A new model for Auckland commercial property yields

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Abstract

Purpose – The purpose of this paper is to extend the studies of commercial property yields by providing a cross-field approach through the implementation of methods used in physics.

Design/methodology/approach – Based on the equations used to describe real gases in physics, the commercial property yields are expressed through a model, as a product of two terms. The first term estimates the influence of the income change and investment on yields. The second estimates the yield variation as a function of property size. Additionally, the model combines the macroeconomic and microeconomic components influencing yield adjustment. Calculation of each component involves procedures developed in physics, with the investment volume being linked to the amount of gas and the microeconomic yield being linked to the gas compressibility.

Findings – The model was applied to the Auckland office and industrial markets, both to the historic and current cycle. At the macro-level, it was found that the use of accumulation of investment over a relevant cycle, results in a high data to model correlation. When modelling the yields at the micro-level, a relationship between the outlying properties and the yield softening was observed.

Practical implications – The paper provides an enhanced modelling power through association of the cyclic and investment activity with the yield change. Moreover, the model may be used to decouple the local and the international investment components and the extent of their influence on the local property market. Furthermore, it may be used to estimate the influence of the property size on the yield.

Originality/value – This research provides a new cross-field application of modelling techniques and enhances the understanding of factors influencing yield adjustments.

Keywords Yields, Cap rates, Commercial yield modelling, Virial expansion, Yield adjustment, Yield components

Paper type Research paper

1. Introduction

The paper presents a comprehensive model of commercial yields, and applies its findings to the Auckland prime and secondary office and industrial markets. The approach builds upon that proposed in Szweizer (2018), where a rental model was presented. Both approaches apply the methodology developed in physics to commercial property markets. Szweizer states that statistical methods developed in physics, which deal with complex multibody interactions, may be re-applied to property markets, providing a statistical toolset which may be used universally. In the work presented here, we employ a similar approach when developing the yield model.

The model expands the equation of state developed in thermodynamics, which is used in the context of real gases. This equation is transformed here to express property related qualities. Two major parts are arrived at. The first part models yield behaviour in the general economic context, and the second describes modifications to yield due to individual buildings' competition. Therefore, a macroeconomic approach is provided by the first component, and the microeconomic approach is contributed through the second.

The paper is organised as follows. First, the general formula is discussed, which splits the yield into macro- and micro-level components. Subsequently, we discuss the macro-level component as Part I of the paper and the micro-level component as Part II. At the completion of Part II, we arrive at a composite formula for yield modelling. Each part contains a discussion and examples of the formulae application to the Auckland office and industrial markets.

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Data sources

The rents, vacancy, yields and transactions data are sourced from CBRE. Interest rates are sourced from Reserve Bank of New Zealand and International Investment Position data series are sourced from Statistics New Zealand.

2. Literature review – property

The standard model is exposed in numerous publications. These include Sivitanides *et al.* (2001), Hendershott and MacGregor (2005), Chen *et al.* (2004), Chichernea *et al.* (2008), Sivitanidou and Sivitanides (1999) and Shilling and Sing (2007). The standard model assumes cap rates to be expressed through an adjustment around equilibrium values, which are determined through real estate fundamentals such as rent levels, rent growth and risk-free interest rates.

Chervachidze *et al.* (2009) revisit the standard model and extend it by incorporating three additional parameters. These are economy-wide risk premium over the risk-free rate, the overall availability of debt in the economy and the investor sentiment. In the extended model, it is assumed that the macroeconomic capital flows and the availability of debt significantly impact capital pricing. It is noted that large capital flows from developing countries to developed ones have become manifest since the early 2000s. These capital flows increased debt availability in the developed countries, and have bid up prices of real estate assets.

In the model lagged capitalisation rate, real rent index and real T-Bond yield (calculated as nominal yield minus inflation rate) are used as standard variables. These are extended by economy-wide risk premium over the risk-free rate, calculated as the difference between Moody's AAA Corporate Bond Index and the 10-year T Bond, and the overall availability of debt in the economy, calculated as a ratio of total net borrowing and lending to the nominal GDP level.

The final version of the model incorporates investor sentiment variable which is to address structural changes to the model performance, especially in the period preceding GFC. It is argued that the real estate investor sentiment, together with the dramatic increase in availability of liquidity, which happened in the early 2000s can be used as an explanation of the "great cap rate compression" observed before 2007.

McAllister and Nanda (2016) investigate the relation between the activity of foreign investors and capitalisation rates in major European office markets. The study involves 28 major cities over the period between 1999 and 2013. The modelling revealed a significant effect of foreign investment on market capitalisation rates. The strategy used employs the base capitalisation rates model as described by Archer and Ling (1997), Chervachidze *et al.* (2009), Chervachidze and Wheaton (2013). The main hypothesis being tested is: all else equal, an increase in the proportion of transactions involving foreign investors will lower capitalisation rates. The model involves: lagged capitalisation rate, real risk-free rate, the real rent ratio as a ratio of the real rent in a year and the same period average of real rent and the availability ratio. All are applied to a specific city and country. Country risk premium and debt availability are also included in the model.

The results suggest a wide spread of foreign investment in major European cities, ranging from as low as 7 per cent in Dublin to as high as 99 per cent in Bucharest and 94 per cent in Warsaw. London is estimated at 50 per cent, Paris at 60 per cent, Amsterdam at 63 per cent and Berlin at 29 per cent. It is important to note that the above study assumed foreign investment relating to entities or persons of foreign nationality investing in a local office market. This definition differs somewhat from the one used in the paper presented here, where we associate the origin of the investment with the country from which the investment funds are derived from.

