Optimal decisions and coordination strategy of a capital-constrained supply chain under customer return and supplier subsidy

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Abstract
Purpose – The purpose of the paper is to explore impacts of financing and supplier subsidy on capital-constrained retailer and the value of returns subsidy contract under a situation where the retailer makes joint operations and finance decisions.

Design/methodology/approach – This paper considers a two-level supply chain, including a retailer and a supplier. Facing problems of capital constraints and even customer returns, the newsvendor-like retailer orders from a well-capitalized supplier. The supplier allows the retailer a delay in payment and provides a subsidy contract to alleviate its problems if it is profitable. Considering their difference of initial capital status, the retailer is assumed to be Follower of Stackelberg Game and the supplier is the Leader.

Findings – The supplier return subsidy contract has some merits for both partners in the chain. And it does not coordinate the supply chain when the retailer has enough initial capital; however, when the retailer is capital constrained, it does. In addition, the retailer’s initial capital level significantly affects the supplier’s subsidy decision.

Research limitations/implications – Return rate is simplified to a fixed proportion of completed demand. In addition, trade credit is only financing source in this paper, and other types of financing methods, such as bank credit, can be taken too.

Originality/value – This paper first incorporates trade credit financing and customer returns into a modeling framework to investigate the capital-constrained retailer’s joint operations and finance decisions and the value of supplier’s subsidy contract.

Keywords Supply chain finance, Trade credit, Capital-constrained, Customer returns, Return subsidy

Paper type Research paper

Notations
\( w_f \) = Supplier’s postponed wholesale price;
\( c \) = Supplier’s production cost;
\( p \) = Retail price, which is normalized to 1;

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$v$ = Salvage value of the product;
$\alpha$ = Customer returns rate, $0 \leq \alpha < 1$;
$\lambda$ = Subsidy level of per unit returned product, $0 \leq \lambda \leq 1$;
$k(Q) = \text{The retailer’s bankruptcy threshold, i.e., the minimal realized demand for the retailer to repay its loan obligation at full;}$
$B = \text{The retailer’s initial capital, } B \geq 0$;
$D = \text{The uncertain future demand;}$
$Q = \text{The retailer’s order quantity;}$
$\pi^R_d = \text{The expected profit of the retailer in a decentralized supply chain;}$
$\pi^S_d = \text{The expected profit of the supplier in a decentralized supply chain;}$ and
$\pi^C = \text{The expected profit of a centralized supply chain.}$

**Introduction**

In recent years, customer returns have become a very ubiquitous phenomenon. Many customers are increasingly inclined to return unwanted products back to the store for diverse reasons such as product quality, the mismatch of customer’s expectations and the regret of impulse buying. With the application of the product traceability system and the Internet of Things technology, it has become possible to grasp the accurate information on production, storage and product flow, which enables the supplier to monitor customer returns accurately. According to Stock *et al.* (2002), the annual value of the returned products exceeded $100bn in the USA. Mostard and Teunter (2006) showed that the return rates of some specific fashion items have reached up to 74 per cent. Because of the difficulty in dealing with returned products, product return significantly challenges the participants in the supply chain. It is noted that the problem becomes more aggravating for small firms because of their financial distress. Bagley (2012) reported that there exist 25-27 million small enterprises in the USA, accounting for 60 to 80 per cent of whole US jobs. Meanwhile, because of the short of collateral and credit history, it is very difficult to obtain loans from financial institutions. Thus, for these small retailers in the supply chain, they are not only up against the difficulty of financing but also confronted with the risk of customer returns.

The absence of financial resources hinders the development of many small retailers seriously, thus affecting the expansion of the supply chain. One of solutions to the problem is to provide trade credit to the retailers facing capital constraints, which draws great interest of researchers on the interface of operations and finance. The researchers take trade credit as the financing method of capital-constrained retailer not only under the setting of EOQ but also under that of Newsvendor. Deterministic (EOQ) and stochastic demand models, multi-period and single-period newsvendor models, symmetric and asymmetric information, risk attitude and the combination and comparison of different financing modes are being increasingly settled (Aggarwal and Jaggi, 1995; Gupta and Wang, 2009; Chen and Cai, 2011; Zhang *et al*., 2015). However, the literature about trade credit has not considered the impact of customer returns on all participants in the supply chain. Meanwhile, the majority of existing researches about customer returns management focus only on the situation where the retailer has sufficient initial capital and will not borrow from the supplier (Su, 2009; Chen and Bell, 2011; Xiao *et al*., 2010). That is, the literature to investigate the customer returns problem of capital-constrained retailer is missing. To summarize, the existing papers investigate either customer returns policies or trade credit, which gives us the motivation to put forward a model in this paper to consider the joint effect of these two aspects. We, in this paper, attempt to investigate the optimal strategies of the supplier and the capital-constrained retailer in the presence of trade credit and customer returns.
In this paper, we study a supply chain composed of one well-capitalized supplier and one capital-constrained retailer. The retailer faces stochastic demand with customer returns. To encourage customers to purchase more products to improve customer satisfaction and cultivate customer loyalty, one of the feasible schemes for the retailer is to offer a full refund to its customers. At the same time, a full refund policy also correspondingly increases the retailer’s operational costs and reduces the optimal order quantity. In the light of this, the supplier will observe further decreased revenues and sales from the retailer’s actions to customer returns and the retailer’s insufficient initial capital. Therefore, to eliminate these adverse effects, there are two aspects of considerations for the supplier. On one hand, the supplier will provide trade credit to ease the financing difficulties and eliminate the cooperation concerns of the capital-constrained retailer before signing, so as to achieve cooperation. Moreover, on the other hand, because of the complexity of the reasons for customer returns, it is difficult to determine the attribution of the responsibility of these returned products. To address the risk of customer returns in the process of cooperation and encourage the retailer to order more, we design a return subsidy contract. Actually, the supplier subsidy contract is equivalent to the buyback contract if the supplier’s buyback is limited to the returned products, which helps to understand impacts of returns on both of the retailer and supplier better by focusing on the solving of returns. The supplier, as the Stackelberg game leader, provides a postponed wholesale price[1] and a subsidy level of per unit returned product first. Given the supplier’s strategy, the retailer, as the follower, then decides the purchase quantity of the products. All participants in the supply chain are assumed to be risk neutral. Furthermore, we also assume that there is no moral hazard, that is to say, participants in the supply chain will not deliberately manipulate the market to obtain some subsidies for returned products.

We extend previous research on trade credit by adding customer returns and explore the following questions. First, under what conditions should the supplier provide subsidies for returned products, and what is the optimal subsidy level? Second, what about the effect of the subsidy level and customer returns on the retailer’s decisions? Third, how to coordinate the supply chain in the presence of trade credit and customer returns? Through the study of the above questions, this paper has the following practical contributions. First, this paper studies problems of decision-making and coordination of the capital-constrained supply chain under the condition of customer returns and also discusses the influence of the customer returns on the capital-constrained supply chain, which has some theoretical significance. Second, this paper examines the impact of customer returns and capital constraints on decisions of supply chain partners. Also, it extends previous deterministic demand case to more general stochastic market demand case, which is more in line with the actual operation of enterprises. Therefore, the conclusions of this paper are more realistic and meaningful.

To address these questions, we investigate a single-period model based on the classic newsvendor model, in which the supplier sells short life cycle products to the retailer at a fixed retail price. It finds that the supplier is not always willing to provide subsidies for these returned products. Specifically, the supplier’s decision is affected by the initial capital of the retailer and the customer returns rate. The retailer’s order quantity increases with the supplier’s subsidy level but decreases with the customer returns rate. Additionally, we further show that the supply chain can be coordinated with a proper postponed wholesale price when full refund policy and trade credit are considered.

The rest of the paper is organized as follows. Section 2 gives a review of related literature. Section 3 describes the model. Section 4 investigates a centralized supply chain and derives some analytical insights. Section 5 studies the strategies of all participants in a decentralized
supply chain and the coordination of the supply chain. To gain some further insights, we use a set of numerical examples in Section 6. Finally, we conclude the paper and propose some directions for further research in Section 7. All proofs are presented in the Appendix.

Literature review
The model setting in this paper is a combination of trade credit and customer returns. In the following, we will do a brief review of literature related to each area.

There is an abundant literature on trade credit. Goyal (1985), Aggarwal and Jaggi (1995) and Su (2012) investigated the economic order quantities under an allowable delay in payment, and among these models the demand is deterministic. Robb and Silver (2006) and Maddah et al. (2004) studied the stochastic inventory models with trade credit. However, these studies only focused on the operational decisions and neglected the financial decisions. There have been some recent tries to model operational and financial decisions jointly. Kouvelis and Zhao (2012) studied a capital-constrained retailer’s and a capital-constrained supplier’s problems using a Stackelberg game under a perfect capital market circumstance. The research showed that the trade credit contract can improve the supply chain’s efficiency. Yang and Birge (2013) investigated two-part trade credit contract with accessible bank financing and further studied the structure of the financing portfolio consisting of trade credit, cash and a bank loan. Caldentey and Chen (2011), whose research had some similarities to ours, allowed the capital-constrained retailer to delay in payment. Other studies on trade credit include Chen and Wang’s (2012), Cai et al.’s (2014) and Chen’s (2015). All of the above papers about trade credit neglected the situation where the retailer faces customer returns, which affect the retailer’s and the supplier’s profits in some sense.

