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Capital immobility and rollover risk in debt markets

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Abstract
This paper aims to develop a credit-risk model in which firms face rollover risk, and the markets for defaulted assets are segmented due to entry costs. The paper shows that reducing the entry costs in this economy may decrease the total surplus of the economy. This outcome can arise because when market barriers are lifted, the gap between the liquidation prices across the markets will shrink, but then the market that would experience a price drop may face more bankruptcies because the rollover risk will increase in that market. The paper describes under which condition such an intervention policy improves or hurts the total surplus.

Keywords Market segmentation, Rollover risk, Contagion, Intervention policy

Paper type Research paper

1. Introduction
Market barriers deterring capital flows have been considered one of the main culprits in liquidity crises such as the 2008 financial crisis and the European debt crisis in the early 2010s. Specifically, when investors cannot move across markets flexibly, shocks to a local market cannot be absorbed by investors in other markets, even if those outside investors have enough liquidity. As a result, the local shocks can disrupt at least that local market, which would not occur if the markets were well connected. Simply put, market segmentation prevents efficient asset allocation and risk sharing, as emphasized by Basak and Cuoco (1998), Gabaix *et al.* (2007), Duffie (2010), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014).

However, markets that enjoy free capital flows have a dark side as well. That is, when markets are closely linked, distress in one market can spread to other markets, amplifying the initial local shocks. This phenomenon is commonly called financial contagion. One well-known contagion mechanism is that when assets are traded among a common pool of either financially constrained or risk-averse investors, shocks to some assets will be transmitted to other assets through those common investors; see, e.g. Kyle and Xiong (2001), Gromb and Vayanos (2002), and Goldstein and Pauzner (2004). When this contagion effect is sufficiently large, such local shocks can cause entire markets to collapse, as we witnessed during the 2008 financial crisis. Longstaff (2010) and Gorton and Metrick (2012) provide empirical evidence for this fact.

These two opposite effects of facilitating capital flows raise the following questions: Will policies seeking to reduce market barriers increase the total welfare of the economy? If not, under what conditions will such policies hurt welfare? More concretely, can the government

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prevent a liquidity crisis by providing information about a distressed industry to investors specializing in other industries? Did the recent development in the securitization market benefit the economy by attracting investors, who mainly participate in capital markets, to mortgage markets? Will removing legal barriers between commercial banks and investment banks increase the market capitalization of the banking sector in spite of the cost of higher instability?

This paper considers these issues by paying particular attention to credit markets and shows that reducing certain barriers between the secondary markets for defaulted assets may reduce the total welfare, especially when firms face severe rollover risk. Specifically, building on He and Xiong (2012b), the paper develops a credit-risk model in which each firm issues finite-maturity debt, and the secondary markets for defaulted assets are segmented. That is, each potential buyer in one market must pay some costs to buy failed assets in the other market. This so-called entry cost can be interpreted as information costs, regulatory costs, search costs and so on because all these frictions hinder efficient capital flows between the markets.

The model makes two more assumptions about potential buyers. First, potential buyers have lower productivities than incumbent firm managers as in Shleifer and Vishny (1992). Second, potential buyers have different productivities even among themselves. Because of these assumptions, liquidation of assets always incurs efficiency losses, but the size of the losses will be lower if the assets are liquidated to high-skilled buyers rather than to low-skilled buyers. As such, total welfare in this economy is determined by two factors: (1) how many assets are liquidated due to bankruptcies and (2) to whom those assets are liquidated among different potential buyers.

The paper then shows the aforementioned main result that policies seeking to reduce the entry costs may decrease total welfare. To see why, note that when the entry costs are lowered, some high-skilled buyers in a relatively more liquid market move to the other market to exploit the gap in the liquidation prices between the markets. Thus, some failed assets in the less liquid market are now liquidated to high-skilled buyers rather than to low-skilled buyers. In other words, assets in default are more efficiently allocated due to the reduction in the entry costs. All else being equal, this asset reallocation will increase total welfare.

However, what matters is that the investor (or capital) outflows from the liquid market to the illiquid market shrinks the liquidation price gap, because arbitrage opportunities would otherwise arise. That is, the liquidation price in the liquid market decreases and the liquidation price in the illiquid market increases. This reduced price gap affects the asset quantity liquidated in the secondary markets in an unusual way. Specifically, equityholders in the liquid market that face a price drop will default earlier to avoid increased rollover risk, causing more asset sell-offs. By contrast, equityholders in the illiquid market will default less aggressively, because rollover risk has been attenuated. In a nutshell, when firms face both default and rollover risks surrounding their debt, the supply curve in the secondary market can be downward sloping.

Due to this uncommon general equilibrium effect, policies of reducing market barriers can either increase or decrease total welfare, unlike in other typical product markets that have upward-sloping supply curves. In particular, when the negative effect arising in the relatively liquid market is sufficiently large, total welfare will decrease. In this regard, we can interpret this negative effect as an adverse contagion effect, because such an effect would not occur if capital flows were completely blocked.

Now, under what conditions does the negative contagion effect outweigh the benefits of expediting capital flows? On the one hand, when the relatively liquid market is sufficiently liquid, reducing the entry costs will increase total welfare. The reason is that in this case, the aforementioned capital outflows push up the asset price in the illiquid market substantially, without causing a sizable price impact in the liquid market. Thus, the adverse contagion effect cannot be overwhelming. On the other hand, when the relatively liquid market is not sufficiently liquid and the other market is severely illiquid, the same policy can decrease total welfare for the opposite reason: The above capital outflows now substantially depress the price in the liquid market while only slightly improving the price in the illiquid market.

In fact, in this economy, multiple equilibria arise under some conditions because the supply curves are downward sloping. When multiple equilibria exist, the effects of any policy changes depend on which equilibrium is selected. Before studying this multiplicity issue, however, note that equilibrium multiplicity is not the most important factor driving the main result of the paper. That is, the aforementioned negative policy outcomes still obtain even when the economy has a unique equilibrium, to be discussed again when analyzing the model.

Turning to the multiplicity issue, the model has multiple equilibria when the relatively liquid market is not sufficiently liquid and the other market is just mildly illiquid. In this circumstance, which equilibrium will arise depends on the market participants' self-fulfilling beliefs. On the one hand, suppose the market participants believe the liquidation prices will be high. Then equityholders choose to default less aggressively, because rollover risk is lower under such optimistic belief. The self-conjectured high prices can thus clear the markets, because the reduced default chances lead to lower asset supply. Put differently, the economy in this equilibrium behaves as if the liquid market is sufficiently liquid; therefore, policies of reducing the entry costs will increase total welfare.

On the other hand, if the market participants believe the liquidation prices to be low, equityholders will behave the opposite way, causing more asset liquidation. Owing to this increased asset supply, the self-conjectured low prices can again clear the markets. That is, the economy now behaves as if the less liquid market is severely illiquid, and therefore, the above same policy may decrease total welfare.

