
- $Q = \{1, 2, 3, \ldots, q_0\}$ be the set of cargoes available for transport between ports of this area. It is assumed that cargoes are compatible with the ship carrying them and can be mixed together on board the ship with ship stability maintained. Each cargo $r \in Q$ has a loading event and a discharging event;
\[ L = \{1, 2, 3, \ldots, l_0\} \text{ is a set of loading events, one for each cargo,} \]
\[ D = \{1, 2, 3, \ldots, d_0\} \text{ is a set of discharging events, one for each cargo,} \]
\[ F = \{f\} \text{ is a one-element set of open event } f. \]
\[ G = \{g\} \text{ is a one-element set of close event } g. \]
\[ E = L \cup D \text{ be the set of load and discharge events, combined.} \]
\[ E_{ij} = E \cup F \text{ be the set of open, load, and discharge events, combined.} \]
\[ E_G = E \cup G \text{ be the set of load, discharge, and close events, combined.} \]
\[ E_{ijz} = E_{ij} \cup G \text{ be the set of open, load, discharge, and close events, combined.} \]
\[ p_i \text{ be port } p \in P \text{ identified at event } i \in E_{ij}. \]
\[ Z = \{1, 2, 3, 4\} \text{ be an index representing two combined positions: \textquoteleft pass or bypass Suez or Panama Canal\textquoteleft as alternative route position, and \textquoteleft laden or ballast\textquoteleft as ship load position.} \]
\[ Z \text{ element of } \textquoteleft 1\textquoteleft represents ship passing canal while in laden position, \textquoteleft 2\textquoteleft represents ship bypassing canal while in laden position, \textquoteleft 3\textquoteleft represents ship passing canal while in ballast position, and \textquoteleft 4\textquoteleft represents ship bypassing canal while in ballast position.} \]
\[ p_{ijz}^k \text{ be the gross profit earned by ship } k \in S \text{ on transport link } ij \text{ while in position } z \in Z \text{. Gross profit equals freight plus demurrage (based on reversible or irreversible calculation), minus cooling/heating cost of cargo } r \in Q \text{ at } i \in L, \text{ minus handling cost of cargo } r \in Q \text{ at } i \in E, \text{ minus dispatch (based on reversible or irreversible calculation), minus port dues of port } p \in P \text{ at } i \in E_{ij}, \text{ where } p_i \neq p_j \text{, and minus canal/strait dues and fuel consumption of main engine when sailing transport link } ij \text{ while in position } z \in Z \text{, where } p_i \neq p_j. \]
\[ T_i^k \text{ be voyage close day of ship } k \in S; \]
\[ C_i^k \text{ be the cost of fuel consumption of auxiliary engine per day plus daily fixed cost of ship } k \in S. \]
\[ C_i^k \text{ be voyage fixed cost of ship } k \in S, \text{ not considered elsewhere; } \]
\[ x_{ijz}^k \text{ be the problem decision variable. It equals 1 if ship } k \in S \text{ sails transport link } ij \text{ while it is in position } z \in Z \text{, and it equals zero otherwise. If } x_{ijz}^k = 1 \text{ and } i \in E, \text{ cargo } r \in Q \text{ is loaded on board ship } k \text{, where } i \text{ is its loading port, or discharged from the ship if } i \text{ is its discharging port. Likewise, if } x_{ijz}^k = 1 \text{ and } j \in E, \text{ cargo } r \in Q \text{ is loaded on board ship } k \text{, where } j \text{ is its loading port, or discharged from the ship if } j \text{ is its discharging port.} \]
\[ y_i \text{ be another problem decision variable, alternative to } x_{ijz}^k. \]
\[ G = \sum_{k \in S} \left( \sum_{i \in E_f} \sum_{j \in E_g} \sum_{z \in Z} p_{ijz}^k x_{ijz}^k - C_i^k T_i^k - C_g T_g \right) / T_g \]
\[ + \sum_{i \in L} \left( P_i y_i - C_{g0} \right) / (t_i y_i + t_{g0}) \]