Another stream of research related to our paper is the research on customer returns. Cohen et al. (1980) considered a model in which returned quantity is a fixed fraction of demand. Chen and Bell (2009) studied a firm facing customer returns, found that customer returns influence the firm’s inventory and pricing decisions. Mostard and Teunter (2006) investigated a similar problem, but they only considered a single-period newsvendor problem and assumed that the undamaged returned products can be sold again before the end of the season. In our paper, we also consider a single-period newsvendor problem with customer returns; the difference is that the retailer is capital-constrained and the supplier’s return policy is further investigated as well. There exists some literature jointly study the retailer’s and the supplier’s strategies with customer returns. Su (2009) investigated a model of different customer returns policies and further studied the situation where the manufacturer promises to buy back the leftover items from the retailer. Chen and Bell (2011) studied a model in which the retailer faces with price dependent stochastic demand and customer returns, the manufacturer provided two different buyback prices and showed how to coordinate the supply chain using buyback policy. Xiao et al. (2010) considered manufacturer buyback policy and customer returns policy together in a model and found that refund amount and the customer’s valuation have an important impact on the supply chain. According to Lawton (2008), only about 5 per cent of returned products from customers were truly defective. For these false failure returns, Ferguson et al. (2006) proposed a target rebate contract and demonstrated that this proposed contract can coordinate the supply chain well. Moreover, the profits of both parties and the whole supply chain are increased. Before the end of the sales period, these false failure returns may be resold by the retailer, and the retailer’s processing cost is much less than that of the supplier. So, for the supplier, a good decision is not to accept the customer returns but to leave these customer returns to the retailer so that the false failure returns can be better handled. Some suppliers have adopted this approach. Therefore, in the face of the fact that false failure
returns occupy a higher proportion of returned products nowadays, we put forward a supplier subsidy contract to induce the retailer to expand the order quantity to optimize the performance of the supply chain. In this contract, the customer returns will not be returned to the supplier but to the retailer that will get some return subsidies from the supplier correspondingly. Our work is complementary to the existing literature by investigating customer returns and trade credit together.

Model description

In this paper, we investigate a supply chain consisting of a supplier (he) and a retailer (she). The supplier produces the single-period short-life products and sells them to the retailer who faces a random demand \( D \), which follows a cumulative distribution function \( F(D) \) with density \( f(D) > 0 \). The retailer sells the product to customers at retail price \( p \), which is normalized to 1 for brevity (e.g. Gerchak and Wang, 2004).

Different from the classic newsvendor model, customer returns are explicitly considered in the proposed model. The retailer promises that products can be returned for a full refund during the sales period. Following Vlachos and Dekker (2003), we assume that customer returns are a fixed proportion of quantity sold, which is defined as \( \alpha \), where \( 0 \leq \alpha < 1 \). Therefore, we define a customer returns function as \( \alpha \min(D, Q) \). At the end of the sales period, all leftover units including the returned products and the remnant inventory are sold at the salvage value \( v \), which is lower than the production cost \( c \), \( 0 < c < 1 \).

Moreover, as the focus of the proposed model, the retailer does not have enough initial capital to order an optimal quantity. Thus, the supplier provides trade credit to the retailer and allows the retailer to pay for the products at the end of the sales period with a postponed wholesale price \( w_T \). According to Pasternack (2008), the manufacturing cost per unit is independent of production volume, and all retailers charge the product at the same fixed price, which is a very common phenomenon in some franchise businesses and other businesses (e.g. dairy products and periodicals). Similarly, we assume that the supplier has many competitors in the market, so he will charge the retailer a fixed postponed wholesale price \( w_T \) because of the competitive pressures. It is obvious to assume that \( 0 \leq v < c < w_T < \beta = 1 \). Furthermore, the supplier has sufficient capital, and he does not charge the retailer any prior payment before the sales season. Also, to ease the retailer’s risks and improve the order quantities, the supplier comes up with a return subsidy contract that promises that once the customer returns, he will give the retailer subsidies for returned products at the end of the sales period. We assume that the subsidy level of per unit returned product is \( \lambda \), \( 0 \leq \lambda < 1 \). We assume that \( \lambda + v < 1 \); otherwise, the retailer will encourage customers to return to obtain some extra profits. We further assume \( c + \lambda \alpha < w_T \), so that the supplier can benefit from this transaction. In addition, we can get \((1 - \alpha) + \alpha(\lambda + v) > w_T \). If not, the retailer is not willing to sell any products.

The sequence of events and decisions is as follows (Figure 1):

![Figure 1. The sequence of events](image-url)
At the start of the sales period (Time 0): the supplier simultaneously announces a postponed wholesale price \( w_T \) and a subsidy level of per unit returned product \( \lambda \). Observing \( w_T \) and \( \lambda \), the retailer determines order quantity \( Q \) and borrows trade credit \( w_T Q \) from the supplier. It is noteworthy that \( w_T \) is exogenously given and will not change during the sales period, which is explained before.

At the end of the sales period (Time 1): the random demand \( D \) is realized, the sales volume is \( \min(D, Q) \). Next, the customer who is dissatisfied with the product will send them back to the retailer. The retailer receives the returned products \( \min(D, Q) \) and pays a full refund \( \alpha \min(D, Q) \) to customers. Right after noticing the customer returns, the supplier will give subsidy \( \lambda \alpha \min(D, Q) \) to the retailer to share the risk of customer returns. Then, the retailer will sell all leftover units \( \min(D, Q) + (Q - D)^+ \) at the salvage value \( v \). Finally, the retailer repays trade credit to the supplier. Similar to other trade credit literature (Jing et al., 2012; Kouvelis and Zhao, 2012), we assume the retailer has limited liability. If the retailer’s revenue (including subsidy) at the end of the sales period is not enough to repay the loan, she only repays all of its revenue and will not be liable for the remainder. If not, the retailer will repay the loan at full.

Following Chen and Cai (2011), we define \( h(D) = f(D)/F(D) \) as the hazard function and \( H(D) = Dh(D) \) as the generalized failure rate of random demand \( D \), in which \( F(D) \) denotes the tail distribution. To guarantee the existence and uniqueness of equilibrium of the problem, we assume that \( F(D) \) has a finite mean and is continuous on \((0, \infty)\) and \( H(D) \), and \( h(D) \) is monotonically increasing in \( D \geq 0 \), i.e. IGFR and IFR. We assume that information among the supplier, the retailer and the customer is symmetric. Key notations are collected in Notation.

Note that we use subscript \( S, R, C \) and \( d \) to denote the supplier, retailer, centralized supply chain and decentralized supply chain, respectively. Besides, we use a variable with superscript * to denote the equilibrium solution. The bold parts of Notation represent the decision variables.

**The centralized supply chain**

In this section, we consider the performance of a centralized supply chain, in which the supplier and the retailer are integrated to make decisions. The expected profit of the centralized supply chain \( \pi_C \) is:

\[
\pi_C = E[(1 - \alpha)\min(D, Q) + v(Q - D)^+ + \alpha v \min(D, Q) - cQ]
\]

\[
= (1 - \alpha - v + v\alpha)\int_0^Q F(D)dD + (v - c)Q
\]  

(1)

Here, \( x^+ = \max(x, 0) \). In equation (1), \( \min(D, Q) \) represents the retailer’s sales revenue; \( \alpha \min(D, Q) \) is the value that the retailer pays to the customer for the returned products; \( v(Q - D)^+ + \alpha v \min(D, Q) \) denotes the sum of the residual value of the returned products and the remaining products at the end of the period and \( cQ \) is the product cost. Revenue subtracts expenditure is the expected profit of the centralized supply chain. Solving the optimal problem, we can come to the following result.

**P1:**

- Under a centralized supply chain, the optimal order quantity \( Q^*_C \) is unique and satisfies \( F(Q^*_C) = \frac{\frac{c - v}{\alpha} + x^+}{1 - \alpha - v + v\alpha} \).
• For any feasible customer returns rate $\alpha$, the optimal order quantity $Q^*_C$ decreases in $\alpha$.

• The optimal order quantities without customer returns are more than that in the case with customer returns.

It is obvious to find that the optimal order quantity $Q^*_C$ is affected by the customer returns rate $\alpha$. It is intuitive that $Q^*_C$ decreases in $\alpha$. As the customer returns rate increases, the centralized supply chain faces higher risk and correspondingly lowers the order quantity.

When $\alpha = 0$, there are no customer returns, the risks faced by the centralized supply chain become much lower than that in the case $\alpha > 0$. Therefore, the optimal order quantities without customer returns are more than that in the case with customer returns.

The decentralized supply chain

In this section, we consider the scenario where the supply chain is decentralized. Both the supplier and the retailer make their own decisions to maximize their individual profits. Backward induction is adopted to derive the equilibrium solutions in the decentralized supply chain; we first compute the retailer’s optimization problem to get $Q(\lambda)$ for given $\lambda$. Then, we take $Q(\lambda)$ into the supplier’s optimization problem to get the optimal $\lambda^*$.

The retailer’s problem

We have supposed that the capital-constrained retailer can use trade credit without any prior payment. Thus, the retailer receives $Q$ units of product at the beginning of the sales period, repays the smaller of its procurement cost $W_T Q$ or $(1-\alpha) \min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\alpha \min(D, Q) + B$ to the supplier at the end of the sales period. Therefore, the retailer’s expected profit can be expressed by:

$$\pi^R_d = E[(1-\alpha)\min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+$$
$$+ v\alpha \min(D, Q) + B - W_T Q]^+ - B$$

(2)

In equation (2), $\lambda \alpha \min(D, Q)$ is the return subsidy paid by the supplier to the retailer and $W_T Q$ represents the retailer’s purchase cost. The retailer’s final wealth can be expressed as $[(1-\alpha) \min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\alpha \min(D, Q) + B - W_T Q]^+$, subtracts the initial capital $B$ to get the retailer’s expected profit as shown in equation (2). If $(1-\alpha) \min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\alpha \min(D, Q) + B \geq W_T Q$, the retailer can fully repay trade credit $W_T Q$ to the supplier; otherwise, the retailer only pays back $(1-\alpha) \min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\alpha \min(D, Q) + B$. Here, there exists a minimal demand level below which the retailer cannot repay the loan obligation, we denote it as the retailer’s bankruptcy threshold $k(Q)$. The following Lemma describes this bankruptcy threshold:

**Lemma 1.** The retailer can fully pay back the loan obligation if and only if the realized demand $D$ is no less than the threshold $k(Q) = \frac{w_T Q - B - vQ}{1-\alpha + \lambda \alpha - v + v\alpha}$.