For example, in the model presented here, if a New Zealand citizen borrows funds from a Chinese bank and invests in the New Zealand property market, we would treat the transaction as a foreign investment, as the funds are obtained from overseas, even though the person executing the purchase is a local citizen. Conversely, a foreigner may borrow funds from a local bank and invest in the New Zealand market. Such a case would be understood as a local investment.

Chaney and Hoesli (2015) extend the standard model through the incorporation of up to 30 variables and compare results obtained with the transaction-based and appraisal-based data sets. The data comprise 3,500 transactions and 8,700 appraisals spanning the period between 1995 and 2010 in Switzerland. The objective of the study is to establish if the appraisal-based yields are smoothed with respect to the transaction-based yields, but also to determine the statistical strength with which each type of variable enters the modelling equation.

The variables are grouped into eight economically meaningful categories. These include refurbishment risk, illiquidity risk, tenant risk, land leverage, commercial tenancy commitment, location, vacancy rate and performance of the economy. The variables are also divided into two groupings with respect to their influence either at the macroeconomic or microeconomic level. The modelling equation is composed of additive parts, which clearly distinguish the variable influence as being either at the macro or the micro level.

The illiquidity risk is associated with a natural logarithm of the property net lettable area (NLA), which in the paper is denoted as the "volume". The inclusion of the property area as a modelling variable is taken after Saderion *et al.* (1994), who used property size, age and location when modelling 500 transactions of apartment complexes in Houston between 1978 and 1988.

In the model, the natural logarithm of the NLA is taken to address the problem of its distribution being very strongly skewed.

As the model is applied to two data sets, the results obtained in each case are compared to the other. The first set of modelling parameters is obtained for investors' data, and the second for appraisers' data and the relevance of each variable for each tested group is shown against the other. It is established that appraisers find macro location and building condition to be of the dominant importance, while investors are highly motivated by the property age, rent to median rent ratio, risk-free rate and the building size.

Szweizer (2018) introduces equations borrowed from physics to property market modelling. The equation of an ideal gas is compared to the formula linking income, capital value and initial yield as the first step. Subsequently, a general treatment of rental cycles is developed. The main feature of the model is the identification of four turning points, labelled as market shocks, during a property cycle. A cycle is divided into four distinct phases responsible for the compression of vacancy, overheated market, market deflation and the market bottom. The approach assumes yields to be constant throughout each cycle, although a variation of yields between each cycle is expected. In the paper presented here, we expand this model by undertaking a detailed analysis of intra-cyclical variations in yields.

3. Literature review – physics

In physics, the gas law may be stated in terms of the compressibility factor Z. When the compressibility is assumed to be equal to unity, the gas is called ideal. For real gases, the compressibility represents intermolecular interactions of gas constituents. The equation of state with compressibility expressed as an infinite series of terms, called virial expansion, was first proposed by Onnes (1902). In the paper, he proposes seven different forms for expansion of the compressibility factor and applies them so to model the behaviour of hydrogen, oxygen, nitrogen and carbon dioxide. The virial expansion is expressed in powers of gas density, but it could also be expressed in terms of temperature.

Schreiber *et al.* (2011) provide an educative paper with a description of how the coefficients are calculated and with some examples of the virial expansion applications. The virial coefficients are shown to be expressible in terms of integrals of Mayer f-functions, and the Mayer expansion is discussed. Mayer (1942) proposes that the virial expansion may be formulated in terms of specific integrals, all depending on the same base function and representing interactions between pairs, triplets and higher combinatoric arrangements of molecules. The integrals may be represented through a collection of graphs, each relating to interactions among increasingly more complex molecule arrangements.

The advantage of the Mayer approach comes through the observation that simultaneous interactions among multiple molecules are of much lower probability than pairwise interactions. Therefore, a rapidly converging series is expected when the virial expansion is expressed in this way.

The Schreiber paper provides examples of second virial coefficient calculation over simple potential models. First, an infinite barrier is discussed, which is used to model a hard sphere interaction. (Two molecules bouncing from each other in a similar way as two snooker balls when colliding). Second, a finite square well potential is discussed to illustrate interactive forces and the second virial coefficient is recalculated over it.

4. The equation of state for real gases

Real gases are described by an equation of state employing compressibility Z, as in Formula 1:

$$\frac{PV}{T} = nZR \tag{1}$$

Here, P stands for pressure, V stands for volume, T stands for temperature, n stands for the amount of gas, Z stands for compressibility factor, and R is the universal gas constant.

When the Formula 1 is translated into property markets one arrives at the following relationship (Formula 2):

$$\frac{Rent \times \ Occupancy}{Capital \ Value} = y_0 \times Z \tag{2}$$

Here, y_0 stands for yield arrived at through macroeconomic analysis and Z stands for yield correction due to microeconomic considerations. In the following section, we construct a model of y_0, and in the subsequent section, the compressibility Z correction is discussed.

5. Part I – macroeconomic component

In Formula 2, the y_0 replaces the amount of gas "n" as used in physics (Formula 1). In property, there are three contributing factors influencing this quantity. The first relates to the cyclic position, and the impact the general economy has on the property market. The second relates to the local availability of liquidity and is modelled through the local interest rates. The third factor represents the international willingness to invest in the local property.

Figure 1 shows the Otto process as employed by Szweizer (2018) when modelling property rental cycle. During the property cycle, the value of the market is influenced by two factors. The first is the change in value due to rental and vacancy changes. In Figure 1 this contribution is represented through vertical arrows, related to the "physical work" input or output, or in property income changes. Thus, the contraction of vacancy and increasing rents result in income growth and sharpening of yields. When the process is reversed between points 3 and 4, yields soften as a result.