Lastly, the paper also examines whether injecting liquidity into the secondary markets can alternatively improve total welfare. For instance, the Term Asset-Backed Securities Loan Facility (TALF) launched by the US government in 2009 can be considered as this liquidity injection program. Unfortunately, this policy may hurt total welfare as well, even in a single isolated market. To see why, suppose the market participants in such a single market believe the asset price will rather drop when the government injects new capital. Equityholders then choose to default earlier because they expect to face bigger rollover risk. But since new capital has been already added, the demand and supply can actually match each other. Therefore, the self-conjectured low price can be sustained as an equilibrium. In other words, the downward-sloping supply curve again causes the negative effect when the government provides more liquidity.

This paper contributes to the finance literature as follows. First, many researchers have intensively studied financial contagion and systemic risk; for instance, see Allen and Gale (2000), Kyle and Xiong (2001), Gromb and Vayanos (2002), Dasgupta (2004), Goldstein and Pauzner (2004), Brunnermeier and Pedersen (2008), Oh (2013), Choi (2014), Liu (2016), and Gromb and Vayanos (2018). But, most of these papers consider a single fully integrated market and focus on the contagion issue only. The exceptions are Dasgupta (2004), Goldstein and Pauzner (2004) and Gromb and Vayanos (2018), who consider partially segmented markets. However, in those papers, investors allocate their capital across different markets by following an exogenously given rule. In the present paper, investors make an endogenous entry decision by taking into account the price discrepancy across the markets. In this regard, the present paper provides a better micro-foundation for capital mobility. Duffie and Strulovici (2012) use a similar setup in which intermediaries move their capital across different markets at some costs. But, their paper focuses on asset-price dynamics rather than welfare issues. Also, their paper does not consider rollover risk in credit markets, which is the key element of the present paper.

This paper also contributes to the credit-risk literature studying interactions among primary markets, secondary markets and interbank markets. He and Milbradt (2014) and

Chen *et al.* (2017) study the feedback effects between a primary bond market and a secondary bond market by analyzing search frictions in the latter market. Liu (2016) studies the interdependence between a primary debt market and an interbank market through an interbank interest rate. The present paper considers two sectors and studies the feedback effects between primary debt markets and secondary asset markets through capital flows across the secondary markets.

Further, many papers emphasize an importance of equilibrium multiplicity in credit markets. Diamond and Dybvig (1983) show that demand-deposit contracts lead depositors to achieve optimal risk sharing if they coordinate well, but otherwise cause panic-driven bank runs. Rochet and Vives (2004) and Goldstein and Pauzner (2005) provide alternative models to show that the inefficient bank runs can actually occur as a unique equilibrium outcome, when creditors have heterogeneous but precise enough information about their firm. He and Xiong (2012a) develop a full dynamic model of debt runs to emphasize the roles of asset volatility and debt maturity. Their paper also derives a unique equilibrium. Liu (2018) builds another model, however, to show that multiple equilibria still emerge even if the above private information is sufficiently accurate. The main mechanism is that a coordination problem among creditors causes asset fire sales, creating a price impact on other banks' assets, which in turn exacerbates the coordination problem. As a result, strategic complementarity among creditors becomes very large, thereby producing multiple equilibria. Kuong (2018) develops a model that generates multiple equilibria, using the interaction between equityholders' risk-taking incentives and market illiquidity. In essence, the mechanism generating multiple equilibria in the present paper has some common features with the mechanisms in Liu (2018) and Kuong (2018). But, their papers focus on the market fragility issue stemming from equilibrium multiplicity, whereas the present paper highlights the market segmentation issue. Moreover, as mentioned above, equilibrium multiplicity is not the key driver for the main result of this paper.

Lastly, regarding the liquidity injection policy, Benmelech and Bergman (2012) show that providing new capital to banks facing incomplete contracts may not generate any effects, because those banks rationally choose to hoard that additional capital instead of lending it out. Bleck and Liu (2018) develop a model to show that injecting liquidity may even hurt the aggregate economy due to a crowding-out effect. The present paper provides another mechanism to explain the negative effects of such a policy, using the self-fulfilling beliefs arising in debt markets facing rollover risk. More importantly, the aforementioned papers show that liquidity injection will be either ineffective or harmful if the government provides liquidity excessively. In the present model, the negative effects occur when the government injects an insufficient amount of capital.

The paper is organized as follows. Section 2 develops a simple model to illustrate the main idea. Section 3 develops a full credit-risk model. Section 4 solves the model. Section 5 discusses the model implications. Section 6 concludes. All technical proofs are included in Appendix.

2. Simple static model

This section presents a simple model to highlight the main idea, in which the supply curves are exogenously given. The paper develops a full credit-risk model in Section 3, where the supply curves are endogenously generated.

2.1 Setup

Consider an economy with two markets (or sectors), market A and market B. A certain asset is traded in each market. The assets in the two markets are identical. There are only two dates, indexed by $t \in \{0, 1\}$. Each market consists of original asset holders and potential asset buyers. All the agents are risk-neutral and have a zero discount rate.

Each one unit of the asset will produce cash flows of 1 at date 1 with certainty. Thus, the present value of the asset at date 0 is equal to 1. Each market has a unit measure of original asset holders, each of whom has one unit of the asset at date 0. If the asset holder continues to hold the asset until date 1, she will earn the cash flows of 1. However, some asset holders receive a liquidity shock at date 0. Those asset holders are forced to liquidate their assets immediately at date 0. The other asset holders rationally keep their assets until date 1, because the original asset holders are the most productive asset users, to be described in detail later.

Regarding the liquidity shock, let p^i denote the asset price in market i. Then, a fraction $q^i(p^i)$ of the original asset holders in market i receives the liquidity shock at date 0. Moreover, $q^i(p^i)$ decreases in p^i. That is, the supply curve is downward sloping. In the main credit-risk model that will be developed in Section 3, the supply curve is endogenously determined. In the present simple model, $q^i(p^i)$ is exogenously given.

Each market also has potential asset buyers who can buy liquidated assets at date 0. But, the markets are partially segmented from each other. Specifically, each potential buyer in market i has to pay fixed costs κ to buy an asset in market $-i$, where $-i$ denotes the opposite index of i. The parameter κ is called the entry cost. To clarify, the original asset holders in market i sell their assets only in market i.

The potential buyers have different productivities, which can be either high or low. High-type buyers will produce α_h from one unit of the asset at date 1, whereas low-type buyers will produce α_l from the same asset, where $\alpha_l < \alpha_h < 1$. Thus, the date-0 value of the asset to the high-type (resp. low-type) buyers is simply α_h (resp. α_l). Moreover, the assumption $\alpha_h < 1$ means that even the high-type buyers are less skilled than the original asset holders. Therefore, liquidation of any assets to potential buyers will incur efficiency losses to some extent. But, the size of the losses will be lower if the assets are liquidated to high-type buyers rather than to low-type buyers. These two facts will crucially affect total welfare in this economy. Each market i has a measure $f^i \in [0, \infty)$ of high-type buyers and an infinite measure of low-type buyers.

Lastly, all potential buyers are financially constrained. To be specific, every potential buyer is allowed to purchase at most one unit of the asset. This simple form of financial constraint that imposes limits on the total asset size rather than on the total budget size is widely used in the literature; for instance, see Duffie *et al.* (2005) and He and Milbradt (2014). But then, due to risk neutrality, each potential buyer will buy only 0 or 1 unit of the asset without loss of generality.