It is obvious that $k(Q) < Q$ recall that $(1-\alpha) + \alpha(\lambda + v) > w_T$. Lemma 1 means that the retailer will not become insolvent when $D \geq k(Q)$. We can further find that if $B > (w_T - v)Q$, then $k(Q) < 0$, so the retailer must not go bankrupt. Therefore, the expected profit of the retailer can be rewritten as:

$$\pi^R_d = \begin{cases} 
E[(1-\alpha)\min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\min(D, Q) - W_T Q], & B > (w_T - v)Q \\
E[(1-\alpha)\min(D, Q) + \lambda \alpha \min(D, Q) + v(Q-D)^+ + v\min(D, Q) + B - W_T Q]^+ - B, & B \leq (w_T - v)Q 
\end{cases}$$

(3)
Solving the optimization problem of equation (3) leads to the following Proposition.

P2. Under a decentralized supply chain, given a subsidy level of per unit returned product \( \lambda \), the retailer’s optimal order quantity \( Q_d^* \) can be characterized by:

- If \( B \geq (w_T - v)F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda \alpha - v + v\alpha}) \), \( Q_d^* = Q_d^1 = F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda \alpha - v + v\alpha}) \);
- If \( 0 \leq B < (w_T - v)F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda \alpha - v + v\alpha}) \), \( Q_d^* = Q_d^2 \), which satisfies

\[
F(Q_d^2) = \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + v\alpha} F\left(\frac{w_T Q_d^2 - vQ_d^* - B}{1 - \alpha + \lambda \alpha - v + v\alpha}\right)
\]

(4)

Here, we use \( Q_d^1 \) and \( Q_d^2 \) to distinguish the retailer’s optimal order quantity under the above two different initial capital scales.

Because of the limited budget, the optimal order quantity of the retailer is influenced by the offered subsidy level \( \lambda \) and the initial capital \( B \). This Proposition divides the retailer’s order decision into two intervals. In the first interval, the retailer’s initial capital is sufficient. However, in the second interval, the initial capital of the retailer is relatively low. As the retail price is normalized to 1, we can notice that \( \frac{1}{C_0} \) is relatively low. As the initial capital level becomes higher, to avoid potentially huge loss, the retailer is inclined to order smaller products.

In Corollary 1, we show that when the retailer is capital-constrained, the retailer’s optimal order quantity decreases in \( B \), and it is larger than the case when the capital is sufficient. It is easy to understand this phenomenon. When the initial capital \( B \) is small, the retailer’s maximal wealth to lose is small because of limited liability, so the retailer is inclined to order more products. Thus, the capital constrained retailer orders more than the retailer with sufficient initial capital. However, as the initial capital level becomes higher, to avoid potentially huge loss, the retailer’s order quantity decreases.

Corollary 1. For a given subsidy level of per unit returned product \( \lambda \), \( Q_d^1 < Q_d^2 \), meanwhile \( Q_d^2 \) decreases in \( B \).

The Corollary 1 shows that when the retailer is capital-constrained, the retailer’s optimal order quantity decreases in \( B \), and it is larger than the case when the capital is sufficient. It is easy to understand this phenomenon. When the initial capital \( B \) is small, the retailer’s maximal wealth to lose is small because of limited liability, so the retailer is inclined to order more products. Thus, the capital constrained retailer orders more than the retailer with sufficient initial capital. However, as the initial capital level becomes higher, to avoid potentially huge loss, the retailer’s order quantity decreases.

Lemma 2. For every \( Q \) and \( \lambda \) that satisfy equation (4), we must have \( k(Q)\leq h(k(Q)) < 1 \), in which \( k(Q) = \frac{w_T Q - B - vQ}{1 - \alpha + \lambda \alpha - v + v\alpha} \).

Lemma 2 is used to prove some subsequent results.

Corollary 2. Under a decentralized supply chain, the retailer’s optimal profit \( \pi^R(Q_d^*) \) and her optimal order quantity \( Q_d^*(\lambda) \) increase in \( \lambda \).

In Corollary 2, we show that both \( \pi^R(Q_d^*) \) and \( Q_d^*(\lambda) \) increase in \( \lambda \). The intuitive explanation for the conclusion is that if the supplier gives a higher subsidy level, the retailer’s risk is lower. Hence, the retailer earns more profit and orders more products.

Corollary 3. The retailer’s optimal order quantity \( Q_d^* \) in the decentralized supply chain decreases in \( \alpha \).

Similar to P1, the explanation of Corollary 3 is that a higher \( \alpha \) results in a higher risk to the retailer. So, the retailer orders smaller products.

The supplier’s problem

In this section, corresponding to the retailer’s problem, we analyze the supplier’s problem in two different situations. In the first situation, the retailer has enough capital, the supplier can
recover all the loans at the end of the sales period; in another situation, the retailer is capital-constrained and may not be able to repay the loan at the end of the sales period fully.

**Situation 1.** The supplier’s problem when the retailer is well-funded.

In this situation, \( B \geq (w_T - v)F^{-1}(\frac{\alpha x - v}{1 - \alpha + \lambda \alpha - v + v \alpha}) \), the retailer has sufficient capital. Given the optimal order quantity of the retailer, the supplier’s problem is:

\[
\pi_d^S = E[w_T Q_1^* - \lambda \alpha \min(D, Q_1^*) - cQ_1^*]
\]

where \( w_TQ \) is the trade credit that the supplier must be able to recover at the end of the sales period. \( \lambda \alpha \min(D, Q) \) is the subsidy that the supplier gives it to the retailer for the customer returns. \( cQ \) is the supplier’s production cost. The optimal subsidy level of per unit returned product \( \lambda^* \) is characterized in the following Proposition:

**P3.** Under a well-funded decentralized supply chain, for any \( \lambda \in [\underline{\lambda}, \overline{\lambda}] \), in which \( \underline{\lambda} = \max\left(\frac{\alpha x - v \alpha + \alpha - 1}{\alpha}, 0\right) \) and \( \overline{\lambda} = \min(1 - v, \frac{\alpha x - c}{\alpha}) \), the supplier’s optimal subsidy level of per unit returned product \( \lambda^* \) is

\[
\lambda^* = \begin{cases} 
\overline{\lambda} & \text{if } L(\lambda) \geq 0 \\
\underline{\lambda} & \text{if } L(\lambda) \leq 0 \\
\lambda & \text{if } L(\lambda) > 0 \text{ and } L(\lambda) < 0
\end{cases}
\]

where \( L(\lambda) = w_T - c - \lambda \alpha F(Q_1^*) - (1 - \alpha - v + \lambda \alpha + v \alpha)h(Q_1^*) \int_0^{Q_1^*} F(D)dD \), which decreases in \( \lambda \), and the unique \( \lambda \) satisfies \( L(\lambda) = 0 \).

If \( L(\lambda) \geq 0 \), then \( \frac{d\pi_d^S}{d\lambda} \geq 0 \), the supplier’s profit increases in \( \lambda \), and hence the supplier will give the subsidy at the highest level \( \overline{\lambda} \). On the contrary, if \( L(\lambda) \leq 0 \), then \( \frac{d\pi_d^S}{d\lambda} \leq 0 \), the supplier’s profit decreases in \( \lambda \). Therefore, the supplier will give the subsidy at the lowest level \( \underline{\lambda} \). However, if \( L(\lambda) > 0 \) and \( L(\overline{\lambda}) < 0 \), we can get \( \frac{d\pi_d^S}{d\lambda} \bigg|_{\lambda=\underline{\lambda}} > 0, \frac{d\pi_d^S}{d\lambda} \bigg|_{\lambda=\overline{\lambda}} < 0 \), the supplier’s profit function is concave in \( \lambda \). Therefore, there exists a unique \( \hat{\lambda} \) satisfying \( L(\hat{\lambda}) = 0 \) that makes the supplier’s profit maximum. P3 gives the optimal subsidy level of the supplier in the sufficient capital supply chain, which indicates that the supplier’s return subsidy strategy can benefit both the supplier and the retailer, and further provides some references for the management decision of real enterprises.

