Economic growth, poverty traps and cycles: productive capacities versus inefficiencies

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Abstract

Purpose – The authors analyse a growth model to explain how economic fluctuations are primarily driven by productive capacities (i.e. capacity utilization driven by innovations and know-how) and productive inefficiencies.

Design/methodology/approach – This study's methodology consists of the combination of the economic growth model, à la Solow–Swan, with a sigmoidal production function (in capital), which may explain growth, poverty traps or fluctuations depending on the relative levels of inefficiencies, productive capacities or lack of know-how.

Findings – The authors show that economies may experience economic growth, poverty traps and/or fluctuations (i.e. cycles). Economic growth is reached when an economy experiences both a low level of inefficiencies and a high level of productive capacities while an economy falls into a poverty trap when there is a high level of inefficiencies in production. Instead, the economy gets in cycles when there is a large level of the lack of know-how and low levels of productive capacity.

Originality/value – The authors conclude that more capital per capita (greater savings and investment) and greater productive capacity (with less lack of know-how) are the economic policy keys for an economy being on the path of sustained economic growth.

Keywords Economic growth modelling, Long waves, Low-level equilibrium and traps, Endogenous fluctuations

Paper type Research paper

JEL Classification — C61, D24, E32, O40

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1. Introduction

The Schumpeter (1939, 1950) notion of “long waves” stresses the importance of innovations in order to avoid fluctuations (cycles) in the economy. It refers to the key role of entrepreneurs who, with their own capabilities or know-how and initiative, create new opportunities for innovations investment in economic growth and employment. Therefore, the decisive factor that fosters innovation, leading to rapid growth, is the profit derived from the innovations provided by the market, i.e. the development of productive capacities and endogenous innovations in firms’ production functions (Lafuente et al., 2020; UNCTAD, 2020). Hence, this paper aims to analyse how economic cycle fluctuations may be very different and influenced by a variety of reasons, such as productive capacities (capacity utilization and know-how), and/or technological progress and innovations, while facing production inefficiencies.

To this aim, we start by taking into account the Solow (1956) and Swan (1956) seminal models for economic growth. By using a strictly concave and constant returns to scale one-sector production function, the Solow–Swan model describes how the evolution of capital stock, labour force and advances in technology interact and affect economic growth. Keeping aside tractability, the main reason why neoclassical production functions became the canonical way of representing technology in macro models is that they generate predictions that are consistent with the Kaldor’s facts (Jones and Romer, 2010). However, some of these facts could be challenged and, hence, it could be argued that an alternative production function, once it is introduced in the Solow model, would generate predictions that might better explain the data. This is what firstly motivates the present research paper.

Indeed, there are situations in which production cannot be described by a strictly concave production function (Jones, 2005; Wu, 2010; Dupuy, 2012; Growiec, 2013). One intuitive example is the case of a firm developing a product that is initially sold in the absence of competition, generating increasing marginal returns. After a while, however, other firms may introduce similar goods affecting the sales of the original product, whose market share shrinks. To face changes in market conditions, and the increasing competitive pressures, economies should have a clear vision of strategies based on the use of new technologies, the introduction of quality requirements, product differentiation, and therefore in their capacity utilization and know-how in manufacturing (Balk, 2001; Christopoulos and McAdam, 2019) [1]. So we may obtain increasing returns to scale when dealing with an economy where capital accumulation under certain circumstances coincides with improved factor reallocation across firms (as in Hsieh and Klenow, 2009) or with higher capacity utilization (Wen, 1998).

Hence, in this research paper our first goal is to propose a production function that represents the various stages of production that economies may face. We present a production function that is no longer strictly concave and may imply a convex-concave technology [2]. A nonconcave production function is also related to the presence of poverty traps or low-level equilibrium, as discussed by Capasso et al. (2012), Grassetti and Hunanyan (2018), and Grassetti et al. (2018a, b) [3]. Thus, the aggregate production function allows us to represent the evolution of a given economy by means of three different stages: a first stage in which returns are decreasing, as they are constrained by certain assets being fixed (e.g. land); a second stage, in which the country begins to industrialize, with increasing marginal returns; and a final stage where such marginal returns become either constant or even decreasing. Within this framework, we analyse the role of the capacity utilization and of measures of inefficiencies on the growth process.

We claim that the deviation from strict concavity of the production function is important to explain economic growth theory. We propose a sigmoidal (convex-concave) production technology, modified to take into consideration capacity utilization (i.e. productive capacity development) versus inefficiencies in production. The former is assumed to be related to the level of know-how within the firm: as capacity utilization represents the relationship between the output produced with the given resources and the potential output that can be produced if
capacity was fully used, we assume that the larger the level of expertise within the firm the larger the capacity utilization. Thus, by means of this novel production function, we allow production to increase both thanks to an exogenous technical change (that we assume Hicks neutral) and to the level of know-how that directly affects capacity utilization. The latter inefficiencies depend on waste of inputs, market and institutional rigidities that affect firm production (see for instance, Bogetoft and Hougaard, 2003). That is, a firm that operates within markets and institutions characterized by high levels of inefficiencies, uses more resources and factor inputs than required by a given technology, thus tying resources to low-productivity activities and reducing the overall allocative efficiency of an economy. Hence, understanding the effect of capacity utilization, know-how, and inefficiencies in production is necessary for an economy to avoid poverty traps and cycles, and to reach a sustained economic growth.

In this vein, we claim that the trajectory of economic growth is determined by capital per capita dynamics driven by productive capacities (for example firms’ ability to operate on the frontier of the production possibility set: Arrow et al. (1961), Debreu (1951) and Farrell (1957), and more recently Kerstens et al. (2019) [4]. Within this framework, we wonder what happens in the presence of inefficiencies in the production process so that economies operate within their own technological constraint, rather than on its frontier.