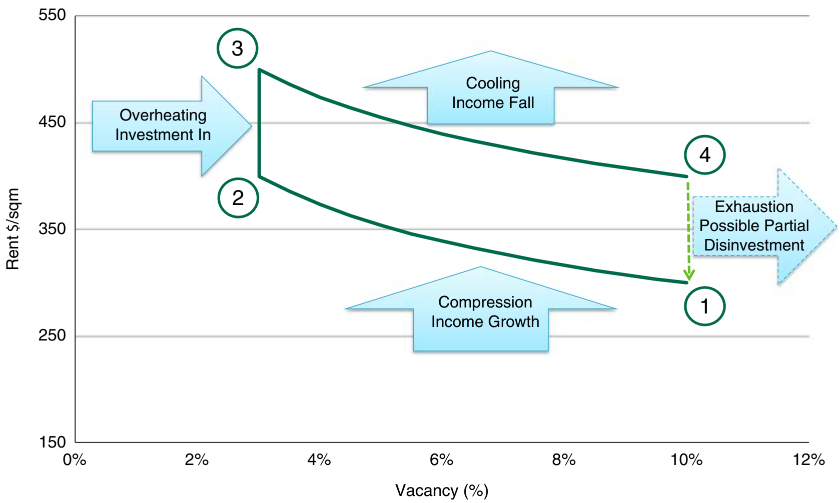

Figure 1.
Rent vacancy curve Income and Investment Flow

The market value is also changed due to the rental changes when the occupancy remains stable (phase between points 2 and 3). This contribution denoted as "heat flow" in physics or investment flow in property is shown with horizontally pointing arrows. The heat flow represents the strength of investment in property, both local and international. The flow in, represented by the left arrow, relates to the increased investment activity, and the flow out, represented by the right arrow, corresponds to disinvestment, which sometimes happens in the Auckland market. The effective change in value due to the cyclic activity corresponds to the area enclosed by the points 1–4.

The presence and magnitude of the disinvestment phase depends on the market and the cycle. In some markets, the disinvestment may be negligible, while in the others quite pronounced. As an example, Auckland experienced substantial outflow of foreign investment prior and following GFC. At that time, the Australian investors sold most of their properties in Auckland and did not return to the New Zealand market for several years.

When developing the model, it was observed that the yields do not respond directly to the individual transactions which happen at any given time, but rather, yields respond to the long-term accumulation of investment in the market. Thus, if we look at Figure 1, yields at point 2 are not influenced by the transactions happening at that time, but are a result of investment accumulation along the curve from point 1 to point 2.

The above observation resulted in the model being developed as relying on the integration of both "work – income growth" and "heat – value growth" over the cyclic progression.

The work – or the income growth component, is integrated with respect to vacancy over the cycle starting at point 1. In the Otto cycle, the rental growth is expressed through the adiabatic function with elasticity gamma. When this is integrated over a period from point 1 to some subsequent point P, the following formula is arrived at:

$$\Delta y_W = A \int_{V_1}^{V_P} R_1 V^\gamma dV = \frac{AR_1}{\gamma+1}\left(V_P^{\gamma+1} - V_1^{\gamma+1}\right), \tag{3}$$

where "A" is a constant weight with which this contribution enters the yield model, V_P is the vacancy at some future point P, at which the yield is being evaluated, R_1 is the rent at point 1, and γ is the elasticity as calculated through the rental model.

The heat – or the investment component, is integrated with respect to rent over the cycle, and is expressed through the following formula:

$$\Delta y_Q = B\int_{R_1}^{R_P} mC_v Cap dR = B\int_{R_1}^{R_P} m\frac{Occ}{\gamma-1}RdR = B\frac{mOcc}{2(\gamma-1)}\left(R_P^2 - R_1^2\right), \quad (4)$$

where C_v is the specific heat at constant vacancy, *Cap* stands for capital value, *Occ* is the occupancy rate, and m is the amount of funds being invested. The constant B denotes the weight with which the investment component enters the model. In the above formula, the amount of funds invested between points 1 and P is assumed to be constant, which limits the applicability of Formula 4 to short time intervals.

The data were provided as a time granulated series. To calculate the accumulation of investment between periods 1 and k the Formula 4 becomes Formula 5:

$$\Delta\% Val_Q = \sum_{i=1}^{k} B\frac{\Delta\% m_{i+1} Occ_{i+1}}{2(\gamma-1)}\left(R_{i+1}^2 - R_i^2\right), \quad (5)$$

where $\Delta\%\mathrm{m}_i$ represents the percentage change of the investment measuring parameter of the iteration, and the $\Delta\%$Val is the percentage change in property value between partitions 1 and k.

Formula 5 was used in two forms. First, by setting $\Delta\%\mathrm{m}_i$ to represent the local interest rates, the contribution of the local investment market is captured. Secondly, by setting $\Delta\%\mathrm{m}_i$ to the percentage change in New Zealand International Investment Position the attractiveness of the local investment against the overseas investment is captured. While this last term does not directly measure the foreign investment in the local property market, it does represent the relative attractiveness of New Zealand investment against the international one.

There is no precise data series available that would capture the amount of foreign investment in the New Zealand commercial property market. Therefore, we could not use such a direct approach when modelling. However, the model only requires the knowledge of the period-by-period percentage change in the relative attractiveness of the New Zealand market to the foreign investment. To estimate this, we used period-by-period percentage change in the International Investment Position as provided by Statistics New Zealand.