**Situation 2.** The supplier’s problem when the retailer is cash-strapped.

In this situation, \( B < (w_T - v)F^{-1}(\frac{\alpha x - v}{1 - \alpha + \lambda \alpha - v + v \alpha}) \), the retailer is capital-constrained. Given the retailer’s optimal order quantity, the supplier’s problem is written as follows:

\[
\pi_d^S = E[\min((1 - \alpha)\min(D, Q_2^*) + \lambda \alpha \min(D, Q_2^*) + v(Q_2^* - D)^+ + v \alpha \min(D, Q_2^*) + B, w_T Q_2^*) - \lambda \alpha \min(D, Q_2^*) - cQ_2^*]
\]

\[
= -(1 - \alpha - v + \lambda \alpha + v \alpha) \int_0^{B_1(Q_2^*)} F(D)dD + \lambda \alpha \int_0^{Q_1^*} F(D)dD + w_T Q_2^* - \lambda \alpha Q_2^* - cQ_2^*
\]

(6)
In equation (6), \( \min[(1 - \alpha)\min(D, Q_2^*) + \lambda \alpha \min(D, Q_2) + v(Q_2^* - D)^+ + v\alpha \min(D, Q_2^*) + B, w_T Q_2^* + B, w_T Q_2] \) denotes the final income of the supplier when the retailer’s order quantity is \( Q_2^* \); the customer return subsidy paid by the supplier and the product cost are \( \lambda \alpha \min(D, Q_2) \) and \( cQ_2 \), respectively. Income subtracts the expenditure, and it is the supplier’s expected profit as shown in equation (6). If \( (1 - \alpha)\min(D, Q_2) + \lambda \alpha \min(D, Q_2^*) + v(Q_2^* - D)^+ + v\alpha \min(D, Q_2) + B \geq w_T Q_2^* \), the supplier earns a profit of \( w_T Q_2^* - \lambda \alpha \min(D, Q_2) - cQ_2^* \); otherwise, the supplier liquidates the retailer and gains \( (1 - \alpha)\min(D, Q_2) + v(Q_2^* - D)^+ + v\alpha \min(D, Q_2) + B - cQ_2 \).

After the above analysis, we can know that the optimal subsidy level set by the supplier satisfies:

\[
\lambda^* = \max_{\lambda \in \lambda_s} \pi_s^* = -(1 - \alpha - v + \lambda \alpha + v\lambda) \int_0^{\lambda(Q_2^*)} F(D)dD + \lambda \alpha \int_0^{Q_2^*} F(D)dD + w_T Q_2^* - \lambda \alpha Q_2^* - cQ_2^*
\]

Because of the difficulty of calculating the optimal subsidy level of per unit returned product in this situation, we give an upper bound on the optimal subsidy level through the analysis below, and further narrow the scope of the subsidy level’s feasible region.

**Lemma 3.** For a capital-constrained decentralized supply chain, the total profit is concave in \( Q \) and independent of \( \lambda \).

Through the proof of Lemma 3, we can get that the total profit first increases in \( Q \) until \( Q = Q_C^* \), then decreases with \( Q \). Therefore, if \( Q_2^* > Q_C^* \), the total profit declines on the interval \((Q_C^*, Q_2^*)\), but the retailer’s profit will increase during this interval, so the supplier’s profit must decline on the same interval. That is, the rational supplier is not willing to offer a subsidy level such that the retailer orders more than \( Q_C^* \).

We use \( Q_2^* = Q_2^*(\lambda^*) \) to denote the retailer’s optimal order quantity given the supplier’s optimal subsidy level \( \lambda^* \), then \( Q_2^* = Q_2^*(\lambda^*) \leq Q_C^* \). By Corollary 2, we can conclude that \( Q_2^* \) increases in \( \lambda \) and hence, when \( Q_2^*(\lambda^*) = Q_C^* \), \( \lambda^* \) reaches its upper bound \( \hat{\lambda} \). Thus, combining the \( PI \) and \( P2 \), we can get \( \hat{\lambda} \) satisfies

\[
\frac{w_T - v}{1 - \alpha - v + \lambda \alpha + v\alpha} F\left(\frac{w_T Q_2^* - vQ_C^* - B}{1 - \alpha - v + \lambda \alpha + v\alpha}\right) = \frac{-v}{1 - \alpha - v + v\alpha} \text{ in which } Q_C^* = F^{-1}\left(\frac{-v}{1 - \alpha - v + v\alpha}\right).
\]

However, considering the feasible region \([\hat{\lambda}, \bar{\lambda}]\) (in \( P3 \)) of \( \lambda \), we can come to the following Proposition.

**P4.** Under a capital-constrained decentralized supply chain, for any \( \lambda \in [\hat{\lambda}, \bar{\lambda}] \), in which \( \hat{\lambda} = \max\left(\frac{w_T - v}{\alpha}, 0\right) \), \( \bar{\lambda} = \min(1 - v, \frac{w_T - c}{\alpha}) \), we have:

- If \( \hat{\lambda} \leq \lambda \), then the upper bound of the supplier’s optimal subsidy level \( \lambda^* = \lambda \), \( \lambda^* = \lambda \);
- If \( \hat{\lambda} < \lambda < \bar{\lambda} \), then the upper bound of the supplier’s optimal subsidy level \( \lambda^* = \lambda \), \( \lambda^* \in [\hat{\lambda}, \bar{\lambda}] \);
- If \( \lambda \geq \bar{\lambda} \), then the upper bound of the supplier’s optimal subsidy level \( \lambda^* = \lambda \), \( \lambda^* \in [\hat{\lambda}, \bar{\lambda}] \).

\( \hat{\lambda} \) is determined by

\[
\frac{w_T - v}{1 - \alpha - v + \lambda \alpha + v\alpha} F\left(\frac{w_T Q_2^* - vQ_C^* - B}{1 - \alpha - v + \lambda \alpha + v\alpha}\right) = \frac{-v}{1 - \alpha - v + v\alpha}, \text{ where } Q_C^* = F^{-1}\left(\frac{-v}{1 - \alpha - v + v\alpha}\right).
\]
For the case $\hat{\lambda} \leq \underline{\lambda}$, $Q^*_2$ is always greater than $Q^*_C$ in the interval $[\underline{\lambda}, \overline{\lambda}]$, the supplier is not willing to provide $\lambda^*$ more than $\underline{\lambda}$, so $\lambda^*$ would be $\underline{\lambda}$. For another case $\hat{\lambda} \geq \overline{\lambda}$, $Q^*_2$ is less than $Q^*_C$ during the interval $[\underline{\lambda}, \overline{\lambda}]$, thus the upper bound of $\lambda^*$ would be $\overline{\lambda}$ and the feasible interval of $\lambda^*$ still be $[\underline{\lambda}, \overline{\lambda}]$. As for the case $\underline{\lambda} < \hat{\lambda} < \overline{\lambda}$, $Q^*_2$ is less than $Q^*_C$ at first until $\lambda^* = \hat{\lambda}$, then $Q^*_2$ becomes greater than $Q^*_C$. According to the above analysis, the supplier will not let the retailer’s order quantity over $Q^*_C$, so we can deduce $\overline{\lambda}^* = \hat{\lambda}$, and then we can further get that the feasible interval of $\lambda^*$ will be reduced to $[\underline{\lambda}, \hat{\lambda}]$.

$P4$ can be illustrated in Figures 2 and 3, which have the following parameters: $B = 5$, $\alpha = 0.5$, $v = 0.1$, $c = 0.3$, $w_T = 0.700$, 0.452 or 0.340, the feasible regions of subsidy level are $[0.3, 0.8]$, $[0, 0.304]$, $[0, 0.08]$ respectively. We further assume that the demand function is subject to $U[0,100]$.

Figure 2 describes the retailer’s optimal order quantity $Q^*$ with respect to $\lambda$, shows that the optimal order quantity of the centralized supply chain $Q^*_C$ remains unchanged on $\lambda$; however, the optimal order quantity of the capital-constrained decentralized supply chain $Q^*_2$ increases in $\lambda$. These discoveries confirm $P1$ and Corollary 2. Moreover, we assume $\lambda = \hat{\lambda}$ at the intersection of $Q^*_2$ and $Q^*_C$.

Figure 3 examines the supplier’s profits in terms of $\lambda$, demonstrates that the supplier’s profits are concave on $\lambda$ for the given $c$. Combine Figures 2 and 3, we can come to the following observations. When $w_T = 0.340$, $Q^*_2 > Q^*_C$, $\hat{\lambda} < \underline{\lambda}$, $\lambda^* = 0 = \underline{\lambda}$, which corresponds to case 1 of $P4$. When $w_T = 0.452$, $Q^*_2 < Q^*_C$ at first until $\lambda = \hat{\lambda}$, then $Q^*_2 > Q^*_C$, $\underline{\lambda} < \hat{\lambda} < \overline{\lambda}$, $\lambda^* = \hat{\lambda}$, which is consistent with case 2 of $P4$. Furthermore, corresponding to case 3 of $P4$, when $w_T = 0.700$, $Q^*_2 < Q^*_C$, $\hat{\lambda} > \overline{\lambda}$, $\lambda^* = \overline{\lambda}$ and $\overline{\lambda}^* = \overline{\lambda}$.
Next, we concentrate on studying the supplier's problem when $B = 0$, that is, the retailer does not have any initial capital.

**Lemma 4.** When the retailer does not have any initial capital, the supplier's expected profit will decrease in $\lambda$.

Using Lemma 4, we can come to the following Proposition:

**P5.** Under the scenario where the retailer has no initial capital, the supplier's optimal subsidy level of per unit returned product $\lambda_N^*$ is given by:

$$
\lambda_N^* = \begin{cases} 
\frac{w_T - v \alpha + \alpha - 1}{\alpha} & \text{if } \alpha \geq 1 - \frac{w_T}{1 - v} \\
0 & \text{if } \alpha < 1 - \frac{w_T}{1 - v}
\end{cases}
$$

The interpretation of the above result is as follows. We have proved in Lemma 4 that the supplier's expected profit decreases in $\lambda$. Therefore, the supplier's optimal subsidy level of per unit returned product $\lambda_N^* = \lambda$, that is, $\lambda_N^* = \frac{w_T - v \alpha + \alpha - 1}{\alpha}$ or $\lambda_N^* = 0$. However, when $\alpha \geq 1 - \frac{w_T}{1 - v}$, the customer returns rate is so high that the retailer faces greater risk. Under this circumstance, if the supplier does not provide return subsidies, the retailer's expected profit is negative and will not order any products. Therefore, to promote the transaction, the supplier's optimal subsidy level of per unit returned product $\lambda_N^* = \frac{w_T - v \alpha + \alpha - 1}{\alpha}$. On the contrary, when $\alpha < 1 - \frac{w_T}{1 - v}$, the customer return rate is relatively low, and the retailer will order products even without supplier's subsidies, so $\lambda_N^* = 0$.