Now, from the empirical point of view, this research article is motivated by facts observed from the data of the Productive Capacities Index (PCI) developed by United Nations Conference on Trade and Development (UNCTAD) which show that differences in socioeconomic development across countries and regions are a consequence of gaps in their productive capacities. The UNCTAD has developed the PCI index to measure how productive resources, entrepreneurial capacities and production interact and determine the capacity of an economy to produce goods and services and to grow.

The PCI covers 193 economies for the period 2000–2018, and the set of productive capacities and their specific combinations are mapped across 46 indicators. The PCI is multidimensional, and its components are human capital, natural capital, energy, transport, ICT, institutions, private sector and structural change. PCI scores and GDP per capita levels are closely intertwined, as there is a highly positive correlation coefficient between PCI and GDP per capita (0.91). This correlation demonstrates the close relationship between productive capacities and GDP overall, thereby propelling a rise in GDP per capita. Indeed, on the other side, the productive capacities help economies to avoid poverty traps [5]. Structurally weak and vulnerable economies, including the least developed countries (LDCs) and landlocked developing countries (LLDCs) perform particularly poorly on PCI (see Figure 1).

Nevertheless, other empirical evidence shows that there is a loss of efficiency and therefore falls in firms productivity, and that the dynamics of productive capacity, innovation and economic performance may be shaped by interconnections and complex relationships between the ability of economies to turn out R&D efforts into successful innovations, and the commitment of economies to invest profits in further technological efforts (see, for instance, Bogliacino and Pianta, 2013). For example, researchers argue that the large capital inflows that southern Europe received during the first decade of the euro have mostly been captured by low-productivity firms, depressing aggregate productivity through a composition effect (Gopinath et al., 2017; García-Santana et al., 2019). However, inefficient management practices have also kept southern European firms from taking full advantage of the IT revolution (Bloom et al., 2012; Pellegrino and Zingales, 2017). An extensive empirical literature documents that IT adoption requires changes in a firm’s organization (Brynjolfsson and Hitt, 2000), and that it induces higher productivity gains in better-managed firms (Garicano and Heaton, 2010; Bloom et al., 2012), because management practices and IT are complements.

Thus, in line with these stylized facts, our model claims that the greater the promotion of productive capacities (the lower the productive inefficiencies) the greater the economic
growth. Within the model dynamics we show that poverty traps could arise in the presence of high productive inefficiencies and low economic growth. Furthermore, the model accounts for complex dynamics characterized by cycles (for the relation between production technologies and cycles see Growiec et al., 2018) when productive capacities and inefficiencies are not sustained in terms of the per capita capital that the economies possess.

Taking into account all these considerations, the contribution of this research paper is to show the dynamic foundations for getting either economic growth, economic fluctuations

**Figure 1.**
The productive capacities index (PCI)

**Source(s):** UNCTAD (2021)
(cycles) or poverty traps, when a convex-concave production function is affected by capacity utilization (driven by know-how) and market and (or) institutional inefficiencies [6]. The concept of capacity utilization has a strong connotation with a production process involving a fairly clear notion of a firm’s full capacity where know-how and capital endowments are crucial determinants for its measurement. Therefore, we present a model in which we internalize both the productive capacities driven by capacity utilization and the level of knowledge in the production process and there are also inefficiencies within this process.

The remainder of the paper is organized as follows. Section 2 develops the model discussing its main properties. Section 3 shows the results on economic growth and poverty traps, while Section 4 discusses the cycles’ results. Section 5 discusses some economic policy recommendations, while Section 6 concludes.

2. Model setup
Solow’s seminal model assumes a supply of goods based on a production function with constant returns to scale. Labor grows at a constant rate, the level of technology is constant over time, the saving rate is constant and capital depreciates at a positive constant rate, that is, at each point in time, a constant fraction of the capital stocks can no longer be used for production. Within the model, the capital stock is a key determinant of the economy’s output, and changes in the capital stock affect economic growth. Although the model predicts the possibility of multiple equilibria, the main findings can be summarized in two stationary states of the economy: one characterized by zero production and capital per capita; the other is achieved as a result of the way the production function is constructed. The decisive assumption concerns the marginal productivity of capital that would tend to zero when capital tends to infinity. Such an assumption assures global stability towards the output-per-person stationary state (Azariadis and Kaas, 2007).

Within this framework, theoretical and empirical analyses often describe aggregate production by means of the Cobb–Douglas production function. While this assumption has the advantage of allowing the economy convergence to a unique steady state, these models are not able to explain neither the existence of cycles in economies nor the differences between countries that grow over years and those that are stuck in poverty traps. In fact, Jones (2005) claims that the shape of the aggregate production function plays a crucial role for the analysis of development economics, showing that a Pareto distribution may drive technological innovation.

Indeed, in order to analyse equilibria and cycles in economic growth, previous literature shows that a model must assume a concave-convex production function (Skiba, 1978; Askenazy and Le Van, 1999; Hung et al., 2009). In this line, in what follows we model an augmented convex-concave production function to analyse and understand the impact on growth dynamics of productive capacity (capacity utilization) and technological inefficiencies.

Let us consider an economy in which a large number of identical firms exist. The representative firm produces per capita output, \( y = Y/L = f(K/L, L/L) = f(k, 1) \geq 0 \), according to the production law given by a sigmoid functional form:

\[
y = f(k \mid A, \beta, \sigma) := \frac{A}{1 + e^{\beta - \sigma k}}. \tag{1}
\]

where \( A > 0 \) is the total output productivity (i.e. Hicks neutral technical change), which depends on technology, and \( k \geq 0 \) stands for per capita capital. Notice that the production function is augmented by the parameter \( \beta \in (0, 1) \) representing inefficiencies in production,
and $\sigma \in (0, 1)$ that stands for a coefficient of productive capacities or capacity utilization. In the model, the larger $\beta$ the larger the production inefficiencies, the larger $\sigma$ the larger the capacity utilization. It is also worth noting that the technical change, captured by $A$ in the production function, is exogenous (that is, Hicks neutral technical change), and while endogenous technical changes aim to address growth problems, our parameter $A$ measures total output productivity, which we can modify numerically and therefore capture a certain technological change.