2.2 Equilibrium definition

This section solves each individual investor's problem and then defines an equilibrium. To start with, every high-type buyer in market i solves the following profit-maximization problem:

$$\max\{0, \alpha_h - p^i, \alpha_h - p^{-i} - \kappa\}, \tag{1}$$

which means she (1) earns nothing if she does not buy any assets, (2) earns $\alpha_h - p^i$ if she buys an asset from market i and (3) earns $\alpha_h - p^{-i} - \kappa$ if she buys an asset from market $-i$. Therefore, the high-type buyer in market i weakly prefers:

$$\begin{cases} \text{not to buy any assets,} & \text{if } \alpha_h \le \min\{p^i, p^{-i} + \kappa\} \\ \text{to buy an asset from market } i, & \text{if } p^i \le \min\{\alpha_h, p^{-i} + \kappa\} \\ \text{to buy an asset from market } -i, & \text{if } p^{-i} + \kappa \le \min\{\alpha_h, p^i\}. \end{cases}$$

Note that in this model, the assets may trade at a price lower than α_h, because every high-type buyer is financially constrained and the number of high-type buyers is limited. High-type buyers can thus make positive profits in such a case where the assets are priced lower than their intrinsic value for the high-type buyers. The literature on limits of arbitrage investigates this type of a phenomenon in depth.

Each low-type buyer in market i solves a similar problem to (1) but with α_l in place of α_h. However, note that both p^A and p^B must lie in $[\alpha_l, \alpha_h]$ in equilibrium; otherwise, the markets cannot clear. Thus, a low-type buyer in market i has no strong incentives to buy an asset from market $-i$. When $p^i > \alpha_l$, she does not have any incentives to buy an asset from market i, either. When $p_i = \alpha_l$, however, she is indifferent between buying an asset from market i and not buying any assets.

Meanwhile, solving an original asset holder's problem is simple. Since p^i lies between α_l and α_h, every original asset holder optimally retains her asset unless she receives the liquidity shock. Therefore, $q^i(p^i)$ indeed represents the supply curve in market i.

An equilibrium is then defined as a price pair (p^A_*, p^B_*) that jointly clears both markets. Section 4.3 provides an equilibrium construction algorithm, which is actually accessible without a need to digest the full credit-risk model. Here, the main result of the paper is first presented via two crucial examples. In those examples, characterizing an equilibrium is fairly straightforward.

2.3 Fully segmented markets

Before describing those examples, this section considers a case in which the two markets are fully segmented, i.e. $\kappa = \infty$. In this case, we can focus on a single market i because no interactions occur between the two markets. In such a single market, at most three equilibria arise. First, $p^i_* = \alpha_h$ can be an equilibrium if $q^i(\alpha_h) \le f^i$. In this equilibrium, the marginal buyer is a high-type buyer, but every high-type buyer is actually indifferent between buying an asset and not buying any assets. Second, p^i_* such that $\alpha_l < p^i_* < \alpha_h$ can be an equilibrium if $q^i(p^i_*) = f^i$. In this equilibrium, the marginal buyer is still a high-type buyer, but every high-type buyer strictly prefers to buy an asset. Third, $p^i_* = \alpha_l$ can be an equilibrium if $q^i(\alpha_l) \ge f^i$. In this equilibrium, the marginal buyer is a low-type buyer, but every low-type buyer is indifferent between buying an asset and not buying any assets.

Figure 1 plots a case in which $q^i(\alpha_h) < f^i < q^i(\alpha_l)$ so that the market has all three equilibria. These equilibria are commonly called self-fulfilling equilibria. That is, if the market

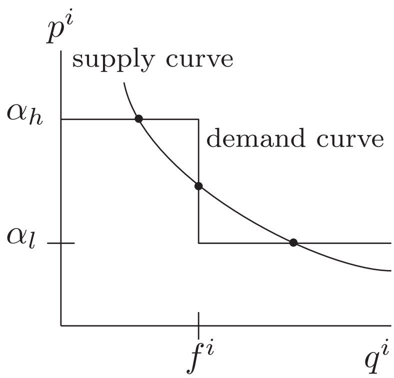


Figure 1.
This figure plots the supply and demand curves in a single isolated market i

participants believe the price will be high, then less of the assets will be liquidated since the supply curve is downward sloping. The self-conjectured high price can then clear the market because the asset supply is low. The other two equilibria that have either intermediate or low price levels can be similarly understood.

Meanwhile, when $f^i > q^i(\alpha_l)$, a unique equilibrium obtains, in which $p^i_* = \alpha_h$. When $f^i < q^i(\alpha_h)$, the economy also has a unique equilibrium in which $p^i_* = \alpha_l$. In other words, a unique equilibrium arises if the market is either sufficiently liquid or severely illiquid.

2.4 Partially segmented markets

This section considers the case of $\kappa < \infty$. Here, we can further assume that $\kappa < \alpha_h - \alpha_l$; otherwise, even the maximum possible price gap cannot exceed the entry cost, bringing us back to the case in which the markets are fully segmented. Also, for ease of exposition, assume that the supply curves in the two markets are identical, i.e. $q^A(p) = q^B(p)$ for every p. In other words, the supply sides in both markets behave the same way. Denote this common supply curve by $q(p)$.

2.4.1 Example 1. Figure 2 describes an example that shows the negative effects of reducing the entry cost. The left panel plots an equilibrium price pair (p^A_∞, p^B_∞), assuming $\kappa = \infty$ for the moment. The right panel plots how the equilibrium changes when κ drops below $\alpha_h - \alpha_l$. When $\kappa = \infty$, the equilibrium is given by $(p^A_\infty, p^B_\infty) = (\alpha_h, \alpha_l)$. In this equilibrium, all failed assets in market A are absorbed by high-type buyers in that market, whereas market B does not have enough high-type buyers who can fully absorb the liquidated assets in that market.

However, when κ falls below $\alpha_h - \alpha_l$, the price pair (p^A_∞, p^B_∞) cannot be sustained any more as an equilibrium, because if it were, all high-type buyers in market A would move to market B to exploit the price gap that is larger than the entry cost. As such, in a new equilibrium, some high-type buyers of measure g in market A must leave for market B as in the right panel. The outflows of these buyers shift the demand curve in market A (resp. B) to the left (resp. right) by g. The reduced demand in market A then pushes down the price in that market to $p^A_* < \alpha_h$. But, the price in market B remains the same at α_l, because $f^B + g$ is still not big enough to fully absorb the liquidated assets in market B.