According to P5, we can conclude that when the retailer does not have any initial capital, the supplier is still likely to provide return subsidies. The reason may be that the supplier wants to facilitate the completion of the transaction, open the product market or enhance the visibility of the enterprise. In real life, the competition between enterprises is increasing day by day. Based on this situation, P5 provides some theoretical guidance about how to open the product market and occupy the market competitive advantage. Also, it provides some business management suggestions for the enterprises.

**Supply chain coordination**

From P5, we know that the optimal order quantity of centralized supply chain is $Q^*_C = F^{-1}_a \left( \frac{c - v}{1 - \alpha + v} \right)$. To coordinate the decentralized supply chain and make it operate in an optimal situation, the supplier must propose a mechanism to induce the retailer to order the quantity $Q^*_C$. The coordination mechanism can be summarized in the following P6:

**P6.** For the optimal subsidy level $\lambda^* \in [\bar{\lambda}, \lambda]$, if $B \geq (w_T - v) F^{-1}_a \left( \frac{w_T - v}{1 - \alpha + v} \right)$, the supply chain cannot be coordinated; conversely, if $B < (w_T - v) F^{-1}_a \left( \frac{w_T - v}{1 - \alpha + v} \right)$, the supply chain can be coordinated as long as $w_T$ satisfies $\frac{w_T}{1 - \alpha + v} = \frac{c - v}{1 - \alpha + v}$, and the optimal order quantity is $Q^*_C = F^{-1}_a \left( \frac{c - v}{1 - \alpha + v} \right)$.

P6 means that the decentralized supply chain cannot be coordinated through the proposed supplier subsidy contract when the retailer has sufficient initial capital.
However, when the retailer is capital-constrained, the supply chain can be coordinated with an appropriate $w_T$. The logic is if the retailer has enough initial capital, trade credit and return subsidy have a small impact on stimulating the retailer to increase its order quantity. However, when the retailer is capital-constrained, trade credit and the proposed supplier subsidy contract can tempt the retailer to order much more products to reach the optimal order quantity of the supply chain, the supply chain is coordinated consequently. This result indicates that the supplier subsidy contract has some redeeming features and deserves to study.

**Numerical analysis**

During this section, we use numerical examples to acquire some further insights. Furthermore, in this section, we only consider the case of $B < (w_T - v)^{F^{-1}}\left(\frac{w_T - v}{1 - \alpha + \lambda - \alpha v + \alpha^{-1}}\right)$ in decentralized supply chain. We assume that $v = 0.1$, $c = 0.3$, $\lambda = 0.5$ as the base values, and the demand function is subject to $U[0,100]$. We first investigate the sensitivity of optimal order quantity $Q_2^*$ with respect to $w_T$, as illustrated in Figure 4. Among this, we set $B = 5$, $\alpha = 0.4$.

Figure 4 shows that the optimal order quantity of the capital-constrained retailer in a decentralized supply chain decreases in $w_T$. The intuitive explanation is that the higher $w_T$ leads to a higher financing cost for the retailer. As a consequence, the retailer will lower her order quantity correspondingly.

We also use Figures 5 and 6 to examine the supplier’s profits in terms of $\alpha$ and $B$. In Figure 5, we suppose that $w_T = 0.700$, $B = 5$. From Figure 5, we can easily find that the supplier’s profit decreases in $\alpha$. It is intuitive that with the increase of $\alpha$, the risk of customer returns which the supplier bears is increased, thus the supplier’s profit must decrease in $\alpha$.
We further investigate the impact of initial capital $B$. Figure 6 depicts the supplier’s profit as a function of $B$ where $B < (w_T - v)^{-1} \left( \frac{w_T - v}{1 - \alpha \lambda (a - v + \delta)} \right)$. Here, we assume $w_T = 0.700, \alpha = 0.4$. Figure 6 indicates that the supplier’s profit increases at first and then decreases as $B$ grows, which is explained as follows. In the initial stage with a small initial capital $B$, the retailer tends to order more products and has a greater risk of default; at the same time, the supplier has to pay larger subsidies. With the increase of $B$, the retailer’s optimal order quantity $Q_2$ and the default risk is reduced, the supplier’s subsidies are relatively reduced as well. On the one hand, the reduction of the retailer’s optimal order quantity will reduce the supplier’s profit; on the other hand, the reduction of the default risk and subsidies will increase the supplier’s profit. On the whole, the increased profit margins higher than the reduced profits, so the supplier’s profit will increase in $B$ at first. However, when $B$ is high, the retailer’s default risk becomes so low that the supplier’s increased profits that have mentioned above turn into smaller than the supplier’s reduced profits caused by the retailer’s lower order quantity, thus the supplier’s profit decreases in $B$ again.

**Conclusion**

In this paper, we investigate a capital-constrained retailer who orders her products from a capital adequacy supplier and is allowed a delay in payment to the end of its sale period, which is equivalent to the supplier’s providing trade credit to the retailer. To alleviate the retailer’s returned products problem, we design a supplier subsidy contract and explore its value for partners of a supply chain. Conclusions show that both of the retailer and the supplier benefit from the supplier subsidy contract, which implies the value of subsidy contract for them. Also, we point out that the supplier’s optimal decision is influenced by the initial capital level of the retailer. That is, when the retailer does not have any initial capital, the supplier’s optimal decision is to set the subsidy level of per unit returned product to its lower bound. But if the retailer has some initial capital, the supplier’s optimal decision varies depending on the demand distribution of the product and the retailer’s capital level. Further, supplier subsidy contract is found infeasible to coordinate the supply chain when the retailer has enough initial capital; however, when the retailer is capital-constrained, it coordinates the supply chain if a proper wholesale price is postponed by the supplier. In addition, from the numerical analysis, we can observe some interesting results. For instance, the supplier’s profit is concave in the retailer’s initial capital level, and that the supplier’s profit decreases in the customer returns rate for a given subsidy level of per unit returned product.

Some possible extensions of this proposed model can be summarized as follows. First, we have reduced the feasible region of the supplier’s optimal subsidy level of per unit returned product when the retailer is capital constrained. Therefore, using algorithms (e.g.
dichotomy) to derive the optimal subsidy level of per unit returned product should be considered in the future. Second, full returns policy has been well studied in our paper; however, investigating the impact of partial returns policies on the suppliers and the capital-constrained retailer's decisions could be another research direction. Finally, our paper assumes that the demand distribution and the customer returns rate are common knowledge. In practice, the retailer may possess more knowledge about customer returns and demand conditions than the supplier. Thus, it is worthwhile to study the case of asymmetric information.

Note
1. It is worth noting that, similar to Yang and Birge (2011), we assume the postponed wholesale price is fixed. Under this setting, trade credit in our model is not channel coordination contract by itself, but the financing means of the capital-constrained retailer.

References


Appendix

Proof of proposition 1
Part (i). Taking derivative with $Q$ in equation (1), we have $\frac{d\pi_C}{dQ} = (1 - \alpha - v + v\alpha)F(Q) + v - c$.

Then we further get $\frac{d\pi_C}{dQ'} = -(1 - \alpha - v + v\alpha)f(Q) < 0$. Therefore, the unique optimal order quantity of the retailer, $Q_C^*$, satisfies the first-order condition $F(Q_C^*) = \frac{\alpha - v}{1 - \alpha - v + v\alpha}$.

Part (ii). From Part (i) we get $F(Q_C^*) = \frac{\alpha - v}{1 - \alpha - v + v\alpha}$. Taking derivative with respect to $\alpha$ on both sides of this equation, we have $f(Q_C^*) \frac{dQ_C^*}{d\alpha} = \frac{c(1 - v)}{(1 - \alpha - v + v\alpha)}$. As $0 \leq v < c < w_T < p = 1$, we know that $c > v$, $1 > v$, so $\frac{dQ_C^*}{d\alpha} = -\frac{c(1 - v)}{f(Q_C^*)(1 - \alpha - v + v\alpha)} < 0$, i.e. $Q_C^*$ decreases in $\alpha$.

Part (iii). Because $0 \leq \alpha < 1$, $\frac{dQ_C^*}{d\alpha} < 0$, thus, $Q_C^*|_{\alpha=0} > Q_C^*|_{\alpha>0}$. Conclusion is proved.

Proof of lemma 1
Under a decentralized supply chain, we can observe from equation (2) that when the retailer is not bankrupt, her operational revenue is $(1 - \alpha)\min(D, Q) + \lambda\alpha\min(D, Q) + v(Q - D)^+ + v\alpha\min(D, Q) + B - w_TQ \geq 0$. Thus, when $D \leq Q$, we can get from the inequality of $(1 - \alpha)D + \lambda\alpha D + v(Q - D)^+ + v\alpha D + B - w_TQ \geq 0$ that the minimal realized demand is $k(Q) = \frac{w_TQ - B - vQ}{1 - \alpha + \lambda\alpha - v + v\alpha}$, i.e. the retailer’s bankruptcy threshold is $k(Q) = \frac{w_TQ - B - vQ}{1 - \alpha + \lambda\alpha - v + v\alpha}$.

Proof of proposition 2
Under a decentralized supply chain, according to the retailer’s initial capital level, we consider the following two cases.