Figure 2 illustrates the fundamental technology mechanisms of the actual convex-concave production function (1). Panel (a) illustrates the impact of capacity utilization on production, showing that for the same capital quantity, output increases as the capacity utilization increases (larger $\sigma$). However, the impact is nonlinear, and tends to disappear the larger capital and $\sigma$. Panel (b) shows the impact of inefficiencies, as captured by the parameter $\beta$, on production given the capacity utilization. For any given level of capital, production is lower the larger the inefficiencies. Again, for high levels of capital, the impact tends to disappear and production tends to converge. These two effects on economic growth will be discussed in detail in the following sections.

Figure 2 also shows that the convex-concave production function (1) violates the Inada conditions [7] since it satisfies

$$f'(0) = \frac{A}{1 + e^\beta} > 0 \quad \text{and} \quad \lim_{k \to 0} f''(k) = \frac{A e^\beta}{(1 + e^\beta)^2}.$$

The violation of the Inada conditions means that the assumption of perfect substitutability is not fulfilled and, at the same time, the idea that a given level of production can be maintained in the presence of an infinitely small pool of resources is not justified through compensation with sufficient capital [8]. In this vein, Capasso et al. (2012) discuss the inappropriateness of a strictly concave production function, arguing that the property $\lim_{k \to 0} f(k) = +\infty$ is far from realistic for economies that might gain infinitely high returns by investing only a small amount of money. When the Inada conditions are not satisfied, the production is driven by increasing marginal product of capital for sufficiently low levels of $k$. Hence, there are states in which growth cannot be described by a strictly concave production function. Mathematically, the corresponding production function will no longer be strictly concave.

Specifically, function (1) is characterized by:

$$f''(k) = \frac{A e^{\beta - \sigma k} (e^{\beta - \sigma k} - 1)}{(1 + e^{\beta - \sigma k})^3}$$

with inflection point $k_f := \beta/\sigma$. This threshold level moves production to diminishing marginal product, so that a higher level of $k$ is required as $\beta$ increases and (or) $\sigma$ decreases.

Figure 3 shows the production function (1) numerically, and the inflection point $k_f = 4.5$. For $k < k_f$ the production functions in convex while for $k > k_f$, $f(k)$ is concave.

2.1 Inefficiencies

Production activities may usually create negative externalities such as pollution, i.e. environmental externalities, which have been largely studied in economics. Less attention has been devoted to the negative consequences over the production systems generated by bad institutions (inefficient government regulations), or inefficient public policies, corruption system, labour market system (large rigidity or extreme flexibilities), or inefficiencies in production given by the labour allocation of productivity gains between leisure and consumption, and/or obstacles for technological progress. These types of market and institution imperfections generate negative externalities that may affect the production
Note(s): Panel (a) shows the impact of capacity utilization: $\beta = 0.9$, yellow line $\sigma = 0.1$, red line $\sigma = 0.5$, blue line $\sigma = 0.99$. Panel (b) shows the impact of inefficiencies: $\sigma = 0.5$, yellow line $\beta = 0.1$, red line $\beta = 0.5$, blue line $\beta = 0.9$
systems, both during the production process (via inefficient use of resources, i.e. misallocation) and in the distribution process with a misallocation of resources across the different nodes of the production network (Restuccia and Rogerson, 2008, 2013).

Among externalities, as Scitovsky (1954) pointed out, there is a distinction between technological externalities and pecuniary ones [9]. The former externalities deal with non-market interactions directly affected by the production function of a firm, while the latter ones refer to interactions through market mechanisms. Our interest is in “technological externalities” in the production function (1) measured by the inefficiency production parameter $\beta > 0$. Hence, firms internalize this negative externality as an inefficiency in the production process. Notice that:

$$v_f(k) v_\beta = -A e^{\beta-k} - \sigma k (1 + e^{\beta-k})^2 < 0;$$

therefore, production decreases as $\beta$ increases: the higher the inefficiencies in production (generated by negative externalities related to bad functioning of markets and institutions), the lower the production levels. Moreover, an increase in $\beta$ moves the inflection point $k_f$ to the right, determining the need of more capital to obtain diminishing marginal product of capital, as shown in Figure 3. For higher $\beta$ the production function has a longer branch that is convex.

This may be interpreted as, for example, the fact that there is an inefficient allocation of production factors (Hsieh and Klenow, 2009) [10]. But, also, the fact that information flows may not be perfect. Asymmetric information between producers and consumers, for instance, could lead to a drastic reduction in market transactions (i.e. the market for lemons problem, Akerlof (1978)), and so lower levels of production. Moreover, inefficient institutions and policies generate negative externalities implying that the distribution of resources cannot be coupled to efficient production (Acemolgu, 2006).

2.2 Productive capacities
Let us now turn to the supportive aspects of the production function (1), namely the role of productive capacities or capacity utilization driven by innovations and know-how (Besanko et al., 2010). The UNCTAD (2006, 2020) created the productive capacities index aiming to measure the productive resources, entrepreneurial capabilities and production linkages which together determine the capacity of a country to produce goods and services and enable
Productive capacities include, for example, the industrial, trade or human capacity facet that is a set of different types of productive, organizational, technological and innovation capabilities embedded in organizations, institutions and infrastructures whose integration determines the capacity of a country to produce goods and services in a competitive global market.