By combining Formulae 3 and 5 we arrive at the following modelling expression for market yields:

$$y_k = y_1\left(1 + A\sum_{i=1}^{k}\Delta y_{Wi} + \sum_{i=1}^{k} B\frac{\% I_{i+1} Occ_{i+1}}{2(\gamma-1)}\left(R_{i+1}^2 - R_i^2\right) + \sum_{i=1}^{k} C\frac{\Delta\% IIP_{i+1} Occ_{i+1}}{2(\gamma-1)}\left(R_{i+1}^2 - R_i^2\right)\right), \quad (6)$$

where $\% I$ stand for the five-year bond rate, and $\Delta\% IIP$ is the international investment position percentage change over the surveying iteration.

When modelling, we used five-year bond rate as the interest rate driving variable. It is customary when modelling property markets, to use the 10-year bond rate, and we tested the model against such setup as well. For completeness, the model was also tested against one-year and two-year bond rates as well as against short-term interest rates in the form of 30-day, 60-day and 90-day bank bill yields. It was found that the five-year bond rate gave the best data fit and therefore this is what is used here. One may wish to observe, that the specific choice of the interest rate measure, as used in the model here, may be related to the New Zealand property market only.

Figures 2–5 show the modelling results when parameters y_1, A, B and C are numerically adjusted to provide for the best fit. The figures represent two cycles which have been fitted separately, but are shown on the charts as a continuous sequence.

The model coefficients and their relative strength as contributors to Formula 6 are shown in Table I. All coefficients are shown as absolute values and are normalised with respect to their total, so to show the proportion with which each component influences yield changes.

The historic cycle ended in the period between 2009 and 2011 depending on the sector. The current cycle has just passed the peak or point 3, as shown in Figure 1.

It is clear from Table I that in most cases the variations in income drive the yield changes. The notable exceptions are historic prime office cycle and current secondary industrial. The current prime industrial yield is also influenced strongly by investment, even though to a lesser degree. When looking at the investment (Q) part of the table it is important to recall that yield changes, as treated by the model, are not influenced by individual sales, but by the accumulation of net investment throughout the entire cycle.

Prime office historic yield shows a substantial influence by local investors, as well as a strong overseas involvement. During the pre-GFC market rush, the local investment in New Zealand was highly influenced by numerous investment funds being created at that time. These investment funds, together with already existing property vehicles, were aggressively buying prime office space which is reflected in Table I. Overseas participation was mostly represented by Australian investors, who entered the Auckland office market in the period between 2003 and 2007. During that time, Australian investors represented a share between 14 and 26 per cent of Auckland prime office purchases by volume. The Australian disinvestment started in 2006 and lasted until 2014 influencing both the historic and the current cycle.

The current prime office cycle, while peaking already, does not show such highly heated state as the one prior to the pre-GFC, which is reflected in much smaller investment contributions shown in Table I. There were large international prime office