Case 1: $B \geq (w_T - v)F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda\alpha - v + v\alpha})$

Under this circumstance, the retailer has sufficient high initial capital. Thus, she must be able to repay trade credit at the end of the sales period. Therefore, the retailer’s problem is

$$\pi_d^R = E\left[(1 - \alpha)\min(D, Q) + \lambda\alpha\min(D, Q) + v(Q - D)^+ + v\alpha\min(D, Q) - w_T Q\right]$$

$$= (1 - \alpha + \lambda\alpha - v + v\alpha)\int_0^Q F(D) dD + vQ - w_TQ$$

Taking the first-order derivative of $\pi_d^R$ on $Q$, we obtain $\frac{d\pi_d^R}{dQ} = (1 - \alpha + \lambda\alpha - v + v\alpha)F(Q) + v - w_T$. Let $\frac{d\pi_d^R}{dQ} = 0$, we get the retailer’s optimal order quantity $Q_1^*$ satisfies $Q_1^* = F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda\alpha - v + v\alpha})$.

Furthermore, we have $\left.\frac{d^2\pi_d^R}{dQ^2}\right|_{Q=Q_1^*} = -(1 - \alpha + \lambda\alpha - v + v\alpha)f(Q_1^*) < 0$.

Therefore, the retailer’s optimal order quantity in a decentralized supply chain is given by $Q_d^* = Q_1^* = F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda\alpha - v + v\alpha})$ and is unique.

Case 2: $B < (w_T - v)F^{-1}(\frac{w_T - v}{1 - \alpha + \lambda\alpha - v + v\alpha})$

In this case, the retailer has a relatively low initial capital level and may not be able to repay the trade credit at the end of the sales period. So, the retailer’s problem is written as:
\[
\pi_d^R = E[ (1 - \alpha) \min(D, Q) + \lambda \alpha \min(D, Q) + v(Q - D)^+ + v\alpha \min(D, Q) + B - w_T Q]^+ - B
\]

\[
= (1 - \alpha + \lambda \alpha - v + v\alpha) \int_{(1-a+\lambda a-v+va)}^{Q} \bar{F}(D) dD - B
\]

Then, we can have \( \frac{d\pi_d^R}{dQ} = (1 - \alpha + \lambda \alpha - v + v\alpha) F(Q) - (w_T - v) F \left( \frac{w_T Q - B - vQ}{1 - \alpha + \lambda \alpha - v + va} \right) \).

Let \( \frac{d\pi_d^R}{dQ} = 0 \), we obtain that the retailer’s optimal order quantity \( Q_2 \) satisfies

\[
\bar{F}(Q_2) = \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} \left( \frac{w_T Q_2 - B - vQ_2}{w_T Q_2 - B - vQ_2} \right). \]

We further get,

\[
\left. \frac{d^2 \pi_d^R}{dQ^2} \right|_{Q=Q_2} = - (1 - \alpha + \lambda \alpha - v + v\alpha) f(Q_2) + \frac{(w_T - v)^2}{1 - \alpha + \lambda \alpha - v + va} \left( \frac{w_T Q_2 - B - vQ_2}{1 - \alpha + \lambda \alpha - v + va} \right)
\]

\[
\times \left[ -(1 - \alpha + \lambda \alpha - v + va) h(Q_2) + (w_T - v) h \left( \frac{w_T Q_2 - B - vQ_2}{1 - \alpha + \lambda \alpha - v + va} \right) \right]
\]

According to \( (1 - \alpha) + \alpha(\lambda + v) > w_T \) and the IFR assumption, we can get \( \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} < 1 \),

\[
h \left( \frac{w_T Q_2 - B - vQ_2}{1 - \alpha + \lambda \alpha - v + va} \right) < h(Q_2), \text{ then } \left. \frac{d^2 \pi_d^R}{dQ^2} \right|_{Q=Q_2} < 0.
\]

So, the optimal order quantity of the retailer in the decentralized supply chain \( Q_d = Q_2 \) is unique and satisfies

\[
\bar{F}(Q_2) = \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} \left( \frac{w_T Q_2 - B - vQ_2}{w_T Q_2 - B - vQ_2} \right).
\]

**Proof of corollary 1**

From P2, we can have \( \bar{F}(Q_2) = \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} \left( \frac{w_T Q_2 - vQ_2 - B}{w_T Q_2 - vQ_2 - B} \right) \) and \( \bar{F}(Q_1) = \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} \)

\( \text{thus } \bar{F}(Q_2) < \bar{F}(Q_1), Q_1 < Q_2. \)

Taking the derivative with respect to \( B \) on both sides of equation (4), we have

\[
-f(Q_2) \frac{dQ_2}{dB} = - \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} f \left( \frac{w_T Q_2 - vQ_2 - B}{1 - \alpha + \lambda \alpha - v + va} \right) \left[ \frac{(w_T - v) \frac{dQ_2}{dB} - 1}{1 - \alpha + \lambda \alpha - v + va} \right].
\]

Rearranging it leads to

\[
\frac{dQ_2}{dB} = - \frac{1}{h(Q_2) - \frac{w_T - v}{1 - \alpha + \lambda \alpha - v + va} h \left( \frac{w_T Q_2 - vQ_2 - B}{1 - \alpha + \lambda \alpha - v + va} \right)} \left[ \frac{(w_T - v) \frac{dQ_2}{dB} - 1}{1 - \alpha + \lambda \alpha - v + va} \right] < 0.
\]

Thus, \( Q_2 \) decreases in \( B \).
Proof of lemma 2
Let \( I(x) = xF(x) \), then \( \frac{dI(x)}{dx} = F(x) [1 - xh(x)] \). If \( xh(x) > 1 \), \( \frac{dI(x)}{dx} < 0 \), \( I(x) \) decreases in \( x \).

Under our circumstance, if we assume \( k(Q) * h(k(Q)) > 1 \), \( I'(k(Q)) < 0 \). So, \( I(Q) < I(k(Q)) \), that is \( QF(Q) < k(Q)F(k(Q)) = \frac{wT - v - vF}{1 - \alpha + \lambda \alpha - v + v\alpha} F \left( \frac{wT - B - vQ}{1 - \alpha + \lambda \alpha - v + v\alpha} \right) \),

which is not consistent with the fact that \( QF(Q) = \frac{wT - vQ}{1 - \alpha + \lambda \alpha - v + v\alpha} F \left( \frac{wT - B - vQ}{1 - \alpha + \lambda \alpha - v + v\alpha} \right) \). Thus, we must obtain \( k(Q) * h(k(Q)) < 1 \).

Proof of corollary 2
Based on the retailer’s initial capital, we investigate the following two cases.

Case 1: \( B \geq (wT - v)F^{-1} \left( \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} \right) \).

From \( P2 \), we get that the retailer’s optimal order quantity satisfies \( F(Q^*_1) = \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} \).

Then, \( \frac{dQ^*_1}{d\alpha} = \frac{(wT - v)\alpha}{(1 - \alpha + \lambda \alpha - v + v\alpha)F'(Q^*_1)} > 0 \), i.e. \( Q^*_1 \) increases in \( \lambda \).

In addition, from the proof of \( P2 \), we can obtain the retailer’s problem as:

\[
\pi^R_d = E \left[ (1 - \alpha)\min(D, Q) + \lambda \alpha \min(D, Q) + v(Q - D)^+ + v\alpha \min(D, Q) - wTQ \right]
\]

\[
= (1 - \alpha + \lambda \alpha - v + v\alpha) \int_0^Q F(D)dD + vQ - wTQ
\]

Put \( Q^*_1 \) into the above equation to get the retailer’s optimal profit as follows:

\[
\pi^R_d = (1 - \alpha + \lambda \alpha - v + v\alpha) \int_0^{Q^*_1} F(D)dD + vQ^*_1 - wTQ^*_1
\]

\[
\frac{\partial \pi^R_d(\lambda)}{\partial \alpha} = \frac{\partial \pi^R_d(\lambda, Q^*_1)}{\partial Q^*_1} \frac{\partial Q^*_1}{\partial \alpha} + \frac{\partial \pi^R_d(\lambda, Q^*_1)}{\partial \lambda} .
\]

i.e.,

\[
\frac{\partial \pi^R_d(\lambda)}{\partial \alpha} = \left[ (1 - \alpha + \lambda \alpha - v + v\alpha)F(Q^*_1) + v - wT \right] \frac{dQ^*_1}{d\alpha} + \alpha > 0
\]

Case 2: \( B < (wT - v)F^{-1} \left( \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} \right) \).

Based on \( P2 \), we have \( F(Q^*_2) = \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} F \left( \frac{wT - vQ^*_2 - B}{1 - \alpha + \lambda \alpha - v + v\alpha} \right) \).