In this vein, we claim that capacity utilization, captured by the $\sigma$ parameter, has a technological role to alter the ability of the firm to combine inputs to produce goods and services, and the pressure on firms to adopt new technologies. Therefore:

$$v_f(k) \sigma = A \kappa^{\beta - \sigma k} (1 + e^{\beta - nk}) > 0,$$

i.e. production levels increase as $\sigma$ increases. Notice that an increase in $\sigma$ decreases the value of the inflection point $k_f$. Likewise, $\sigma$ has no effect on $f(0)$ since $f(0)$ is the production given by only labour and no capital.

### 2.3 The model map

The Solow (1956) growth model is defined by, [11]

$$k_{t+1} = J(k_t) := (1 - \delta)k_t + sf(k_t) \frac{1 + n}{1 + n},$$

where $t \in \mathbb{N}$, $\delta \in (0, 1)$ is the depreciation rate of capital, $s \in (0, 1)$ is the saving rate, and $n > 0$ is the labour force growth rate. Following Wen (1998), we assume that the depreciation rate of capital is not constant and depends on capacity utilization, that is:

$$\delta = \theta \sigma_t, \quad \theta \in (0, 1).$$

The rate of capital depreciation increases according to in the intensity of capacity utilization. In this vein, Nelson and Caputo (1997), Angelopoulou and Kalyvitis (2012), show that the rate of capital depreciation is an increasing function of the degree of capacity utilization, concluding that such an increase in the rate of depreciation has, in turn, a negative impact on accumulation and, hence, on the equilibrium rate of growth. That is, they show that if a higher rate of capital utilization implies that it depreciates more rapidly, the positive effect on capital accumulation of a higher level of demand (a higher degree of capacity utilization) can be at least partly offset by the larger share of total investment to be devoted to the replacement of the worn-out capital.

Since $\sigma$ increases production but also increases the depreciation of capital, in this economy, the representative firm should choose the level of capacity utilization that maximize the capital per capita at time $t + 1$, i.e. it defines the optimal level of $\sigma$ depending on $\max_{\sigma} J(k_t)$ [12]. Notice that

$$\frac{\partial k_{t+1}}{\partial \sigma} = F(\sigma) := \frac{k_t}{1 + n} \left[ \frac{A \kappa^{\beta - nk}}{(1 + e^{\beta - nk})^2} - \theta \right]$$

has a maximum point in $\beta/k_t$ for $k_t > \beta$, therefore the representative firm fixes $\sigma = \sigma_M = \beta/k_t, \forall k_t > \beta$. By substituting $\delta = \sigma_M, \forall k_t > \beta$, the capital per capita evolves according to

$$V(k_t) := \frac{1}{1 + n} \left( k_t - \theta \beta + \frac{As}{2} \right).$$
For $k_t \leq \beta$, $F(\sigma)$ is strictly increasing, therefore $\sigma = 1$ maximizes $k_{t+1}$. Nevertheless, we assume that, for low levels of $k$, the representative firm is not able to exploit all its productive capacity due to a “lack of know-how”. Therefore, for all $k_t < \beta$, we consider

$$\sigma = 1 - q, \quad q \in (0, 1)$$

where $q$ represents the lack of know-how or, similarly, that there is no learning-by-doing in production.

Consequently, for $k \leq \beta$, the capital per capita dynamics follows

$$I(k_t) := \frac{1}{1 + n} \left\{ [1 - \theta(1 - q)]k_t + \frac{A_s}{1 + e^{\theta(1-q)k_t}} \right\}. \quad (5)$$

The evolution of the economy may, hence, be described by the piecewise map:

$$C(k_t) := \begin{cases} I(k_t) & k_t \leq \beta \\ V(k_t) & k_t > \beta \end{cases} \quad (6)$$

Considering this map, in what follows we analyse the dynamic multiple equilibria that may be reached.

### 3. Economic growth and poverty traps

In this section we analyse the existence of equilibria and poverty traps, and conditions in order to make an equilibrium selection, to overcome the poverty trap and to obtain economic growth [13]. The convex-concave production function (1) describes an important economic issue, i.e. depending on the initial level of capital per capita $k$, the economy may converge to the high steady state or decline to the low steady state. Thus, there are critical levels of capital stock that allow the economies to escape from poverty traps (see Azariadis and Drazen, 1990; Askenazy and Le Van, 1999; Akao et al., 2011).

### 3.1 The low-level equilibrium: poverty traps

Firstly, let us consider a situation characterized by a high level of productive inefficiencies (large $\beta$) and/or a low level of capital per capita (small $k$) so that $k \leq \beta$, i.e. the levels of capital per capita are lower than the levels of productive inefficiency. Recall that the Solow model with variable saving rate shows that when $k$ is very low, the savings rate is likely to be low, or even negative, because very poor people need to consume all of their income just to survive (Romer, 2019). However, saving increases with higher levels of output (income) per worker (capita), forming an “S” shaped function (Sachs (2005)). In our case, we demonstrate that when a poverty trap exists, productive inefficiencies are so large that they exceed capital per capita levels and saving rates. The next proposition holds.

**Proposition 1.** For $k \leq \beta$, map $C$ given by (6) has a unique and locally stable fixed point $k^*_1 \leq \beta$ iff $\beta(1 + e^{\beta}) > \frac{A_s}{n + \theta(1 - q)}$.

**Proof.** Assume

$$M(k) = k(1 + e^{\beta(1-q)k})$$

then fixed points for $k \leq \beta$ verifies

$$M(k) = \frac{A_s}{n + \theta(1 - q)}.$$
Being $M(0) = 0$, $M(\beta) = \beta(1 + e^{\beta})$ and $M(k) = 1 + e^{\beta(1-\theta)q}(1+(q-1)k) > 0\forall k \leq \beta$. It follows that $M(k)$ intersects once the positive and constant function $\beta > \frac{B_s}{n+\theta(1-q)}$ iff

$$\beta(1 + e^{\beta}) > \frac{B_s}{n+\theta(1-q)}.$$

Moreover it has $I(0) = \frac{A_s}{1+\theta}$ and $I'(k) = \frac{1}{1+q} \left\{ 1 + (1-q) \left[ \frac{A_s e^{\beta(1-\theta)q}}{(1 + e^{\beta(1-\theta)q})^2} - \theta \right] \right\} > 0$. Therefore, when a fixed point exists, it is always locally stable.