Subject to:
Flow constraints

Using the above-mentioned denotations, the flow constraints can be formulated as follows:

The flow constraints which restrict the flow of transport links for each ship originating from open event to only one link at most, given by:

$$\sum_{j \in E} \sum_{z \in Z} x_{jxz}^k \leq 1, \quad k \in S,$$

(2)

Flow constraints which restrict the flow of transport links for each ship towards event $e \in E$ to be equal to the flow of transport links outward from this event, given by:

$$\sum_{i \in E} \sum_{z \in Z} x_{izx}^k = \sum_{j \in E} \sum_{z \in Z} x_{jxz}^k, \quad e \in E, \text{ and } k \in S,$$

(3)

Flow constraints which restrict the flow of transport links for each ship towards load event $l \in L$ of cargo $r \in Q$ to be equal to the flow of transport links towards discharging event $d \in D$ of same cargo, given by:

$$\sum_{i \in E} \sum_{z \in Z} x_{ilz}^k = \sum_{i \in E} \sum_{z \in Z} x_{idz}^k, \quad l \in L, d \in D, \text{ and } d \text{ and } l \text{ are of same cargo } r \in Q, \text{ and } k \in S,$$

(4)

Flow constraints which prohibit the flow of transport link of each ship in two opposite directions, given by:

$$\sum_{z \in Z} x_{ijz}^k + \sum_{z \in Z} x_{jiz}^k \leq 1, \quad i, j \in E, \text{ and } k \in S,$$

(5)

Flow constraints which restrict the flow of transport links of all ships towards loading event $l \in L$ of cargo $r \in Q$ plus their alternative decision of acquiring a charter-in ship, to only one at most, given by:

$$\sum_{k \in S} \sum_{i \in E} \sum_{z \in Z} x_{ilz}^k + h_l y_l \leq 1, \quad l \in L, h_l$$

$$= 1 \text{ if } y_l \text{ is taken as an alternative decision and } h_l$$

$$= 0 \text{ otherwise}$$

(6)

Capacity constraints

Let:

- $w_i$ be weight of cargo $r \in Q$ at event $i \in E$, in mt,
- $v_i$ be volume of cargo $r \in Q$ at event $i \in E$, in cum (if non-container),
- $n_i$ be number of TEU of cargo $r \in Q$ at event $i \in E$ (if container),
- $W_k^0$ be the remaining dwt capacity of ship $k \in S$ after load or discharge of cargo $r \in Q$ at event $i \in E$, in mt,
- $W_k^0$ be the min weight remaining on board ship $k \in S$ which keeps the ship in laden position,
- $V_k^0$ be the remaining volume capacity of ship $k \in S$ after load or discharge of cargo $r \in Q$ at event $i \in E$, in cum (if non-container),
\(N_k^E\) be the remaining TEU capacity of ship \(k \in S\) after load or discharge of cargo \(r \in Q\) at event \(i \in E\) (if container),

\(W_k^E\) be the dead weight capacity of ship \(k \in S\),

\(V_k^E\) be the volume capacity of ship \(k \in S\) (if non-container); and

\(N_k^E\) be the TEU capacity of ship \(k \in S\) (if container).

Using the above-mentioned denotations, the capacity constraints can be formulated as follows:

Load remaining weight constraints which restrict remaining weight on board each ship at end event \(j \in E\) to be at least equal to remaining weight at start event \(i \in L\) of any transport link minus weight of cargo \(r \in Q\) at event \(i \in L\), given by:

\[
W_k^j \geq W_k^i - w_i \sum_{z \in Z} x_{ijz}^k, \quad i \in L, j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x_{ijz}^k = 1,
\]

(7)

Constraints (7) can be re-written as follows:

\[
M \left(1 - \sum_{z \in Z} x_{ijz}^k\right) + W_k^j \geq W_k^i - w_i \sum_{z \in Z} x_{ijz}^k, \quad i \in L, j \in E \text{ and } k \in S,
\]

where \(M\) is a big number. So \(W_k^j \geq W_k^i - w_i \sum_{z \in Z} x_{ijz}^k\) will hold true only when \(\sum_{z \in Z} x_{ijz}^k = 1\).