Note that g is endogenously determined in this model. Specifically, in equilibrium, the high-type buyers remaining in market A must earn the same profits as the other high-type buyers who buy assets from market B. So, after a measure g of high-type buyers move to market B, the new price gap $p^A_* - p^B_*$ must be equalized to the entry cost κ. In this example,

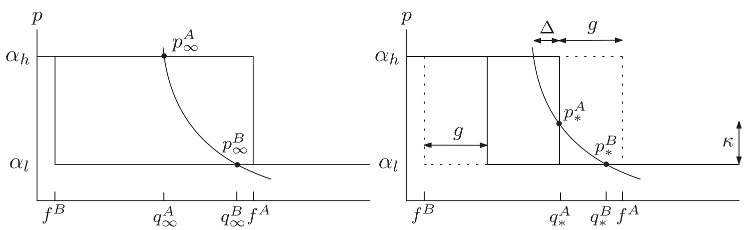


Figure 2. The economy in Example 1

Note(s): The left panel describes the case of $\kappa = \infty$. The right panel plots the case of $\kappa < \alpha_h - \alpha_l$. In both panels, the black and orange lines denote the demand curves in market A and market B, respectively there is a unique g satisfying this condition because $f^A - q^A_\infty < q^A_\infty - f^B$, where $q^A_\infty = q(\alpha_h)$, as shown in the figure.

The key feature of this example is that the asset quantity liquidated in market A increases by Δ, whereas that quantity in market B remains the same. This outcome is obtained because although market A is relatively more liquid, it is not sufficiently liquid, whereas market B is severely illiquid. That is, under this market condition, the reduction in the entry cost decreases the price in market A, but does not improve the price in market B, as seen before. Then, since the supply curve is downward sloping, such price changes trigger more liquidation in market A, but do not affect market B. The welfare implications of this outcome are discussed in Section 2.5.

2.4.2 Example 2. Figure 3 depicts another example that shows the positive effects of lowering the entry cost. As before, the left panel plots an equilibrium (p^A_∞, p^B_∞) for the case of $\kappa = \infty$, whereas the right panel plots an equilibrium (p^A_*, p^B_*) for the case of $\kappa < \alpha_h - \alpha_l$. When $\kappa = \infty$, the equilibrium is again given by $(p^A_\infty, p^B_\infty) = (\alpha_h, \alpha_l)$. But, when the entry cost drops below $\alpha_h - \alpha_l$, some high-type buyers of measure g in market A move to market B, where g is endogenously determined as before. That is, after a measure g of high-type buyers in market A move to market B, the adjusted price gap $p^A_* - p^B_*$ must be equalized to the entry cost κ. This example also has a unique g satisfying this condition, because $f^A - q^B_\infty > q^B_\infty - f^B$, where $q^B_\infty = q(\alpha_l)$, as shown in the figure.

Unlike in the previous example, the price in market A now remains the same at the highest level α_h, whereas the price in market B is pushed up to p^B_*. This different outcome is obtained because market A is sufficiently liquid in the present example. In other words, even after losing some high-type buyers, market A still has enough high-type buyers who can fully absorb the liquidated assets in that market. As a result, the asset quantity liquidated in market A remains unchanged, whereas that quantity in market B decreases by Δ. We discuss the welfare implications of this outcome in the next section.

2.5 Welfare

In this risk-neutral world, total welfare is defined as the present value of total outputs in the economy net of the total entry costs incurred. Here, the entry costs are assumed as deadweight costs, but this assumption does not change the main result of the model qualitatively. Also, total welfare is interchangeably called total surplus in what follows. Specifically, let (p^A_*, p^B_*) denotes an equilibrium price pair. Then, $q(p^i_*)$ units of the asset are liquidated in market i. Among those assets, let q^i_h denote the asset quantity liquidated to high-

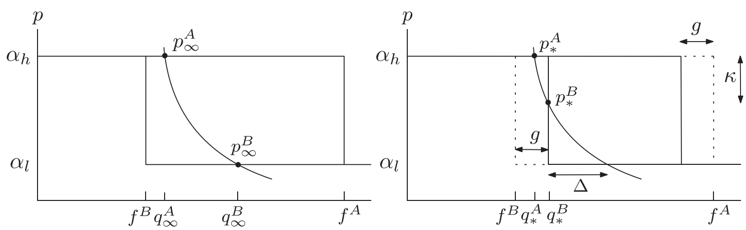


Figure 3.
The economy in Example 2

Note(s): The left panel describes the case of $\kappa = \infty$. The right panel plots the case of $\kappa < \alpha_h - \alpha_l$. In both panels, the black and orange lines denote the demand curves in market A and market B, respectively type buyers, some of whom might have immigrated from market $-i$. Then, $q_l^i := q(p_*^i) - q_h^i$ is the asset quantity liquidated to low-type buyers. Moreover, let g denote the total number of potential buyers who leave their own market. Total welfare $W(\kappa)$ is then equal to:

$$W(\kappa) = 2 - \underbrace{\left[\kappa g + \sum_{i \in \{A,B\}} \{(1-\alpha_h)q_h^i + (1-\alpha_l)q_l^i\}\right]}_{\text{efficiency losses}}. \quad (2)$$

The first term on the right-hand side is the maximum possible total surplus that can be achieved when all potential buyers have full productivity of 1. The second term κg is the total entry costs paid by immigrating potential buyers. The third term $(1-\alpha_h)q_h^i$ measures the efficiency losses incurred by high-type buyers who purchase assets from market i. The fourth term $(1-\alpha_l)q_l^i$ measures the efficiency losses incurred by low-type buyers who purchase assets from market i.

2.5.1 Example 1. Returning to the first example in Figure 2, recall that when $\kappa = \infty$, all failed assets in market A are fully absorbed by high-type buyers in that market. Meanwhile, in market B, f^B units of the asset are liquidated to high-type buyers, and the remaining $q_\infty^B - f^B$ units of the asset are liquidated to low-type buyers. Also, no potential buyers quit their own market, because $\kappa = \infty$. Thus, the definition in (2) implies that total welfare equals:

$$W_1 = 2 - \left[(1-\alpha_h)q_\infty^A + (1-\alpha_h)f^B + (1-\alpha_l)\left(q_\infty^B - f^B\right)\right].$$

When the entry cost drops to κ such that $\kappa < \alpha_h - \alpha_l$, the asset quantity liquidated in market A increases from q_∞^A to $q_\infty^A + \Delta$. But all those failed assets are still fully absorbed by high-type buyers, as shown in Figure 2. Meanwhile, in market B, due to the influx of high-type buyers of measure g, $f^B + g$ units of the asset are liquidated to high-type buyers, whereas $q_\infty^B - f^B - g$ units of the asset are still liquidated to low-type buyers. Total welfare is therefore given by:

$$W_2 = 2 - \left[\kappa g + (1-\alpha_h)(q_\infty^A + \Delta) + (1-\alpha_h)\left(f^B + g\right) + (1-\alpha_l)\left(q_\infty^B - f^B - g\right)\right],$$

again according to the definition in (2).

From the above two results, the net change in total welfare is equal to:

$$\Delta W = W_2 - W_1 = \underbrace{\underbrace{-\kappa g}_{\text{entry costs}} + \underbrace{(\alpha_h - \alpha_l)g}_{\text{asset reallocation}}}_{\text{always positive}} - \underbrace{(1-\alpha_h)\Delta}_{\text{more liquidation}}. \quad (3)$$

The first term $-\kappa g$ is the total entry costs incurred after the change in the entry cost. The second term $(\alpha_h - \alpha_l)g$ is the efficiency gains created by asset reallocation, i.e. g units of the asset in market B are now liquidated to high-type buyers rather than to low-type buyers. The third term $(1 - \alpha_h)\Delta$ is the efficiency losses caused by an increment in the asset quantity liquidated in market A.