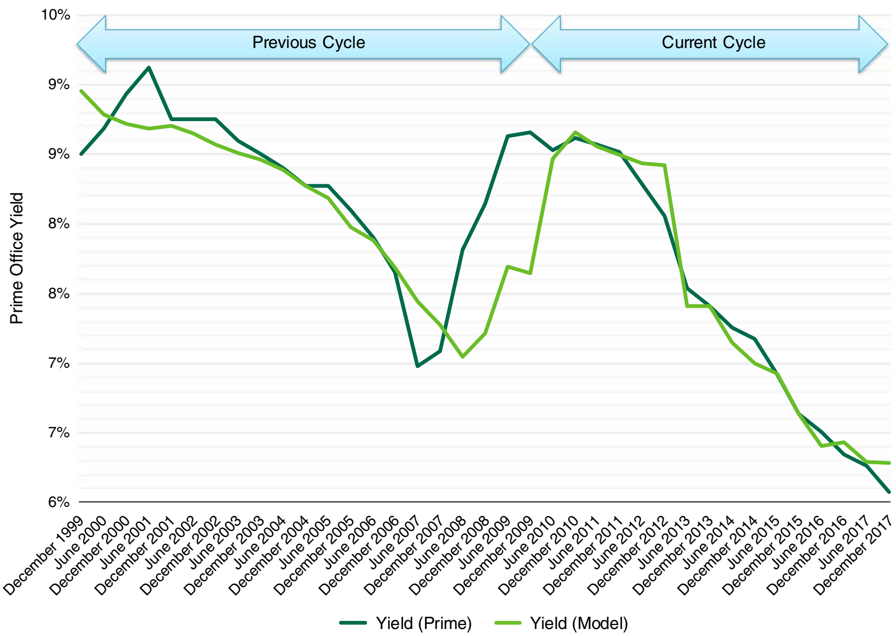

Figure 2.
Auckland Prime Office Yield 1999-2017

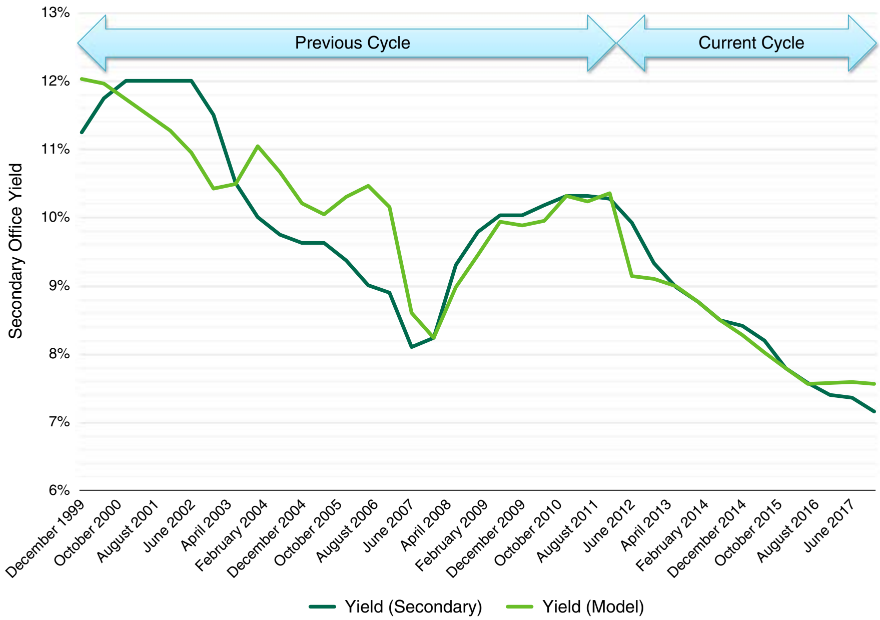

Figure 3.
Auckland Secondary Office Yield 1999

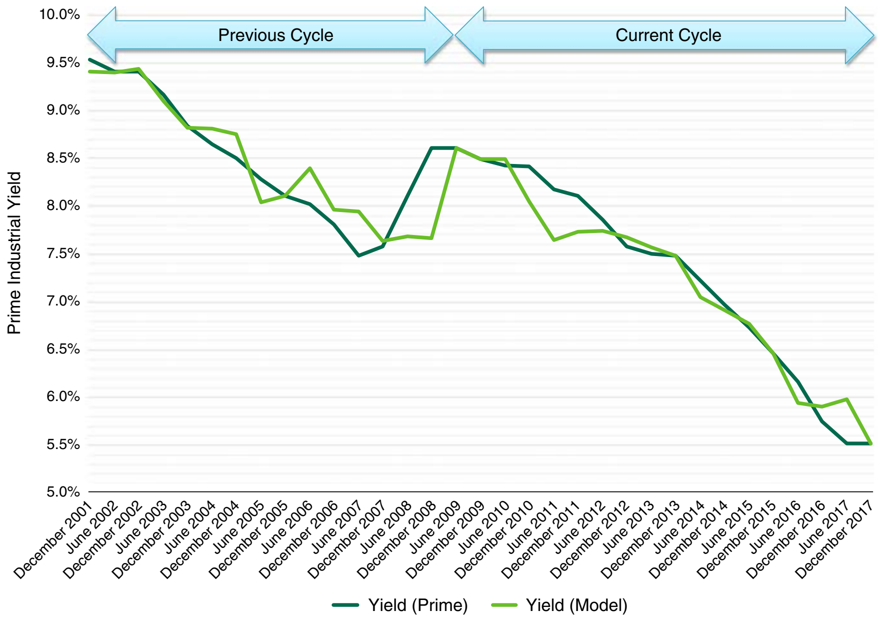

Figure 4.
Auckland Prime Industrial Yield 2001-2017

transactions in the current cycle, but these were between overseas vendors and purchasers, and therefore do not contribute to the net accumulation of the overseas investment.

In recent years, Chinese and Indian banks increased their presence in New Zealand. During the 2013–2014 period, three Chinese banks opened operations in New Zealand.

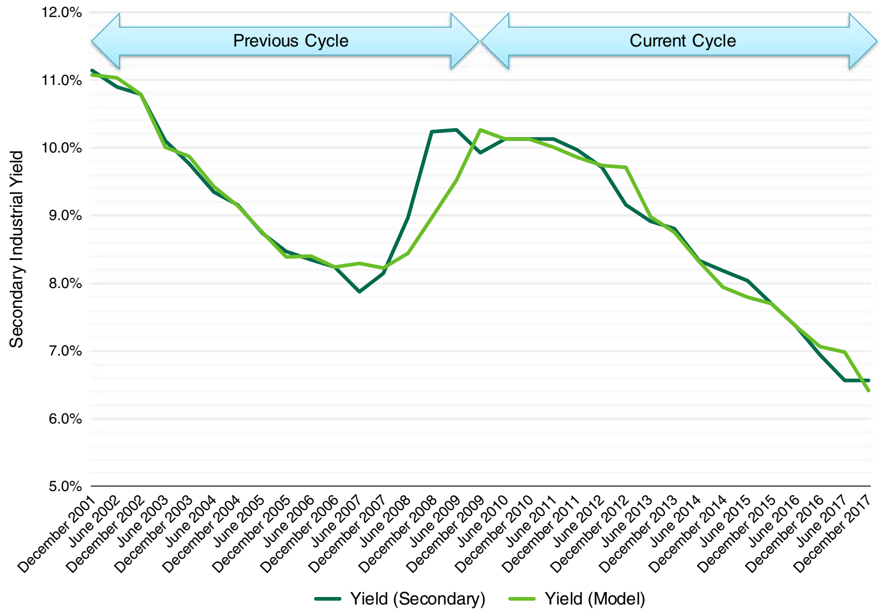

Figure 5.
Auckland Secondary Industrial Yield 2001-2017

Table I.
The macro-level model coefficients

Cycle	Yield (P_1) (%)	Income (%)	*Q*-Local (%)	*Q*-Overseas (%)	R^2 (%)
Prime Office Historic	9.00	19.9	72.6	7.5	54.4
Secondary Office Historic	12.04	99.2	0.4	0.4	66.3
Prime Industrial Historic	9.41	91.5	3.9	4.7	75.8
Secondary Industrial Historic	11.07	94.9	4.8	0.3	85.0
Prime Office Current	8.36	92.7	6.2	1.1	97.9
Secondary Office Current	9.33	95.9	3.8	0.3	93.8
Prime Industrial Current	8.49	64.2	24.1	11.6	95.5
Secondary Industrial Current	9.65	0.7	73.1	26.2	97.3

These include Industrial and Commercial Bank of China (ICBC), China Construction Bank and Bank of China. The locally established overseas banks provide funds to investors of local origin and those who have recently re-settled in New Zealand from other countries. The effect is growth in overseas-funded investment, especially in the industrial space.