Load remaining volume constraints which restrict remaining volume on board each non-container ship at end event \(j \in E\) to be at least equal to remaining volume at start event \(i \in L\) of any transport link minus volume of cargo \(r \in Q\) at event \(i \in L\), given by:

\[
V_k^j \geq V_k^i - v_i \sum_{z \in Z} x_{ijz}^k, \quad i \in L, j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x_{ijz}^k = 1,
\]

(8)

Load remaining TEU constraints which restrict remaining TEU on board each container ship at end event \(j \in E\) to be at least equal to remaining TEU at start event \(i \in L\) of any transport link minus TEU of cargo \(r \in Q\) at event \(i \in L\) given by:

\[
N_k^j \geq N_k^i - n_i \sum_{z \in Z} x_{ijz}^k, \quad i \in L, j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x_{ijz}^k = 1,
\]

(9)

Discharge remaining weight constraints which restrict remaining weight on board each ship at end event \(j \in E\) to be at least equal to remaining weight at start event \(i \in D\) of any transport link plus weight of cargo \(r \in Q\) at event \(i \in D\), given by:

\[
W_k^j \geq W_k^i + w_i \sum_{z \in Z} x_{ijz}^k, \quad i \in D, j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x_{ijz}^k = 1,
\]

(10)

Discharge remaining volume constraints which restrict remaining volume on board each non-container ship at end event \(j \in E\) to be at least equal to remaining volume at start event \(i \in D\) of any transport link plus volume of cargo \(r \in Q\) at event \(i \in D\), given by:

\[
V_k^j \geq V_k^i + v_i \sum_{z \in Z} x_{ijz}^k, \quad i \in D, j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x_{ijz}^k = 1,
\]

(11)
Discharge remaining TEU constraints which restrict remaining TEU on board each container ship at end event \( j \in E \) to be at least equal to remaining TEU at start event \( i \in D \) of any transport link plus TEU of cargo \( r \in Q \) at event \( i \in D \), given by:

$$N^k_j \geq N^k_i + n_i \sum_{z \in Z} x^k_{ijz}, \ i \in D, j \in E, \text{ and } k \in S, \text{ where } \sum_{z \in Z} x^k_{ijz} = 1,$$

(12)

Weight capacity constraints which restrict remaining weight on board each ship after discharge of all cargoes at end event \( g \in G \) so that it does not exceed ship dwt capacity, given by:

$$W^k_i \geq W^k, \ i \in D, \text{ and } k \in S, \text{ where } \sum_{z=3,4} x^k_{igz} = 1, g \in G,$$

(13)

Volume capacity constraints which restrict remaining volume on board each non-container ship after discharge of all cargoes at end event \( g \in G \) so that it does not exceed ship volume capacity, given by:

$$V^k_i \geq V^k, \ i \in D, \text{ and } k \in S, \text{ where } \sum_{z=3,4} x^k_{igz} = 1, g \in G,$$

(14)

TEU capacity constraints which restrict remaining TEU on board each container ship after discharge of all cargoes at end event \( g \in G \) so that it does not exceed ship TEU capacity, given by:

$$N^k_i \geq N^k, \ i \in D, \text{ and } k \in S, \text{ where } \sum_{z=3,4} x^k_{igz} = 1, g \in G,$$

(15)

Laden-or-ballast load position constraints which restricts ship load position to either laden or ballast. Ship is assumed to be in laden position on transport link \( ij \) if \( i \in L \), and is considered so if \( i \in D \) and remaining weight on board the ship at this event is greater or equal to the min remaining weight \( W^k_0 \), which is given by:

$$W^k_i \geq W^k_0, \ i \in D, \text{ and } k \in S, \text{ where } \sum_{z=1,2} x^k_{ijz} = 1, j \in E,$$