Note that $-\kappa g + (\alpha_h - \alpha_l)g$ is always positive because $\kappa < \alpha_h - \alpha_l$. That is, when the market barriers are lowered, the efficiency gains generated by asset reallocation always dominate the deadweight costs incurred by capital movements. In this regard, all else being equal, total welfare must increase. However, recall that the capital outflows from market A to market B trigger more asset sell-offs in market A, and therefore, welfare within market A decreases as expressed in the third term above.

Crucially, the net change in total welfare ΔW can be negative, especially when f^A is slightly larger than q^B_∞ and κ is sufficiently small. As an extreme example, suppose $f^A = q^B_\infty + \epsilon$ and $\kappa = 0$, where ϵ is sufficiently small. Then, similarly as in Figure 2, the economy obtains an equilibrium such that $p^A_* = p^B_* = \alpha_l$, $g = \epsilon$ and $\Delta = q^B_\infty - q^A_\infty$. The formula in (3) then implies $\Delta W = (\alpha_h - \alpha_l)\epsilon - (1 - \alpha_h)\Delta < 0$, because ϵ is small but Δ is sizable. The same result holds when κ is sufficiently small but not exactly equal to 0.

We can understand this main result as follows. When f^A is slightly larger than q^B_∞ and κ is sufficiently small, the price in market A drops substantially, even when only a small number of high-type buyers in market A move to market B. The price gap between the markets would then shrink substantially as well. But, because the entry cost κ itself is small, such a small price gap is enough to attract some high-type buyers in market A to market B. Yet, the benefits created by such capital movements are minuscule, because only a small number of buyers leave market A. However, the aforementioned large price drop in market A causes substantially more assets to be liquidated; therefore, total welfare decreases.

We have thus far considered only a scenario in which the entry cost is reduced from ∞ to some finite number $\kappa < \alpha_h - \alpha_l$. But, using a similar argument, we can also show that total welfare $W(\kappa)$ decreases as the entry cost κ is reduced continuously, especially when the entry cost itself is small. The proof for this result is included in Theorem 2.1.

Theorem 2.1. Suppose that $f^A - q^A_\infty < q^A_\infty - f^B$ and f^A is slightly larger than q^B_∞. Then, the economy has a unique equilibrium, and moreover, (1) $W(0) < W(\infty)$ and (2) $W'(\kappa) > 0$ when κ is small enough.

Proof. See Appendix A.1 for an omitted proof.

2.5.2 Example 2. In the second example in Figure 3, when $\kappa = \infty$, the total welfare W_1 is the same as that in the previous example, i.e.:

$$W_1 = 2 - \left[(1-\alpha_h)q^A_\infty + (1-\alpha_h)f^B + (1-\alpha_l)\left(q^B_\infty - f^B\right)\right]. \tag{4}$$

However, when the entry cost drops to κ such that $\kappa < \alpha_h - \alpha_l$, the asset quantity liquidated in market A remains the same, whereas that quantity in market B decreases from q^B_∞ to $q^B_\infty - \Delta$. Moreover, all those failed assets in market B are now liquidated to high-type buyers of measure $f^B + g$. That is, $q^B_\infty - \Delta$ is equal to $f^B + g$ in this example. Total welfare is thus given by:

$$W_2 = 2 - \left[\kappa g + (1-\alpha_h)q^A_\infty + (1-\alpha_h)\left(f^B + g\right)\right].$$

by the definition in (2).

The above two results imply that the net change in total welfare is equal to:

$$\Delta W = W_2 - W_1 = \underbrace{\underbrace{-\kappa g}_{\text{entry costs}} + \underbrace{(\alpha_h - \alpha_l)g}_{\text{asset reallocation}}}_{\text{always positive}} + \underbrace{(1-\alpha_l)\Delta}_{\text{less liquidation}},$$

which used the fact that $q^B_\infty - \Delta = f^B + g$ again. The sum of the first two terms is positive as in the previous example. The only difference is the third term, which measures the efficiency gains generated by a decrement in asset liquidation in market B. Because this term is positive as well, ΔW must be positive. Lastly, even when the entry cost is reduced continuously, total welfare increases. The proof for this result is omitted, because we can similarly follow the proof arguments used in Theorem 2.1.

Theorem 2.2. Suppose $f^A - q^B_\infty > q^B_\infty - f^B > 0$. Then, a unique equilibrium is obtained, and moreover, total welfare increases as the entry cost decreases.

3. Full credit-risk model

This section develops a full credit-risk model with two markets (or sectors), in which each individual firm issues short-term debt, by extending He and Xiong (2012b) [1]. The two markets are called market A and market B. Each market consists of a primary debt market and a secondary market for assets in default. The secondary markets are segmented from each other. Each primary debt market faces perfect competition as in Leland (1994).

Time flows continuously over $[0, \infty)$. Each market $i \in \{A, B\}$ consists of a continuum of firms, indexed by (i, j). All the firms in the economy are *ex ante* identical. Each firm has a representative equityholder and many small creditors. Each secondary market is populated with potential asset buyers. All the market participants are risk-neutral and have a discount rate r. The model focuses on a steady-state equilibrium for tractability.

3.1 Firm assets

Each firm (i, j) has a risky asset that generates cash flows $x^{ij}_t dt$ per unit time interval $[t, t + dt)$. The corporate taxes are ignored. The cash flow x^{ij}_t evolves according to:

$$\frac{dx^{ij}_t}{x^{ij}_t} = \mu dt + \sigma dZ^{ij}_t,$$

where μ is the growth rate, σ is the volatility and Z^{ij}_t is a standard idiosyncratic Brownian motion. We hereafter interpret the cash flow x^{ij}_t as the size of the asset at time t. All assets are perfectly divisible.

The asset does not live forever. Instead, the asset dies exogenously at a random date that arrives with Poisson intensity $\phi > 0$. This exogenous death does not indicate a bankruptcy event, but indicates a situation where the firm's machines or equipment have reached the end of their lives. This assumption is needed to obtain a steady-state equilibrium.

In this setting, the firm's unlevered value at time t is equal to:

$$F\left(x^{ij}_t\right) = E_t\left[\int_t^\infty \phi e^{-\phi(s-t)}\left(\int_t^s e^{-r(u-t)} x^{ij}_u du\right) ds\right] = \frac{x^{ij}_t}{\rho},$$

where $\rho = r + \phi - \mu$. To ensure this value is finite, assume $r + \phi > \mu$. The indexes i, j or t will be often omitted when doing so that they do not cause any confusion.