During the current cycle, Auckland industrial space constitutes one of the most attractive investment options. This is due to persistently low vacancies, especially in the prime sector. Therefore, there are three main drivers behind the current surge in industrial investment.

First, very low prime vacancy, even against persistently large supply of industrial space, drives the investment with locally and internationally borrowed money. Second, the strong presence of overseas banks in New Zealand, which lend to the local investors, results in increased overseas investment accumulation. Third, the migrant population, which is interested in conducting secondary retail, purchases secondary industrial space, which is used as supportive warehousing to their retail activity. These factors may be seen in Table I through increased values of current prime and secondary industrial investment components.

6. Part II – microeconomic component

The real gas consists of molecules. These molecules interact through a combination of attractive and repulsive forces. When two molecules are further apart the attractive force is dominant. When they are close together the repulsive force pushes them away resulting in a collision and change of relative motion. To take these interactions into account the compressibility factor Z is added to the ideal gas equation.

When properties are valued, the yield achieved by one property influences the valuation of another. Like in the case of molecules, the properties interact with each other within the market context.

Compressibility factor Z is evaluated through the virial expansion. In the realm of property markets this is expressed as:

$$Z = 1 + B_2(T)\rho + B_3(T)\rho^2 + \ldots = 1 + B_2(T)\rho\left(1 + \frac{B_3(T)}{B_2(T)}\rho + \frac{B_4(T)}{B_2(T)}\rho^2 \ldots\right), \tag{7}$$

where T stands for the capital value and ρ stands for density.

In the case of gases, the density ρ is the number of molecules per unit volume. In the case of the property market, we define ρ as the number of properties in a NLA- related basket.

The density

All statistical distributions used in the model depend on exponential functions of the NLA. Therefore, to obtain a linear distribution of data points when calculating densities, we transform the set of NLA's into a set consisting of natural logarithms of these. Thus, the density is defined as the number of properties in a basket partition, which is based on equally split hyperspace of natural logarithms of properties' NLAs. The basket width is defined as two standard errors of the such transformed NLA data set.

We define density space as:

$$\textit{Density space}: ln_{NLA} = \{0, \quad \ln(\max(NLA))\},$$

$$\textit{Basket size}: \textit{basket} = 2\ \text{Standard Error}(\ln_{NLA}).$$

Convergence of virial expansion

The infinite series in Formula 7 involves constants B which are evaluated through multidimensional integrals of the Mayer f-functions. These integrals progress in complexity with each interaction. In general, it is not feasible to perform a summation of the series in any analytical manner.

We postulate that the sale events occur continuously, that the probability of a sale taking place is independent of other sales, and that the sale events occur at a constant average rate. Therefore, we postulate without proof, that the infinite series in Formula 7, converges to an exponential distribution, providing for the following formula:

$$Z = 1 + B_2(T)\rho e^{-\frac{\epsilon a}{kT}\rho}, \tag{8}$$

where T stands for the capital value, ϵ stands for the energy (potential) of building to building interaction, ρ stands for the density, k originally stands for the Boltzmann constant which is set to unity here and "*a*" stands for the extent of the density space and is equal to the natural logarithm of the NLA of the largest property.

The potential

The strength of the interaction between properties is described through the potential $\epsilon(r)$, where r accounts for the difference between any two properties. When evaluating the model, the function $\epsilon(r)$ is chosen in such a way as to first, replicate modelled reality in a reasonably consistent manner, and second is simple enough so that the integrals depending on it may be integrated readily. Square wells and/or barriers are a common choice as these compare well with the modelled reality and can be integrated easily.

In the model presented here a square barrier was selected as follows:

$$\epsilon(r) = \begin{cases} \epsilon 0 \leqslant r \leqslant a \equiv \ln(\max(\text{NLA})) \\ 0\ r > a \equiv \ln(\max(\text{NLA})) \end{cases}, \tag{9}$$

where ϵ is a constant positive number.

Therefore, we assume that properties repel each other with a constant force given by the potential ϵ. Such definition may be understood as representing competition among similar properties.

B_2 coefficient

Assuming a constant barrier potential, as given above, the coefficient B_2 may be evaluated through integration with respect to the distance over one-dimensional space:

$$B_2 = -\frac{1}{2}\int_0^{a=\ln(\max(NLA))} \left(e^{\frac{-\epsilon}{kT}}-1\right)dr = \frac{1}{2}\left(1-e^{\frac{-\epsilon}{kT}}\right)a, \tag{10}$$

In Formula 10, the integrand is the Mayer f-function.