The interpretation of Proposition 1 is straightforward. When the level of capital per capita is very low, and (or) productive inefficiencies are high, so that $k \leq \beta$, then the economy may converge to a unique stable stationary state that is the poverty trap. Therefore, we can affirm that the economy is not at risk of falling into a poverty trap if: (1) productive inefficiencies ($\beta$) are reduced, through for example economic policies aimed at reducing market and institutional imperfections; (2) physical capital per capita ($k$) increases, through economic policies aimed at fostering firm investment; (3) productive inefficiencies ($\beta$) are reduced, and physical capital per capita ($k$) increases, by means of a policy mix. Moreover, the economy falls into a poverty trap (a stable fixed point $k^*_1 < \beta$) if and only if inequality of Proposition 1 holds. However, the economy may escape from a poverty trap if the country increases the total output productivity (i.e. A technical change increases, and $\beta$ decreases) and (or) governments adopt economic policies intended to increase the level of investments ($s$ increases).

### 3.2 The high-level equilibrium: economic growth

Regarding the existence and stability of steady states for $k > \beta$ the following proposition holds.

**Proposition 2.** For $k > \beta$, map $C$ given by (6) has a unique and locally stable fixed point

$$k^*_2 = \frac{A_s}{2n} - \frac{6\theta}{n} \iff \beta < \frac{A_s}{2n}.$$

**Proof.** Fixed points for $k > \beta$ satisfy $V(k) = k$. From a simple algebraic computation follows $k = \frac{A_s}{2n} - \frac{6\theta}{n}$ that belongs to the interval $(\beta, +\infty)$ iff

$$\beta < \frac{A_s}{2n}.$$ Moreover $V'(k) = \frac{1}{1+q} \in (0, 1) \forall k$ therefore when the fixed point exists, it is always locally stable. $\square$

Notice that, as $\beta$ increases, the fixed point $k^*_2 > \beta$ might disappear. Moreover, as previously discussed, if $\beta$ is sufficiently high a poverty trap exists. Therefore, a high level productive inefficiencies may lead the economy to a globally attractive poverty trap, as Figure 4 shows: in the two panels, all the parameters are fixed, except for $\beta$. Starting from the same initial condition, if the level of productive inefficiencies is low, the economy reaches the equilibrium with high capital per capita (panel (a)), while if the level of productive inefficiencies is high, the equilibrium moves to the poverty trap (panel (b)).

Notice that the conditions for the existence of $k^*_1$ and $k^*_2$ may be simultaneously satisfied only if $\beta > \ln\left(\frac{n+(1-q)\theta}{n+(1-q)\theta}\right)$. This consideration leads to the following Remark 1 regarding multistability.

**Remark 1.** Let map $C(k)$ as given in (6) and consider

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Then map $C$ has two locally stable fixed points and a multistability phenomenon occurs.

As Figure 5 shows, when a multistability phenomenon occurs, starting from different initial conditions, an economy may reach the higher capital per capita equilibrium level $k^*_2$ or may lie in the poverty trap $k^*_1$. More precisely, the two basins of attraction are given by $B(k^*_1) = [0, \beta]$ and $B(k^*_2) = (\beta, +\infty)$.
Remark 1 states that there is an interval of productive inefficiencies $\beta$, such that if the economy is at this value of inefficiencies (items (1) and (2) of Remark 1), then both situations occur, that is the poverty trap and the high-level equilibrium. Again, the important consideration here is to implement economic policies aimed at reducing the basin of attraction of the poverty trap (which is equivalent to enlarging the basin of attraction of the high-growth equilibrium). Since the poverty trap attracts all the trajectories starting below $\beta$, an economic policy that would push $k$ above $\beta$ could allow the economy to escape from the poverty trap. So, the lower the value of $\beta$, the smaller the basin of attraction of the poverty trap.

However, the model also suggests that two countries characterized by the same level of inefficiencies $\beta$, could converge to different steady states if they have different levels of technology, savings, and capital.

Let us now make an important observation that formally highlights the discontinuities presented in our economy. Such discontinuities give rise to the origin of cycles behaviours. Remark 2. Notice that map $C(k_t)$ has a discontinuity point in $k_t = \beta$ whenever
\[
\frac{q(e^\omega+1)}{e^\omega-1} \neq \frac{\Delta q}{2q_0^p}.
\]
The discontinuity is caused by the change in the law that governs the productive capacities, i.e. capacity utilization $\sigma$. Moreover a multistability phenomenon may only appear if $I(\beta) < \lim_{k_t \to \beta} V(k_t)$. That is
\[
\frac{q(e^\omega+1)}{e^\omega-1} < \frac{\Delta q}{2q_0^p},
\]
being the lhs of previous inequality increasing in $q$, a low level of know-how could rise $I(\beta)$ and create a jump downward for the evolution of capital, when the economy switches the law for the capacity utilization.

Remark 2 states that as far as the lack of know-how increases (larger $q$), then the capacity utilization reduces (smaller $\sigma$), and so, though the levels of capital per capita are equal to the inefficiencies in production, the economies experiment cycles behaviours, as discussed in the next section.