Taking derivative with respect to \( \lambda \) on both sides of the equation and simplifying it, we obtain

\[
\frac{dQ^*_2}{d\alpha} = \frac{(wT - v)\alpha}{(1 - \alpha + \lambda \alpha - v + v\alpha)F(k(Q^*_2)) - \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} f(k(Q^*_2)) \frac{k(Q^*_2)\alpha}{1 - \alpha + \lambda \alpha - v + v\alpha}}
\]

\[
= \frac{1 - k(Q^*_2)h(k(Q^*_2))}{(1 - \alpha + \lambda \alpha - v + v\alpha)h(Q^*_2) - (wT - v)h(k(Q^*_2))}
\]

Since \( k(Q) * h(k(Q)) < 1 \) (Lemma 2), \( h(k(Q)) < h(Q) \) (IFR), then we obtain \( \frac{dQ^*_2}{d\alpha} > 0 \).
Next, according to the proof of P2, we get the retailer’s optimal problem as:

$$\pi_d^R = E[(1-\alpha)\min(D,Q) + \lambda \alpha \min(D,Q) + v(Q - D^+) + \lambda \alpha \min(D,Q) + B - wTQ]^+ - B$$

$(1-\alpha + \lambda \alpha - v + v\alpha) \int_{wT-v<q<2} F(D)dD - B$

Substituting $Q = Q_2$ into the above equation, the retailer’s optimal profit is rewritten as:

$$\pi_d^{R*} = (1-\alpha + \lambda \alpha - v + v\alpha) \int_{wT-v<q<2} F(D)dD - B \cdot \frac{\partial \pi_d^{R*}(\lambda)}{\partial \lambda} = \frac{\partial \pi_d^{R*}(\lambda, Q_2)}{\partial Q_2} \frac{\partial Q_2}{\partial \lambda} + \frac{\partial \pi_d^{R*}(\lambda, Q_2)}{\partial \lambda}$$

$$\frac{\partial \pi_d^{R*}(\lambda)}{\partial \lambda} = (1-\alpha + \lambda \alpha - v + v\alpha) \left[ F(Q_2) - \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha} F(k(Q_2)) \right] \frac{\partial Q_2}{\partial \lambda}$$

$$+ \alpha \int_{k(Q_2)}^{Q_2} F(D)dD + ak(Q_2)F(k(Q_2))$$

$$= \alpha \int_{k(Q_2)}^{Q_2} F(D)dD + ak(Q_2)F(k(Q_2))$$

$$> \alpha (Q_2^* - k(Q_2))F(Q_2) + ak(Q_2)F(Q_2) = \alpha Q_2^*F(Q_2) > 0$$

Thus, both the retailer’s optimal profit and its optimal order quantity increase in $\lambda$ under a decentralized supply chain.

**Proof of corollary 3**

According to P2, when $cB \geq (wT - v)F^{-1}(\frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha})$, the retailer’s optimal order quantity satisfies $F(Q_1) = \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha}$. Then, we have $\frac{dQ_1}{d\alpha} = \frac{(wT-v)(\lambda + v - 1)}{(1-\alpha + \lambda \alpha - v + v\alpha)^2}$. Therefore, based on $0 \leq v < c < wT < p = 1$ and $\lambda + v < 1$, we obtain $\frac{dQ_1}{d\alpha} < 0$.

When $B < (wT - v)F^{-1}(\frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha})$, we obtain $F(Q_2) = \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha} F\left(\frac{wT-vQ_2}{1-\alpha + \lambda \alpha - v + v\alpha} - B\right)$.

Taking derivative with respect to $\alpha$ on both sides of the above equation, we get

$$-f(Q_2^*) \frac{dQ_2^*}{d\alpha} = \frac{(wT-v)(\lambda + v - 1)}{(1-\alpha + \lambda \alpha - v + v\alpha)^2} F(k(Q_2^*))$$

$$- \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha} f(k(Q_2^*)) \left[ \frac{(wT-v)(\lambda + v - 1)}{(1-\alpha + \lambda \alpha - v + v\alpha)^2} \right]$$

Make some transposition about the above equation, then we have the following equation,

$$\frac{dQ_2^*}{d\alpha} = \frac{(\lambda + v - 1)}{h(Q_2^*) \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha}} \frac{[1-H(k(Q_2^*))]}{h(k(Q_2^*))}$$

For $0 < \frac{wT-v}{1-\alpha + \lambda \alpha - v + v\alpha} < 1$, $\lambda + v < 1$, $H(k(Q_2^*)) < 1$ (Lemma 2), $h(k(Q_2^*)) < h(Q_2^*)$ (IFR), so $\frac{dQ_2^*}{d\alpha} < 0$. That is, $Q_2^*$ decreases in $\alpha$. 

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**Proof of proposition 3**

From $0 \leq v < c < w_T < b = 1$ and $(1 - \alpha) + \alpha(\lambda + v) > w_T$, we have $\Lambda = \max\left(\frac{w_T - v\alpha + \alpha - 1}{\alpha}, 0\right)$.

What’s more, according to $\lambda + v < 1$ and $c + \lambda \alpha < w_T$, we obtain $\Lambda = \min\left(1 - v, \frac{w_T - c}{\alpha}\right)$.

From equation (3), we can obtain the following first-order derivative:

$$
\frac{d\pi_s}{d\lambda} = \left[w_T - c - \lambda \alpha + \lambda \alpha F(Q_1^*)\right] \frac{\partial Q_1^*}{\partial \lambda} - \alpha Q_1 + \alpha \int_0^{Q_1^*} F(D)dD
$$

Note that from the proof of Corollary 2, $\frac{dQ_1^*}{d\lambda} = \frac{Q_1^* - (w_T - v)/\alpha}{\alpha(1 + \lambda \alpha + v\alpha)}$.

Then, we have

$$
\frac{d\pi_s}{d\lambda} = \left[w_T - c - \lambda \alpha F(Q_1^*) - (1 - \alpha + \lambda \alpha + v\alpha - v)h(Q_1^*)\int_0^{Q_1^*} F(D)dD\right] \frac{\partial Q_1^*}{\partial \lambda}
$$

Take $L(\lambda) = w_T - c - \lambda \alpha F(Q_1^*) - (1 - \alpha + \lambda \alpha + v\alpha - v)h(Q_1^*)\int_0^{Q_1^*} F(D)dD$ to get $\frac{d\pi_s}{d\lambda} = L(\lambda) \frac{dQ_1^*}{d\lambda}$.

Based on the proof of Corollary 2, we get $\frac{dQ_1^*}{d\lambda} > 0$, $Q_1^*$ increases in $\lambda$. Therefore, $h(Q_1^*)$ and $\int_0^{Q_1^*} F(D)dD$ increase in $\lambda$ as well. In addition, it is intuitive that $(1 - \alpha + \lambda \alpha + v\alpha - v)h(Q_1^*)\int_0^{Q_1^*} F(D)dD$ which is the last term of $L(\lambda)$ increases in $\lambda$.

Let $\eta(\lambda) = \lambda \alpha F(Q_1^*)$. In $P2$, we have $F(Q_1^*) = \frac{w_T - v}{1 - \alpha + \lambda \alpha + v\alpha}$ and according to the proof of Corollary 2, $\frac{dQ_1^*}{d\lambda} = \frac{w_T - v}{(1 - \alpha + \lambda \alpha + v\alpha)}$, i.e.

$$
\frac{d\eta(\lambda)}{d\lambda} = \alpha F(Q_1^*) \frac{dQ_1^*}{d\lambda}.
$$

Thus,

$$
\frac{d\eta(\lambda)}{d\lambda} = \alpha(1 - \alpha + \lambda \alpha + v\alpha)(1 - \frac{\Lambda}{1 - \alpha + \lambda \alpha - v + v\alpha})
$$

What’s more, $1 - \alpha - v + \lambda \alpha + v\alpha = (1 - \alpha)(1 - v) + \lambda \alpha > 0$, so $\frac{\Lambda}{1 - \alpha + \lambda \alpha - v + v\alpha} < 1$, then $\frac{d\eta(\lambda)}{d\lambda} > 0$, i.e. $\eta(\lambda)$ increases in $\lambda$.

From the above, $L(\lambda) = w_T - c - \lambda \alpha F(Q_1^*) - (1 - \alpha + \lambda \alpha + v\alpha - v)h(Q_1^*)\int_0^{Q_1^*} F(D)dD$ decreases in $\lambda$.

Next, we will discuss the following three different cases:

1. **Case 1:** $L(\bar{\lambda}) > 0$. In this case, $L(\Lambda) > L(\lambda) > L(\bar{\lambda}) > 0$. In addition, referring to the proof of Corollary 2, we have $\frac{dQ_1^*}{d\lambda} > 0$. Therefore, we can show that $\frac{d\pi_s}{d\lambda} > 0$ and the supplier’s optimal subsidy level is $\lambda^* = \bar{\lambda}$.

2. **Case 2:** $L(\Lambda) < 0$. In this case, $L(\bar{\lambda}) < L(\lambda) < L(\Lambda) < 0$, then $\frac{d\pi_s}{d\lambda} < 0$, and the supplier’s optimal subsidy level is $\lambda^* = \Lambda$.

3. **Case 3:** $L(\Lambda) > 0, L(\bar{\lambda}) < 0$. In this case, $L(\Lambda)|_{L(\Lambda) > 0} > L(\lambda) > L(\bar{\lambda})|_{L(\bar{\lambda}) < 0} > 0$. We can show that $\frac{d\pi_s}{d\lambda}|_{\lambda = \Lambda} > 0, \frac{d\pi_s}{d\lambda}|_{\lambda = \bar{\lambda}} < 0$. So, we can show that there is a unique $\hat{\lambda}$ which satisfies $\frac{d\pi_s}{d\lambda}|_{\lambda = \hat{\lambda}} = 0$. Then, the supplier’s optimal subsidy level is $\lambda^* = \hat{\lambda}$.
Proof of lemma 3
According to the proof of $P2$, when the retailer has a relatively low initial capital level, \( \pi^R = (1 - \alpha + \lambda \alpha - v + v\alpha) \int_{wT_0 - \beta - Q}^{Q} F(D) dD - B \). To add this equation to equation (5), we have
\[
\pi_R + \pi_S = (1 - \alpha + \lambda \alpha - v + v\alpha) \int_{wT_0 - \beta - Q}^{Q} F(D) dD - B
\]
\[
- (1 - \alpha + \lambda \alpha - v + v\alpha) \int_{wT_0 - \beta - Q}^{Q} F(D) dD + \lambda \alpha \int_{0}^{Q} F(D) dD + wTQ
\]
\[
- \lambda \alpha Q - cQ
\]
\[
= (1 - \alpha - v + v\alpha) \int_{0}^{Q} F(D) dD + (v - c)Q = \pi_C
\]
That is, \( \pi_R(Q, \lambda) + \pi_S(Q, \lambda) = \pi_C(Q) \). From this equation, we find the total profit of the decentralized supply chain is independent of \( \lambda \). Furthermore, referring to the proof of $P1$, we can further show that \( \pi_C(Q) \) is concave in \( Q \). Therefore, the total profit is concave in \( Q \) and independent of \( \lambda \).