Combining Formulae 8 and 10 the formula for compressibility over a constant barrier potential becomes:

$$Z = 1+\frac{1}{2}\left(1-e^{\frac{-\epsilon}{kT}}\right)a\rho e^{-\frac{\epsilon a}{kT}\rho}. \tag{11}$$

7. Application of the model

The model was applied to the data comprising both office and industrial prime sales spanning the period from 2012 to 2017. When fitting the model two parameters were varied: the macroeconomic yield and the potential ϵ. The fitting parameters are shown in Table II and the fit quality shown in Figures 7 and 9. The distribution of the data points over the natural logarithm of the NLA is shown in Figures 6 and 8.

It was found that the best fit was achieved when the term ϵ/kT was set to the weighted average yield as obtained through the macroeconomic part of the model (described in part I). The weighting was done with respect to the number of sales, but also with the age of the sale through an exponential moving average. Therefore, the sales which were older were contributing less to the weighted yield average than the new sales.

Table II. Micro-level model best fit parameters

	Office	Industrial
ϵ/kT (%)	6.56	6.31
a	10.68	11.09
y_0 (%)	6.56	6.31
Cap value (T) (\$psm)	5,979	2,287
ϵ (\$psm)	392	144
Average rent (\$psm)	387	140
Error (%)	−1.5	−3.0

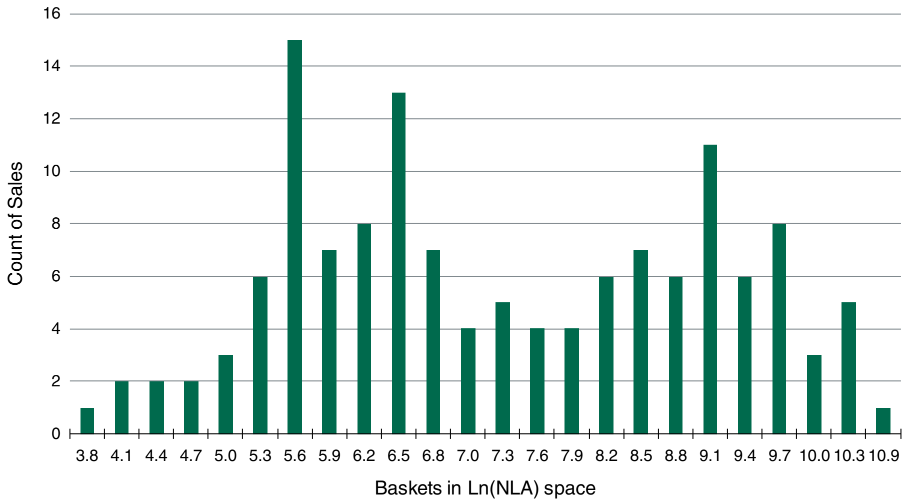

Figure 6.
Auckland prime office histogram

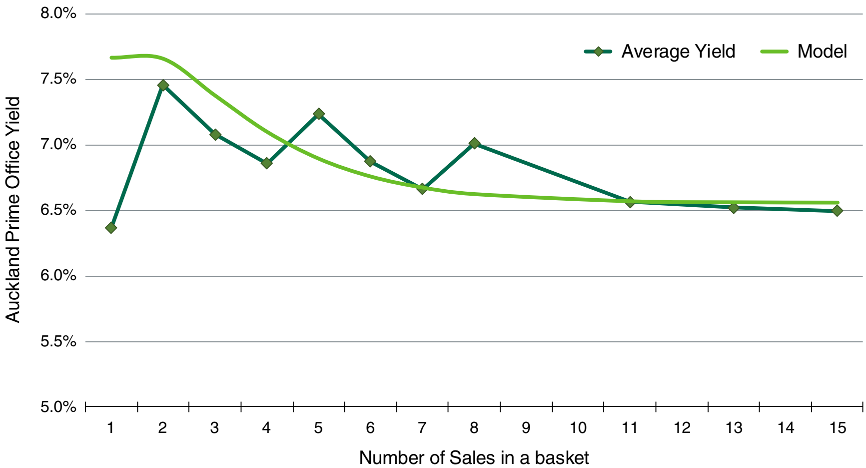

Figure 7.
Micro-level fit to Auckland office yield data

The best-fit parameters are shown in Table II. The fitting process was done in two steps. While fitting the data the ϵ/kT and y_0 parameters were varied. The parameter "a" is the logarithm of the NLA of the largest property and thus stayed constant. It was of interest to estimate the value of interaction potential ϵ. To do so, the capital value (T) was taken as the average of capital values of the entire sample data for each sector, and the constant k was set to unity. The potential ϵ was calculated, resulting in values, agreeing closely with the average net effective rent of the data set.

The similarity of ϵ/kT and y_0 suggests that the average effective rent may be used as the potential ϵ. In such a case, the model reduces to the following formula:

$$yield = y_0\left(1+\frac{1}{2}(1-e^{y_0})a\rho e^{-y_0 a\rho}\right), \tag{12}$$