4. Cycles dynamics
So far we have analysed how capacity utilization and inefficiencies may lead to poverty traps. In this Section we study their effect on the generation of boom and bust periods. Goodwin (1967) is a pioneer growth model, with an income distribution focus (although not exactly on poverty traps), and hints at chaotic dynamics, which were later developed in his book Chaotic economic dynamics (1990). Kuznets (1930, 1955) offers an empirical interpretation of the process of economic growth and development, arguing that cycles are determined by the growth and development of new industries. Similarly, the Schumpeter (1939, 1950) concept of “long waves”, stresses the importance of innovations in order to avoid fluctuations in the economy. The basic idea of J. Schumpeter theory refers to the key role of entrepreneurs who, with their own capabilities and initiative, create new opportunities for investment in economic growth and employment. The decisive factor that fosters innovation, leading to rapid growth, is the profit derived from the innovations provided by the market. The literature has discussed how economic cycle fluctuations may be very different and influenced by a variety of reasons, such as increasing demand, physical capital, technological progress or innovations (see Levy and Hennessy, 2007; Bhamra et al., 2010).

Recently, Growiec et al. (2018) by means of a micro-founded endogenous growth model with aggregate CES technology and factor augmenting technical change formally emphasize medium-term swings of the labour share, focussing on their technological explanation, and allowing the possibility that these swings are driven by endogenous, stable limit cycles.

In our model, as previously discussed, cycles may appear because of the discontinuity in $k_t = \beta$ and this leads to important considerations that concern economies.
(1) The cycles appear for values of $k$ that are not high enough (not in the frontier), so they are cycles regarding emerging/developing economies,

(2) Since they can only appear when $I(\beta) > \lim_{k_t \to \beta} V(k_t)$ and since $I(\beta)$ increases in $q$ while a movement in $q$ does not affect $\lim_{k_t \to \beta} V(k_t)$, it follows that a high level of know-how may avoid cycles, i.e. fluctuations in the economy,

(3) When $\lim_{k_t \to \beta} V(k_t) < \beta < I(\beta)$, the interval $P = \left[ \frac{A_2}{2(\theta+q)}, \frac{A_3(1+q)^{\theta^{-1}}}{n+1-q, \sigma} \right]$ is an invariant and globally attractive set. Notice that the invariant set may exists only if $\beta < \ln \left( \frac{\kappa + (1+q, \sigma d)}{\theta + (1-q, \sigma)} \right)^{\frac{1}{\theta}}$.

Hence, the generation of cycles is not only affected by the savings rate and the technological progress, as it also depends on the level of productive inefficiencies and know-how. Certainly, the factors that determine know-how and these efficiency measures are identified with the firm characteristics such as the endowment of human capital, and/or investment in R&D activities (so the lack of know-how is low, and the capacity utilization is high), and imperfections and frictions that affect the function of markets and institutions.

In the next Figures we numerically show how cycles and more complex dynamics may emerge. In Figure 6, panel (a), a two period cycle is depicted: the economy alternates boom and bust periods. However, as shown in panel (b), by increasing the level of inefficiencies and the lack of know-how (from 0.5 to 0.55 and from 0.3 to 0.78, respectively) a three period cycle arises.

The cycle chartogram in Figure 7 shows the long run evolution of the economy: each point in the chartogram represents a combination of $\beta$ and $q$, while the colour of the point describes the long-run attractor, given the parameters values (e.g. dark blue for a fixed point, light blue for a 2-period cycle, dark red for more complex dynamics). Combinations of $\beta$ and $q$ that lie in the dark blue region correspond to the existence of an attractive fixed point for the economy (that might be a poverty trap or a higher capital per capita equilibrium). For sufficiently high values of $\beta$, map C shows cycles of any order (the orange region corresponds to a 6-period cycle, the yellow region to a 16-period cycle and so on). Moreover the region (and, hence, the combination of $\beta$ and $q$) in which cycles emerge is wider for higher values of $q$ (i.e.: smaller productive capacity, $\sigma$).

![Figure 6](image_url)

**Figure 6.**

Cycles dynamics for the map $(C)$. Panel (a) 2-period cycle for $\beta = 0.5$, $q = 0.3$, Panel (b) 3-period cycle $\beta = 0.55, q = 0.78$

**Note(s):** Parameter values: $n = 1.2$, $\theta = 0.9$, $A = 10$, $s = 0.2$
Notice that for low and high values of $\beta$ the economy converges to a fixed point that might be the high fixed point $k^*_2$ or the poverty trap $k^*_1$. In order to understand the complete behaviour of the model we set

$$U(q, \beta) := \beta(1 + e^{\beta}) - \frac{As}{n + \theta(1 - q)}$$

and

$$E(q, \beta) := \beta - \frac{As}{2(\theta + n)}$$

that are the two bifurcation curves deriving respectively from Proposition 1 and 2.

As Figure 8 shows, the high capital per-capita equilibrium level may be reached only if $\beta$ is sufficiently low, while for high values of $\beta$ the economy lies in the poverty trap. On the other hand, a low level of know-how, i.e. a high level of $q$, may generate fluctuations in the economy.

Said differently, when the level of inefficiencies is sufficiently low (high), the economy always reaches a high equilibrium (poverty trap), regardless of the lack of know-how. Then, for intermediate values of $\beta$, the economy may experience cycles, depending on $q$. The region of cycles is larger, the larger $q$.