(16)

Time constraints

Let:

- \( a_i \) be laycan open day of cargo \( r \in Q \) at event \( i \in E \).
- \( b_i \) be laycan close day of cargo \( r \in Q \) at event \( i \in E \).
- \( t^k_i \) be the number of days taken to handle cargo \( r \in Q \) at event \( i \in E \) by ship \( k \in S \) plus waiting days at port \( p \in P \) at event \( i \in E \).
- \( t^k_{ij} \) be the number of days taken to sail the transport link from event \( i \in E_f \) to event \( j \in E_g \) by ship \( k \in S \), while it is in position \( z \in Z \), plus waiting days at sea, where \( p_i \neq p_f \).
- \( T^k_i \) be the arrival day of ship \( k \in S \) at event \( i \in E_f \) assuming \( T^k_f = 0 \),
- \( T^k_0 \) be voyage fixed days of ship \( k \in S \), not considered elsewhere,
- \( T^k_s \) be voyage slack days of ship \( k \in S \), if it arrives earlier than \( a_{ri} \), aggregated for all \( r \in Q \) and \( i \in E \),
- \( T^k \) be total allowable days of ship \( k \in S \),

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Using the above-mentioned denotations, the time constraints can be formulated as follows:

Event arrival time constraints which restrict arrival day at end event $j \in E_g$ to be at least equal to arrival day at start event $i \in E_f$ of any transport link plus handling days of cargo $r \in Q$ at $i \in E_f$, waiting days in port $p \in P$ at $i \in E_f$, sailing days on link $ij$, and waiting days at sea, given by:

$$T^k_j \geq T^k_i + t_i + \sum_{z \in Z} t^k_{ijz} x^k_{ijz}, \quad i \in E_f, j \in E_g, \quad \text{and} \quad k \in S, \quad \text{where} \quad t^k_{ijz} = 0, \quad \text{and} \quad \sum_{z \in Z} x^k_{ijz} = 1,$$

(17)

Event time precedence constraints which control arrival times so that arrival day at discharge event $d \in D$ succeeds arrival day at load event $l \in L$ of cargo $r \in Q$, given by:

$$T^k_d \geq T^k_l, \quad l \in L, d \in D, l \text{ and } d \text{ are of same cargo } r \in Q, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{i \in E} \sum_{z \in Z} x^k_{idz} = 1$$

(18)

Time window constraints which restrict ship arrival day at event $j \in E$ so that it does not violate cargo laycan open and close days at this event, given by:

$$T^k_j \geq a_i, \quad j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{i \in E_f} \sum_{z \in Z} x^k_{ijz} = 1,$$

(19)

$$T^k_j \leq b_i, \quad j \in E, \quad \text{and} \quad k \in S, \quad \text{where} \quad \sum_{i \in E_f} \sum_{z \in Z} x^k_{ijz} = 1,$$

(20)

Closing time constraints which restrict final closing day for each ship so that it equals total cargo handling days and waiting days in port, sailing days and waiting days at sea, waiting days before cargo open day, and voyage fixed days, given by:

$$\sum_{i \in E_f} \sum_{j \in E_g} \sum_{z \in Z} \left( t^k_i + t^k_{ijz} \right) x^k_{ijz} + T^k_s + T^k_0 = T^k_g, \quad k \in S,$$

(21)

Allowable closing time constraints which restrict closing day for each ship to a maximum allowable days, given by:

$$T^k_g \leq T^k, \quad g \in G, \quad k \in S, \quad \text{where} \quad \sum_{z \in Z} x^k_{idz} = 1 \text{ and } i \in D,$$

(22)

**Non-negativity and integrality constraints**

Non-negativity constraints of continuous variables, given by:

$$W^k_i, V^k_i, N^k_i, T^k_i \geq 0, \quad i \in E_g, \quad k \in S, \quad T^k_s \geq 0, \quad k \in S,$$