3.2 Firm liability and default

Each firm has a continuum of bonds of one unit. Each bond pays a coupon cdt per unit time and a principal P at the maturity date. The maturity date is not predetermined. Instead, each bond matures at a random date that arrives with Poisson intensity λ, independently of any other events. Thus, a fraction λdt of the firm's outstanding bonds are retired at every time. In other words, the average debt maturity is $m = \frac{1}{\lambda}$. Whenever a bond matures, the firm issues a new bond under the same contract terms as all other existing bonds. Therefore, the total units of the bonds remain the same. All these assumptions are commonly used in the literature for simplicity; see Leland and Toft (1996), Hackbarth *et al.* (2006) and He and Xiong (2012a).

The net cash flow to the firm at time t is then given by:

$$x_t - c + \lambda(D(x_t) - P),$$

where $D(x_t)$ is the firm's debt value at time t. The first term is the cash flows from the asset, and the second term is the coupon payments. The third term $\lambda(D(x_t) - P)$ indicates the rollover gains. That is, at each point in time, the firm pays λP as the principal payment and receives $\lambda D(x_t)$ by issuing new bonds.

An equityholder of the firm, who has a deep pocket, can default at any point in time, because she has limited liability. Specifically, the equityholder keeps servicing the debt payment as long as the equity value is positive. But, when the equity value hits zero, the equityholder decides to default. As such, we can reasonably postulate that there are two thresholds, x_D^A and x_D^B, such that each firm (i, j) optimally defaults when its cash flow x_t^{ij} hits x_D^i. The default thresholds will be endogenously determined.

When a firm in market i defaults, its creditors take over the firm's existing asset. The creditors then liquidate the asset in the secondary market within the same sector i. In this regard, the liquidity shock in the static model exactly corresponds to the default event in the present model. After default, the firm exits the economy.

3.3 New entrants

Recall that every firm exits the economy when its asset dies or its equityholder decides to default. To keep stationarity of the economy, the model assumes that a new firm is born whenever such an event happens. Every new firm invests in a new asset of size x_N by issuing both equity and bonds, where x_N is exogenously given. Specifically, every new firm issues one unit of a bond that has the same contract terms as the bonds described above. Equityholders of the firm are residual claimants by definition and make the default decision as above. Both bonds and equity are issued at the break-even prices. Moreover, x_N can be always chosen to be larger than x_D^i for every i, because the maximum possible default threshold can be expressed explicitly.

3.4 Secondary markets

The secondary markets behave almost the same way as in the simple static model. Specifically, the secondary market in each sector i consists of high-type buyers of measure f^i and low-type buyers of infinite measure. Each high-type buyer has productivity α_h, meaning that the outputs of an asset will be reduced by a fraction $1 - \alpha_h$ under her management. Thus, she values one unit of the asset as $\frac{\alpha_h}{\rho}$. Similarly, each low-type buyer has productivity α_l, meaning that she values the same asset as $\frac{\alpha_l}{\rho}$. A high-type buyer (respectively low-type buyer) is called h-type buyer (resp. l-type buyer).

Let p^i denote the liquidation price per unit asset size in market i. Put differently, the creditors of a failed firm in market i receive $p^i x_D^i$ as the liquidation proceeds, because each failed asset in that market is of size x_D^i. Taking the price pair (p^A, p^B) as given, at each point in time, every potential buyer in market i makes one of three decisions: (1) she can buy one unit of the asset from market i at the price p^i, (2) she can buy one unit of the asset from market $-i$ at the price p^{-i} plus the entry cost κ or (3) she can choose not to buy any assets. That is, every potential buyer in market i can buy at most one unit of the asset at each point in time but must pay κ additionally to buy an asset from market $-i$ [2].

In this setting, every k-type buyer in market i, where $k \in \{l, h\}$, solves the following profit-maximization problem:

$$\max\left\{0, \frac{\alpha_k}{\rho} - p^i, \frac{\alpha_k}{\rho} - p^{-i} - \kappa\right\}, \tag{5}$$

which can be understood the same way as in the static model. As such, the potential buyers in this dynamic model essentially behave the same way as the potential buyers in the static model.

3.5 Equilibrium

An equilibrium in this economy is defined as a collection of default thresholds (x^A_{D*}, x^B_{D*}) and liquidation prices (p^A_*, p^B_*) such that (1) given p^i_*, the default threshold x^i_{D*} is individually optimal for every equityholder in market i, (2) given (p^A_*, p^B_*), every potential buyer behaves optimally, and (3) the price pair (p^A_*, p^B_*) jointly clears both secondary markets.

4. Model solutions

This section solves the model. We first analyze the supply side in each market and then characterize an equilibrium.

4.1 Equity and debt values

Let $E(x; p^i)$ and $D(x; p^i)$ denote the equity value and debt value in market i, respectively, for any liquidation price p^i. Also, let $x^i_D = x_D(p^i)$ be an optimal default threshold in market i, which also depends on p^i. Then, we can first compute the debt value as follows. The required return on debt must be the same as the risk-free rate in the risk-neutral world. A standard continuous-time technique can then be used to show that $D(x)$ satisfies the following Hamilton–Jacobi–Bellman (HJB) equation:

$$rD(x) = c + \lambda(P - D(x)) - \phi D(x) + \mu x D_x(x) + \frac{\sigma^2}{2} x^2 D_{xx}(x), \tag{6}$$

subject to $D(x^i_D) = p^i x^i_D$. The left-hand side denotes the required return on debt. On the right-hand side, the first term is the coupon payment. The second term is the principal payment net of the continuation value of debt. The third term denotes the exogenous asset death event. The remaining terms explain how the debt value changes due to the fluctuations in the asset size. The boundary condition indicates the liquidation proceeds the creditors receive when the firm defaults.

The closed-form solution for the above ordinary differential equation is given by:

$$D(x) = \frac{c + \lambda P}{r + \lambda + \phi} + \left(p^i x^i_D - \frac{c + \lambda P}{r + \lambda + \phi}\right)\left(\frac{x}{x^i_D}\right)^{\xi},$$

where

$$\xi = \frac{-\mu + \frac{\sigma^2}{2} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r + \lambda + \phi)}}{\sigma^2} < 0.$$

To clarify, both p^i and x^i_D are taken as given when calculating the debt value.

Next, the equity value $E(x; p^i)$ is computed as follows. As before, a standard continuous-time method says that $E(x)$ satisfies the following HJB equation:

$$rE(x) = x - c + \lambda(D(x) - P) - \phi E(x) + \mu x E_x(x) + \frac{\sigma^2}{2} x^2 E_{xx}(x), \tag{7}$$

subject to $E(x_D^i) = 0$ and $E_x(x_D^i) = 0$. The left-hand side is the required return on equity. On the right-hand side, the first three terms indicate the net cash flows to equity. The fourth term denotes the exogenous death event. The remaining terms explain how the equity value changes due to the fluctuations in the asset size. The first boundary condition means that the equityholder gets nothing upon default. The second boundary condition is the so-called smooth-pasting condition, meaning that the equityholder is indifferent between defaulting and not defaulting when $x = x_D^i$.