Proof of proposition 4
By Corollary 2, we can conclude that \( Q^*_2 \) increases in \( \lambda \), and through the proof of Lemma 3 we get \( Q^*_C \) is independent of \( \lambda \). Then, we draw three diagrams, in which each diagram is corresponding to each case in $P4$.

Figure 7 corresponds to the case 1 of $P4$, in which \( Q^*_2 > Q^*_C \) during \( [\underline{\lambda}, \bar{\lambda}] \). Combined with the analysis above $P4$ in Section 5, the supplier will not provide subsidy level more than \( \bar{\lambda} \). However, taking into account the feasible region of the supplier's optimal subsidy level \( [\underline{\lambda}, \bar{\lambda}] \) and \( \hat{\lambda} \leq \underline{\lambda} \), the supplier's optimal decision would be set his subsidy level at the lower bound of \( [\underline{\lambda}, \bar{\lambda}] \), that is \( \lambda^* = \underline{\lambda} \).

Similar to Figure 7, Figure 8 corresponds to the case 2 of $P4$. We can find that in the interval \( [\underline{\lambda}, \bar{\lambda}] \), \( Q^*_2 < Q^*_C \) at first, then \( Q^*_2 > Q^*_C \) and \( Q^*_2 = Q^*_C \) at the point \( \lambda = \bar{\lambda} \). As the supplier is not willing to provide subsidy level more than \( \bar{\lambda} \), so the rational supplier will reduce the feasible region to \( [\underline{\lambda}, \lambda^*] \).

Similarly, Figure 9 corresponds to the case 3 of $P4$. We can observe that \( Q^*_2 < Q^*_C \) among \( [\underline{\lambda}, \bar{\lambda}] \). Under this circumstance, \( \lambda^* \geq \bar{\lambda} \), as for the rational supplier who will not provide subsidy level more than \( \bar{\lambda} \), he will still choose \( [\underline{\lambda}, \bar{\lambda}] \) as the feasible region and \( \bar{\lambda} \) as the upper bound of the feasible region.

Proof of lemma 4
We denote \( Q^*_N \) as the retailer's optimal order quantity when \( B = 0 \), then we can get
\[
\bar{F}(Q_N) = \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} \bar{F}(\frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha}) Q_N \]from $P2$. Taking derivative with respect to \( \lambda \) on both sides of this equation, we have
\[
\frac{dQ_N}{d\lambda} = \frac{\frac{\partial}{\partial \lambda} \bar{F}(\frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha}) Q_N}{\frac{\partial}{\partial \lambda} \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha}} = \frac{\beta}{\beta \cdot \beta Q^2 - \beta Q} < 0
\]
in which we assume \( \beta = \frac{wT - v}{1 - \alpha + \lambda \alpha - v + v\alpha} \) and \( \beta' = \frac{\partial}{\partial \lambda} = \frac{- (wT - v) \alpha}{(1 - \alpha + \lambda \alpha - v + v\alpha)^2} < 0 \).
We define a function \( g(Q) = Q^N \), as illustrated in Figure 10. Then, we can get 
\[
\frac{dg(Q)}{dQ} = F(Q) \left( 1 - Q^N F(Q) \right) = F(Q) (1 - H(Q)).
\]
Because \( D \) meets the assumption of IGFR, so we can show that \( g(Q) \) is a unimodal function during \( Q \in (D, T) \). There exists a unique \( \hat{Q} \) satisfies \( H(\hat{Q}) = 1 \), because of \( H(D) = 0 \) and \( H(T) = \lim_{D \to T} H(T) = \infty \). Therefore, \( g(Q) \) increases in \( Q \in (D, \hat{Q}) \) and decreases in \( Q \in (\hat{Q}, T) \), just as shown in Figure 10. According to the first order condition
\[
F(Q_N) = \frac{w_T - v}{1 - \alpha + \lambda a - v + v\alpha} F \left( \frac{w_T - v}{1 - \alpha + \lambda a - v + v\alpha} Q_N \right),
\]
we can get \( g(\beta Q_N) = g(Q_N^*) \). Based on \((1 - \alpha) + \alpha(\lambda + v) > w_T\) we can get \( \beta = \frac{w_T - v}{1 - \alpha + \lambda a - v + v\alpha} < 1 \), \( \beta Q_N^* < Q_N^* \), then \( \beta Q_N^* \in [D, \hat{Q}] \) and \( Q_N^* \in (\hat{Q}, T) \), so we must have \( 1 - H(Q_N) < 0 \) and \( 1 - H(\beta Q_N^*) > 0 \). In addition, through the assumption of IFR, we have \( h(\beta Q_N^*) < h(Q_N^*) \). From the above, we obtain \( \frac{dQ_N^*}{d\alpha} > 0 \).

Bring \( Q_N^* \) to equation (8) and simplify it, we have
\[
\pi_S = - (1 - \alpha - v + v\alpha) \int_{Q_N^*}^{Q_N} F(D) dD - (1 - \alpha + \lambda a - v + v\alpha) F(Q_N) F(D) dD - (\lambda a - w_T) Q_N^* - cQ_N^*.
\]
Taking derivative with respect to \( \alpha \) on both sides of this equation, then
\[
\frac{d\pi_S}{d\alpha} = \frac{\partial Q_N^*}{\partial \alpha} - \alpha Q_N^* \beta - \alpha Q_N^* F(Q_N) - \alpha \int_{Q_N^*}^{Q_N} F(D) dD
\]
\[
= \frac{\partial Q_N^*}{\partial \alpha} \left\{ \left[ (1 - \alpha + \lambda \alpha - v + v\alpha) F(Q_N) + (1 - v)(1 - \alpha) F(Q_N) \right. \right.
\]
\[
+ ((1 - \alpha + \lambda a - v + v\alpha) + \lambda \alpha - w_T + c)
\]
\[
+ \frac{1 - \alpha + \lambda \alpha - v + v\alpha}{Q_N^*} \int_{Q_N^*}^{Q_N} F(D) dD \left[ H(\beta Q_N) - 1 \right]
\]
\[
\left. + \left[ (1 - \alpha + \lambda \alpha - v + v\alpha) F(Q_N) + (w_T - v) \right. \right.
\]
\[
+ \frac{1 - \alpha + \lambda \alpha - v + v\alpha}{Q_N^*} \int_{Q_N^*}^{Q_N} F(D) dD \right\} \left[ 1 - H(Q_N^*) \right] / \left[ 1 - H(\beta Q_N^*) \right] < 0
\]
Hence, the supplier’s expected profit decreases in \( \lambda \).

**Proof of proposition 5**

From Lemma 4, the supplier’s expected profit decreases in \( \lambda \), so we can deduce that the supplier will set its optimal subsidy level of per unit returned product \( \lambda_N^* \) as \( \lambda = \max \left( \frac{w_T - v + v\alpha - a}{a}, 0 \right) \).

If \( \alpha \geq \frac{1 - w_T}{1 - v\alpha} \), then \( \frac{w_T - v + v\alpha - a}{a} \geq 0 \), \( \lambda = \frac{w_T - v + v\alpha - a}{a} \), so we can get \( \lambda_N^* = \frac{w_T - v + v\alpha - a}{a} \).

If \( \alpha < \frac{1 - w_T}{1 - v\alpha} \), then \( \frac{w_T - v + v\alpha - a}{a} < 0 \), \( \lambda = 0 \), so the optimal subsidy level \( \lambda_N^* = 0 \).
Proof of proposition 6

Part (i): For the optimal subsidy level \( \lambda^* \in [\underline{\lambda}, \overline{\lambda}] \), we can get \( c + \lambda^* \alpha < w_T < (1-\alpha) + \alpha(\lambda^* + v) \), then \( w_T - c > \lambda^* \alpha \) and \( 1-\alpha + \alpha v > c - v \). In addition, when \( B \geq (w_T - v)F^{-1}\left(\frac{w_T - v}{1-\alpha + \lambda^* \alpha + v}\right) \), from \( P1 \) and \( P2 \), we obtain \( \lambda^* \alpha < w_T - c \) and \( 1-\alpha + \alpha v - v > c - v \). In addition, when \( B/C_1 \leq w_T/C_0 (\lambda^* \alpha + v) \), then \( \lambda^* \alpha < w_T - c \) and \( 1-\alpha + \alpha v - v > c - v \). Further, we simplify the equation to get \( \lambda^* \alpha = w_T - c \) and \( 1-\alpha + \alpha v - v > c - v \) respectively.

Comparing \( Q_1 < Q_C \), and the supply chain cannot be coordinated under this case.

Part (ii): When \( 0 \leq B < (w_T - v)F^{-1}\left(\frac{w_T - v}{1-\alpha + \lambda^* \alpha + v}\right) \), through \( P2 \) we observe \( Q_2 \) that may be equal to the coordination order quantity \( Q_C \). Therefore, if the postponed wholesale price allows the equation \( Q_2 = Q_C \) holds, which is implied by \( \lambda^* \alpha < w_T - c \) and \( 1-\alpha + \alpha v - v > c - v \). Further, we simplify the equation to get \( \lambda^* \alpha = w_T - c \) and \( 1-\alpha + \alpha v - v > c - v \) respectively. Then the proposed supplier subsidy contract can coordinate the capital-constrained supply chain with customer returns.

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