(23)

Integrality constraints of integer variables, given by:

$$x^k_{ijz} = 0, 1, \quad i \in E_f, j \in E_g, k \in S,$$

(24)
The chance-constrained version of the above-mentioned model can be described using the following simple denotations, assuming one ship and one cargo. The transport demand of this cargo is unconfirmed, assumed to be random variable having a known probability distribution. The probability distribution is the *marginal* distribution of demand. Let:

- $d$ be the deterministic cargo transport demand, expressed in quantity units.
- $D$ be the random cargo transport demand, expressed in quantity units.
- $P$ be the least probability ship owner stipulates to transport cargo within $D$.
- $y$ be the quantity of cargo to be transported.

Transport demand constraint implied by the model is given by:

$$ y \leq d $$ (27)

In the chance-constrained model, this constraint reads: the probability of transporting cargo within demand; $\text{Prob.}\{y \leq D\}$, has to be greater or equal to $P$, as indicated by:

$$ \text{Prob.}\{y \leq D\} \geq P $$ (28)

The chance constraint is considered when the cargo transport demand is a random variable, where $y$ is the quantity of cargo to be transported, $D$ is the cargo transport demand, and $P$ is a probability value. In other words, it says: the probability of transporting a cargo within its demand has to be at least equal to $P$. If $P$ can be anticipated, then following the argument mentioned next, the value of $y$, say $y'$ can be determined, which is called the deterministic-equivalent value of $y$.

Constraint (28) is called ‘chance-constraint’. If at $D = d$ the descending cumulative probability of transport demand of cargo has a value just greater or equal to $P$, then (28) in this case implies:

$$ y \leq d $$ (29)

Constraint (29) is the deterministic-equivalent constraint to (28). It is different from constraint (27). The difference is that $d$ in (27) is the quantity of cargo $r$ confirmed offer, while $d$ in (29) is a deterministic-equivalent quantity of cargo random demand, as described earlier. To illustrate, assume for discrete cargo demand $D$, Prob. $\{D < 5 \text{ units}\} = 0.0$, Prob. $\{D = 5 \text{ units}\} = 0.2$, Prob. $\{D = 10 \text{ units}\} = 0.5$, Prob. $\{D = 15 \text{ units}\} = 0.3$, and Prob. $\{D = 15 \text{ units}\} = 0.0$. According to the additive rule of the probability theory, the demand descending cumulative probability distribution reads:

- $\text{Prob.}\{D \geq 5 \text{ units}\} = 0.2 + 0.5 + 0.3 + 0.0 = 1.0$, $0.8 \leq \text{Prob.}\{D \geq 10 \text{ units}\} < 1.0$, and $0.3 \leq \text{Prob.}\{D \geq 15 \text{ units}\} < 0.8$. Now suppose $P = 0.9$. This value falls in second class, which implies a deterministic-equivalent demand value of 10 units (neither 5 nor 15 units), i.e. at $d = 10$.

As defined earlier, the chance-constrained model is exactly (1) to (25) after converting implied constraint (27) to (29). Use the same illustration mentioned above to convert quantities in Table IV to deterministic-equivalent quantities as shown in Table V.

The model is solved by the state-of-the-art Block-Angular Linear Ratio Programming algorithm (El Noshokaty, 2014). In this algorithm, the problem mathematically takes a block-angular form, with a block of objective and constraints assigned to each ship. The model is transformed to a linear form and solved by a modified Mixed Continuous 0-1 Linear Programming algorithm. In this algorithm, a modified Branch and Bound technique is used to solve the mixed continuous 0-1 linear program. At each node in the branch, the problem is decomposed into sub-problems, one for each ship, and then solved by a modified Simplex method.
as indicated by the algorithm. The reason for formulating the model the way mentioned earlier is
that it is amenable to the above-mentioned techniques and methods, which are highly efficient
and reliable, even if the model has a very large number of variables and constraints.

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