The closed-form solutions for $E(x)$ and x_D^i are given by:

$$E(x) = \frac{\pi c}{r+\phi} + \frac{x}{r+\phi-\mu} + Ax^{\eta} - \frac{c+\lambda P}{r+\lambda+\phi} - \left(p^i x_D^i - \frac{c+\lambda P}{r+\lambda+\phi}\right)\left(\frac{x}{x_D^i}\right)^{\xi},$$

$$x_D^i = \frac{-\xi(r+\phi-\mu)(c+\lambda P)(r+\phi)}{(1-\eta+(\eta-\xi)(r+\phi-\mu)p^i)(r+\phi)(r+\lambda+\phi)}, \tag{8}$$

where

$$\eta = \frac{-\mu + \frac{\sigma^2}{2} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\phi)}}{\sigma^2} \in (\xi, 0). \tag{9}$$

The coefficient A is computed in Appendix A.2. Importantly, formula (8) implies that an optimal default threshold x_D^i decreases in p^i. This result is intuitively clear: When the liquidation price goes down, the debt value decreases, and therefore, the equityholder defaults earlier to avoid increased rollover risk. This fact will be used to generate a downward-sloping supply curve in the next section.

4.2 Aggregation

In this model, the supply curves in the two secondary markets are identical, because all firms are *ex ante* identical. Let $q(p)$ denote such a common supply curve, i.e. $q(p^i)$ measures the asset quantity liquidated in any single market i per unit time for any given liquidation price p^i. An optimal default threshold in market i is still denoted by $x_D^i = x_D(p^i)$.

4.2.1 Steady-state distributions. Let $m(x; p^i)$ indicate the steady-state distribution of the asset size in market i for any given price p^i. That is, $m(x; p^i)$ is the number of firms in market i whose asset size is equal to x. A standard continuous-time technique then says that $m(x)$ satisfies the following Kolmogorov forward equation:

$$0 = -\phi m(x) - \frac{\partial}{\partial x}[\mu x m(x)] + \frac{\partial^2}{\partial x^2}\left[\frac{\sigma^2}{2}x^2 m(x)\right], \quad \forall x \neq x_N, \tag{10}$$

subject to $m(x_D^i) = 0$; see Stokey (2009). Specifically, the first term on the right-hand side is the number of firms exiting the market because of the exogenous death events. The other two terms indicate how $m(x)$ changes due to the fluctuations in the asset sizes of individual firms. In a steady state, the sum of all those terms must be zero, except for $x = x_N$. Equation (10) does not hold at $x = x_N$ because some firms newly enter the market with new assets of size x_N. Nonetheless, $m(x)$ satisfies the value-matching condition at $x = x_N$. The boundary condition $m(x_D^i) = 0$ means that every firm exits the market immediately upon default. Lastly, the following fact will be used several times later: The number of firms in market i that default over unit time interval is equal to $\frac{\sigma^2}{2}(x_D^i)^2 m_x(x_D^i)$. See Luttmer (2012) for heuristic derivation of this well-known formula.

Note that any multiples of $m(x)$ satisfy the same equation (10). As such, $m(x)$ needs to be normalized to examine the effects of any policy changes in a fair manner. Remind that in the static model, each market achieves the maximum possible total surplus of 1 in case where all potential buyers have a full productivity of 1. We similarly normalize $m(x)$ here to induce each market to have the maximum possible total surplus of $\frac{1}{r}$ when all potential buyers have a full productivity of 1. That is, the following normalization condition is imposed:

$$\frac{1}{r}\left[\underbrace{\int_{x_D^i}^{\infty} xm(x)dx}_{\text{from existing firms}} + \underbrace{\frac{\sigma^2}{2}\left(x_D^i\right)^2 m_x\left(x_D^i\right)F\left(x_D^i\right)}_{\text{from failed assets}}\right] = \frac{1}{r}. \tag{11}$$

The first term in the bracket denotes the total outputs produced by all existing firms in market i per unit time. Regarding the second term, recall that $\frac{\sigma^2}{2}\left(x_D^i\right)^2 m_x\left(x_D^i\right)$ counts the total number of defaulted firms in market i per unit time. Thus, the second term above denotes the maximum possible present value of future outputs produced by the assets liquidated today. In a steady state, the left-hand side in (11) then equals the maximum possible date-0 value of the total future outputs in market i. This quantity is normalized to $\frac{1}{r}$. In other words, the maximum possible total surplus in each market per unit time is normalized to 1.

The closed-form solution for $m(x)$ satisfying all the above conditions is given by:

$$m(x) = \begin{cases} A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x \in \left[x_D^i, x_N\right] \\ A_3 x^{\eta_2}, & \text{if } x \in [x_N, \infty), \end{cases}$$

where the expressions for A_1, A_2, A_3, η_1 and η_2 are included in Appendix A.3. The model assumes $\eta_2 < -2$ to ensure that the left-hand side in (11) is finite.

4.2.2 Supply curves. From the steady-state distribution $m(x; p)$, the common supply curve $q(p)$ is computed as:

$$q(p) = \underbrace{\frac{\sigma^2}{2}(x_D(p))^2 m_x(x_D(p); p)}_{\text{number of defauted firms}} \times \overbrace{x_D(p)}^{\text{asset size of a failed firm}} . \tag{12}$$

That is, in any single market with a liquidation price p, $q(p)$ equals the number of defaulted firms per unit time in that market multiplied by the asset size of each failed firm. Importantly, $q(p)$ indeed decreases in p, i.e. the supply curve is downward sloping. Appendix A.3 provides a technical proof for this property. Intuitively, when the liquidation price decreases, equityholders default earlier to avoid increased rollover risk, and therefore, more assets will be liquidated.

4.3 Equilibrium construction

This section characterizes an equilibrium. When the two markets are completely segmented, we can find an equilibrium as in Section 2.3 by replacing α_k in that section with $\frac{\alpha_k}{\rho}$ for each $k \in \{l, h\}$. Thus, this section focuses on the case where the markets are partially segmented, i.e. $\kappa < \frac{\alpha_h - \alpha_l}{\rho}$.

Without loss of generality, we can look for an equilibrium in which $p_*^A \geq p_*^B$. In such an equilibrium, if any, some high-type buyers of measure g in market A purchase assets from market B, where $g \in [0, f^A]$, but no potential buyers in market B move to market A. To find this endogenous variable g, we proceed as follows.

First, guess $g = 0$ and find an equilibrium price p_*^i in each market i, assuming this market is fully isolated. Then, examine whether (p_*^A, p_*^B) satisfies $p_*^A - p_*^B \leq \kappa$. If this condition holds, the potential buyers in market A indeed have no incentives to enter market B, justifying $g = 0$ can be sustained in equilibrium.

Second, guess $0 < g < f^A$ and find an equilibrium price p_*^A in market A, assuming this market is fully isolated but has high-type buyers of measure $f^A - g$ instead of f^A. Similarly, find an equilibrium price p_*^B in market B, assuming this market is fully isolated but has high-type buyers of measure $f^B + g$. Then, examine whether (p_*^A, p_*^B) satisfies $p_*^A - p_*^B = \kappa$. If this condition holds, every high-type buyer in market A is indeed indifferent between buying an asset from market A and from market B, justifying $g \in (0, f^A)$ can be sustained in equilibrium.