5. Economic policy recommendations
The above results suggest that to obtain sustained economic growth (outside of poverty traps and cycles) it is essential to have high levels of productive capacities (driven by capacity

Note(s): Parameter values: $n = 0.8, \theta = 0.9, A = 7, s = 0.2$
utilization and know-how, without inefficiencies). Once again, using the UNCTAD Productive Capacities Index and data for the Real GDP (per capita) at constant 2017 national prices (in mil. 2017US$) over the period 2000–2018 we can confirm there is a positive although nonlinear relationship between these two variables, i.e. productive capacity and GDP. In Figure 9 we provide evidence for USA, major European countries (France, Germany and Italy) and two Latin America countries (Brasil and Mexico) as representative economies characterized by different GDP (levels and growth) and heterogeneous degrees of inefficiencies and market imperfections. We obtain some stylized facts. Firstly, economies characterized by higher PC indexes (USA, Germany and France) are also characterized by higher GDP. Secondly, within the selected European economies, Italy seems to be the country having the weakest relationship between PC index and GDP. This finding is in line with previous analyses (Calcagnini et al., 2021) that show that in economies equipped with greater growth potential (i.e., the presence of efficient markets and institutions) positive shocks turn into stronger GDP growth. Thirdly, while Mexico and Italy may be caught in cycles, the other countries seem to be on the path of growth due to productive capacity. It is likely that these findings are driven by the interplay of technical change, know-how and inefficiencies in determining output and growth.

Therefore some policy implications may be proposed. Firstly, we recognize the UNCTAD’s (2020) seven main pillars to engine productive capacity, i.e.: (1) industrial infrastructure, (2) productive resources (natural resources, human capital), (3) private sector development, (4) regional integration (regional integration through development of regional transportation networks, improved trade facilitation, and strengthened connectivity), (5) financing productive

\[ k_1^* \] is the poverty trap while \( k_2^* \) is the high fixed point.
capacity building, (6) science, technology and innovation, (7) institutions, policy and regulations (industrial policy for structural transformation and efficient institutions). These pillars interact in a complex fashion in such a way that successful implementation (there are no inefficiencies in production), the know-how of each pillar of the strategy, depends and determines the success of implementation of the other pillars, in order to avoid poverty traps and cycles behaviours.

Secondly, more capital per capita (more savings and investment) are the key for attaining a high-level equilibrium of economic growth. However, this must be done carefully, because if the economy is characterized by a lack of know-how and such low levels of capacity utilization, a poverty trap may arise, although if capital per capita and inefficiencies in production are at the same level, then cycles are attained.

Hence, we propose that to achieve the real possibility of endogenous growth, economies must combine capital accumulation with enhancement of technical progress, so the lack of

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**Source(s):** Data from UNCTAD (https://unctad.org/topic/least-developed-countries/productive-capacities-index), and from Penn World Table (https://www.rug.nl/ggdc/productivity/pwt/)

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**Figure 9.** Scatter plot of the productive capacities index (y-axis) and log GDP per capita (x-axis), data availability for the years 2000–2018
know-how diminishes and the productive capacity increases as well as a decrease in the inefficiencies in production. Moreover, unceasing technical progress counteracts a tendency of diminishing marginal returns to capital. For this purpose, economies must encourage investments in physical and human capital, reinforce R&D activities, as well as expand technology transfers. Economies should enhance technology transfers, raise production capabilities, and carry out their own R&D.

Furthermore, economic policies should aim at fostering productivity to overcome the poverty trap and/or to attain a high-level equilibrium of growth economic policies should improve productivity. In this direction, for example, the promotion of a tax system that provides incentives for firms’ innovation activities, could increase the capital per capita in the economy. Finally, policies that lead to better education and training to improve skills, flexibility, and mobility, are likely to improve labour productivity, hence fostering growth.

Thus, policy makers have a key role to play in determining the proper policy mix to avoid or escape from a poverty trap. Indeed, economic growth and productivity depend upon a combination of investment in physical and human capital, knowledge and technical progress (Sanchez-Carrera et al., 2021). However, factors such as the quality of institutions, market efficiency, the degree of openness and the flexibility of the economy may considerably weaken the economic growth of a country (Calcagnini et al., 2015). Therefore, policies have to ensure that the fundamental influences of efficiencies (and that productive inefficiencies do not occur) in production are conducive to an economy attaining economic growth. Government influences are driven by economic policies, for example to enhance the investment in productivity-boosting initiatives such as R&D activities, education and industrial infrastructure; institutional settings, which govern how governments, firms and individuals interact; and social capability, which refers to the orientation of a people to effect change to bring about productivity growth. For instance, to foster investment in a country characterized by low capital accumulation, foreign aid can help to finance investment until countries develop saving rates high enough to place them on a trajectory that escapes from the poverty trap. In this vein, microfinance loans appear to have large impacts on poverty if individuals could borrow enough to finance the lumpy production technology needed to move them out of a trap (Kraay and McKenzie, 2014). Therefore, among others, policies aimed at developing financial markets are worth undertaking.

6. Concluding remarks

In this research paper we modify the Solow–Swan growth model by means of a sigmoidal production function that takes into consideration the effect of inefficiencies and capacity utilization on production. The resulting model is a piecewise nonlinear map able to describe the evolution of a non-developed, developing and developed economy in which we prove that poverty traps may coexist with economic growth when the level of inefficiencies is sufficiently high. Moreover, we show that cycles arise when the lack of know-how is significant. Therefore, if an economy aims to avoid poverty traps and/or undesired fluctuations in terms of cycles behaviours, then firms’ structures must be characterized by low levels of productive inefficiencies and high levels of capacity utilization (and know-how).

Further research may address the methodological concern that stems from the fact that our model apparently only recognizes cycles in economies with low capital accumulation levels. The definition of low equilibrium levels and poverty traps is essentially correct, but we believe that our model should be more complete and realistic if it recognized the possibility of high-income economies falling into economic crisis situations. Would it be possible to include these mechanisms? We would initially argue that high levels of accumulation (or high capacity utilization) can also bring unstable economic environments, and therefore economic fluctuations (uninterrupted technical progress is ideal, but is indeed unattainable, from what
is observable from experience). Furthermore, deficient institutions, in the sense that these can hamper indefinite growth, can also appear in high-income economies. While modelling institutions is never an easy task. Perhaps further research could shed some more light on their definition of institutions and their deficiencies. In short, further research may be driven by acknowledging “high-equilibrium level” traps, and perhaps further elaborating on its parametric concept of ‘institutions.’ Future research may also focus, for instance, on a production function including human capital as a mechanism to mitigate productive inefficiencies and/or enhance their capacity utilizations and know-how. However, in any case, human capital must also be accompanied by an investment in R&D activities such that there is complementarity. As a feasible result, higher human capital may increase the productivity of innovative firms, and firms’ R&D activities may reinforce the human capital stock through positive economic externality, and thus sustained economic growth may be obtained in the long run. To sum up, we believe that the grounds for some discussion regarding the present model have been set.