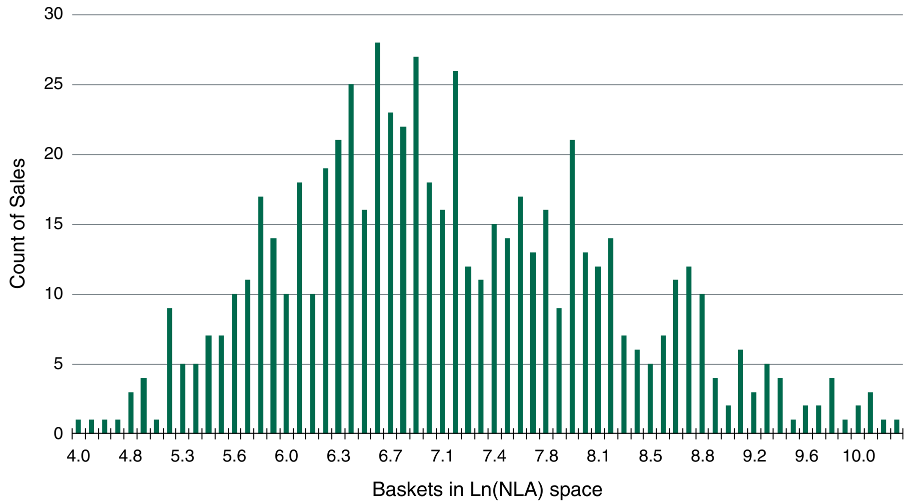

Figure 8.
Auckland prime industrial histogram

where y_0 is the macroeconomic yield, as obtained in the first part of the paper, "a" is the ln (max(NLA)) and ρ is the number of properties in a basket as defined above.

Formula 12 represents the combined macro- and micro-economic model as presented here. The value of y_0 is obtained through the macroeconomic expression shown in Formula 6, and the term in brackets, represents the microeconomic correction. This may be compared to Formula 2, where the RHS term consists of the macroeconomic yield, which is multiplied by the microeconomic correction (the Z factor). Thus, the microeconomic correction depends on all: the building size, the density but also the macroeconomic yield itself.

It is clear from Figures 7 and 9 that the yield stabilizes in the region where the number of properties per basket is large, but is softer when the properties are distinct by their size. Thus, the very large and the very small properties which differ in this way from the bulk of properties attain softer yields.

Figure 9.
Micro-level fit to Auckland industrial yield data

The model was tested against individual sales data points. Formula 12 was used with y_0 set to the macroeconomic yield for each year, as obtained through the macroeconomic model described in the first part of the paper. The other two variables "a" and "ρ" were applied as specified in Table II, that is the overall data sample size and the individual basket association spanning throughout the entire period were used.

Subsequently, the difference between factual yield and the modelled was calculated. The standard error shown in Table III represents, averaged over each year, the discrepancy between the actual yield achieved through the sales and the yield estimated through the model.

8. Conclusion and discussion

The technique presented in the paper attempts to provide for a comprehensive commercial yield model. It is assumed, based on the real gas equation, that yield may be expressed as a product of two terms. The first term represents the macro-level approach and the second term the micro-level approach.

When compared to the standard model several observations may be entertained. The standard model is an adjustment model with an expectation that the yields may be modified with respect to some underlying equilibrium value. Here, the yields are expected to evolve through subsequent cycles with no reference to any fixed base value. The standard model represents yields as proportionally dependent on both the rents and the risk-free interest rates, with the rents scaled through an elasticity factor, and the risk-free rate through an exponential contribution. Here, the rents contribute through a quadratic dependence, which is a result of a different approach to the understanding of the market dynamics.

In the standard model, yields are assumed to be responding to the current market conditions. Therefore, the current rental changes and the current availability of debt is factored into the model. In the approach presented here, we expect the yields to respond to the accumulation of both the income generated by the property and the investment in the market, both taken over the period of the entire cycle. This approach results in an evolution of yields, with yields becoming increasingly sharper in subsequent cycles, just because the amount of the total investment in the property markets accumulates over longer periods of time. In this way, yields correspond to the total accumulated value of the market, and if a substantial disinvestment was to happen, the yields would soften as a result. To calculate the accumulation of income and investment, these need to be integrated over the entire period considered. The process of integration results in the quadratic dependence on rents.

The most important observation utilised in the first part of this paper is the assumption that yields do not respond to individual sales data, but rather correlate with the long-term accumulation of investment in the property market. Three main components contributed to the macro-level model. It was assumed that yield changes are due to the accumulation of income generated by the property, the accumulation of local investment and the

	Prime office		Prime industrial	
Year	No. of sales	SE (%)	No. of sales	SE (%)
2012	7	0.254	79	0.127
2013	17	0.240	73	0.116
2014	34	0.158	102	0.121
2015	31	0.224	133	0.119
2016	22	0.180	137	0.094
2017	24	0.259	118	0.098
Total	135	0.093	642	0.048

Table III. The model applied to the Auckland prime stock accumulation of foreign investment. All three components were calculated over the duration of a property cycle.

The macro-level model allows for further splitting of the relative strength with which the three components contribute to the yield changes. This allowed for the identification of the overseas derived funds contribution. In New Zealand, the direct identification of the strength of such contribution is very difficult if approached directly and the model provides for a clear advantage in this area.

When analysing micro-level yield variation, the buildings' sizes were used to assess yield distribution. It showed that properties which differ substantially from the bulk of properties, acquire softer yields. Therefore, very large or very small buildings, which differ substantially from the mainstream constructions are penalised through softer yields.

It should be noted that the choice of the NLA as the density factor in compressibility expansion, was used due to the density used in the original physics approach to compressibility. Replacement of density with other factors pertaining to property markets would seem to be of potential interest. For example, as noted in the literature review, building age could constitute a promising research avenue leading to the expansion of this study.

The model presented here has a potential to be used as a forecasting tool. This depends on the ability to forecast the main independent variables used by the model. Therefore, the interest rates, IIP changes, or any other variable which could be used in its place as a measure of international attractiveness of the market, would need to be forecasted first. Also, the ability to construct a forecast of the rental model, which is used as a basis for the income generated component of the model, would be required.

The other difficulty is associated with the establishment of the location of the market turning points. When forecasting, these need to be predicted, which constitutes a considerable challenge. On the other hand, the ability to locate future turning points, may be a prime objective of forecasting.

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