Notes


2. Griliches (1957) laid the groundwork for this type of research, analysing sigmoid capacity utilization and production functions (i.e. convex-concave technologies). Plata et al. (2007) analyse the classic models of Solow–Swan and Ramsey taking up the possibility of different trajectories in the production function. Taking the Richards (sigmoidal) production growth function, they show that the Solow and Ramsey model, with a neoclassical production function, represents only specific cases of the same models with the Richards production function, with poverty traps appearing naturally.

3. The theory of Low Level Equilibrium Trap developed by Nelson (1956) states that when per capita income has increased above the minimum specific level, population tends to increase. But when the growth rate reaches an upper physical limit as the per capita income increases, the growth starts declining. That is, if the per capita income is increased above the specific level through saving and investment, it leads to population growth. As a result, the increase of population lowers per capita income to its stable level of equilibrium. Thus, the economy is caught in a low level equilibrium trap. To exit from this trap of income growth must be higher than the rate of population growth.

4. The pioneering work of Farrell (1957), which calculates the efficient unit isoquant through the use of linear programming deriving the measure of efficiency, is the initial framework of the extensive literature on efficiency production.


6. The links among economic growth and cycles, the elasticity of capital, and factor shares have been subject to unceasing controversies (see for example, Arrow et al., 1961; Schenk-Hoppé, 2005; Gordon and Vaughan, 2011).

7. Inada requires $f(0) = 0$, that is you cannot produce without capital. We remove this assumption since in an economy, at the very beginning of initial growth, we may consider that there is no capital and it is human work that produces output to be used as capital later on. This is argued also in Grassetti et al. (2018a). Inada requires $\lim_{k \to 0} f'(k) = +\infty$, that is you would have infinitely high returns investing just a small amount. We reject this assumption since you cannot get infinitely high returns in a poor economy that has $k$ close to zero (no infrastructures, no streets, . . . ) and you invest a small amount (see, Capasso et al., 2010, 2012).

8. Baumgartner (2004) provides a demonstration of the conflict between economic-mathematical formalization and physical constraints by showing that the application of Inada’s conditions (one of
9. The key feature is that even though the individual firm’s capital stock may be subject to diminishing marginal physical product, the presence of an aggregate production positive externality enhances its productivity so that in equilibrium the economy is able to sustain a steady growth rate. Typically, the externality is specified by the presence of the aggregate capital stock in individual production functions, which serves as a proxy for knowledge, as in Romer (1986), or by the presence of productive government expenditure, as in Barro (1990) and Turnovsky and Liu (2004).

10. Hsieh and Klenow (2009) investigate the relationship between resource allocation and aggregate TFP. Using microeconomic data from manufacturing establishments, they quantify the potential extent, in terms of aggregate productivity, of misallocation of resources in China and India, relative to the prevailing allocation of resources in the United States. Their results show considerable gaps in the marginal products of capital and labour between plants within well-defined industries in these two countries, always in relation to the situation observed in the United States. When capital and labour are hypothetically reallocated to achieve the same level of efficiency seen in the United States, these authors estimate a hypothetical gain in manufacturing TFP of 30 to 50% in China and 40 to 60% in India.

11. The Solow growth model is characterized by the following set of equations,

\[ Y_t = F(K_t, A_t L_t), \]
\[ K_{t+1} = (1 - \delta)K_t + I_t, \quad K_0 = K_0, \]
\[ I_t = sY_t, \quad 0 < s < 1, \]
\[ A_t = (1 + n)A_0, \quad A_0 = \bar{A}_0, \]
\[ L_t = 1, \quad (or \quad L_t = (1 + n)L_0). \]

where: i) savings (equal to investment, \( I_t \)) are a constant fraction, \( s \) of output, \( Y_t \), and ii) \( F(\cdot) \) is a production function in capital, \( K_t \), and labour, \( L_t \), where \( A_t \) represents the state of the production technology. Let \( k_t = K_t / L_t \), with \( F(\cdot) \) homogenous of degree one, then the equilibrium of the model is described by: \( k_{t+1} = \frac{(1 - \delta)k_t + k(\cdot)}{1 + \sigma} \) a first-order nonlinear difference equation.

12. Here the firm is not interested in maximization of profits; profit maximization depends on how the production \( f(k) \) is allocated between workers and shareholders. Despites so, the decision over profits does not affect capital accumulation since the savings rate is unique and, as visible in equation (2), the capital at \( t + 1 \) does not change, depending on how the output is allocated. The choice of the firm refers, instead, to the opposite effect of capacity utilization: since \( \sigma \) increases new production but it also increases the depreciation rate of existing capital, a capital per capita maximization problem needs to be solved.

13. Kraay and McKenzie (2014) summarize the concept of poverty trap with the help of low savings and investments. The study assumes that aggregate output per capita (\( y \)) depends on the state of technology and the level of capital per capita (\( k \)). The study further assumes that \( k \) depends on the level of investments (\( I \)) while investments are financed by savings (\( s \)). Although, there is a large literature discussing the existence of poverty traps (Barrett et al., 2019), yet it provides inconclusive results regarding the validity of poverty traps.

References


Further reading

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