Lastly, guess $g = f^A$ and find p_*^A and p_*^B as in the second case. Then, examine whether (p_*^A, p_*^B) satisfies $p_*^A - p_*^B \geq \kappa$. If this condition holds, every high-type buyer in market A indeed prefers to buy an asset from market B rather than from market A, justifying $g = f^A$ can be sustained in equilibrium.

Using these three steps, all equilibria can be pinned down at least numerically. In particular, the economy has a unique equilibrium when $q_\infty^B < f^A < 2q_\infty^A - f^B$ or $0 < q_\infty^B - f^B < f^A - q_\infty^B$, as seen in Examples 1 and 2. One more example is presented in the next section, which has multiple equilibria. This example will be used to show that the effects of any policies can be different even qualitatively, depending on which equilibrium is selected. Regarding the existence of equilibrium, Theorem 4.1 shows that this economy has at least one equilibrium.

Theorem 4.1. This economy has at least one equilibrium.

Proof. See Appendix A.4.

4.3.1 Example 3. Figure 4 shows an example that has multiple equilibria unlike in Examples 1 and 2. Multiple equilibria emerge in this example, because market A is not sufficiently liquid and market B is mildly illiquid. The left panel in the figure implies that g cannot be chosen to be 0, because if it were, the price gap would be larger than κ. The other three panels exhibit all three equilibria of this example.

When multiple equilibria exist, the market participants' self-fulfilling beliefs determine which equilibrium arises, as discussed via Figure 1. That is, when the market participants believe the prices will be high as in the second panel of Figure 4, the markets behave in a way that justifies such optimistic beliefs can be sustained as rational expectations. As a result, the

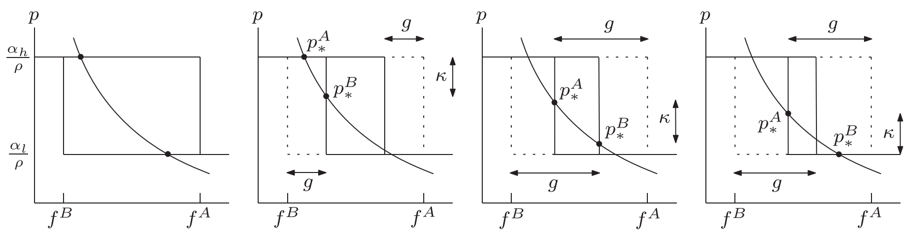


Figure 4.
The economy in Example 3

Note(s): The first panel describes the case of $\kappa = \infty$. The other panels exhibit all three equilibria for the case of $\kappa < \frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho}$

price in market A remains at the highest level $\frac{\alpha_h}{\rho}$ when the entry cost is reduced from ∞ to some finite number $\kappa < \frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho}$. In this regard, this equilibrium is called the best equilibrium.

By contrast, when the market participants believe the prices will be low as in the fourth panel, the markets again behave in a certain way that justifies such pessimistic beliefs can be sustained as rational expectations. Therefore, the price in market B remains at the lowest level $\frac{\alpha_l}{\rho}$ when the entry cost is lowered from ∞ to some finite number $\kappa < \frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho}$. This equilibrium is thus said to be the worst equilibrium. The equilibrium in the third panel, which has intermediate price levels, can be similarly understood. This equilibrium is termed the intermediate equilibrium.

4.4 Welfare

Total welfare in this economy is similarly defined as in the static model. That is, since we focus on a steady-state equilibrium, total welfare can be defined as the total outputs from both existing firms and failed assets per unit time, net of the total entry costs incurred per unit time. Specifically, let (p_*^A, p_*^B) denote an equilibrium price pair such that $p_*^A \geq p_*^B$ without loss of generality. Then, by definition, $q(p_*^i)$ units of the asset are liquidated in market i per unit time. Among those assets, let q_h^i denote the asset quantity liquidated to high-type buyers. Then, $q_l^i := q(p_*^i) - q_h^i$ is the asset quantity liquidated to low-type buyers. More explicitly,

$$q_h^A = \begin{cases} q\left(p_*^A\right), & \text{if } \frac{\alpha_l}{\rho} < p_*^A \leq \frac{\alpha_h}{\rho} \\ f^A - g, & \text{if } p_*^A = \frac{\alpha_l}{\rho}, \end{cases} \qquad q_h^B = \begin{cases} q\left(p_*^B\right), & \text{if } \frac{\alpha_l}{\rho} < p_*^B \leq \frac{\alpha_h}{\rho} \\ f^B + g, & \text{if } p_*^B = \frac{\alpha_l}{\rho}, \end{cases}$$

where g is the number of high-type buyers in market A who purchase assets from market B. From the normalization condition in (11), total welfare is then given by:

$$W = 2 - \underbrace{\left[\kappa g + \sum_{i \in \{A,B\}} \left\{ \frac{(1-\alpha_h)q_h^i}{\rho} + \frac{(1-\alpha_l)q_l^i}{\rho} \right\}\right]}_{\text{efficiency losses}}, \tag{13}$$

as in the static model.

5. Model implications

This section studies model implications. We first discuss the effects of reducing the entry cost and then examine the effects of liquidity injection as well. Interestingly, the liquidity injection program may also hurt total welfare.

5.1 Parameter values

Table 1 summarizes the baseline parameter values that are used in this model. The risk-free rate r is set to 4%, because the one-year treasury rate over the period from 1998 to 2007 was around 3.80%. As in He and Xiong (2012b), we set the debt maturity m to one year to highlight the effects of rollover risk facing firms issuing short-term debt. As the average time-to-maturity of non-financial firms is around three years according to Custódio *et al.* (2013) and the maturity of commercial papers issued by financial firms is generally less than nine months, this parameter choice is reasonable. The principal payment P is normalized to

100, and the coupon size c is set to 9. In the literature, a coupon rate close to 10% is widely used to represent BB-rated corporate bonds; see, e.g. He and Xiong (2012b). The exogenous death rate ϕ is set to 4% as in Miao (2005). In fact, according to Dunne *et al.* (1988), the annual turnover rate in the US manufacturing industry is around 7%. Together with the endogenous default events, the present model produces a similar turnover rate. The asset growth rate μ is set to $\mu = 3\%$, which is close to the value commonly used in the literature; see Hackbarth *et al.* (2006). The asset volatility is chosen as $\sigma = 20\%$, because the average volatility of BB-rated firms is around 21%; see Zhang *et al.* (2009). The model uses $\alpha_h = 80\%$ and $\alpha_l = 40\%$, because according to Chen (2010), the average recovery rates during the booms and recessions are around 80 and 40%, respectively. The size of a new asset, x_N, is set as 7.4 to generate a quantitatively reasonable supply curve. Here, note that for any given p, the value-weighted default rate in market i is equal to $\frac{q(p)}{\int_{x_D^i}^{\infty} x m^i(x)dx}$. This formula implies that the highest possible default rate equals 8.6%, and the lowest possible default rate equals 0.58% in the model. Thus, the choice of $x_N = 7.4$ is reasonable enough. The other parameters such as f^A, f^B and κ that represent the demand side are appropriately chosen in the next sections, depending on which policies will be analyzed.

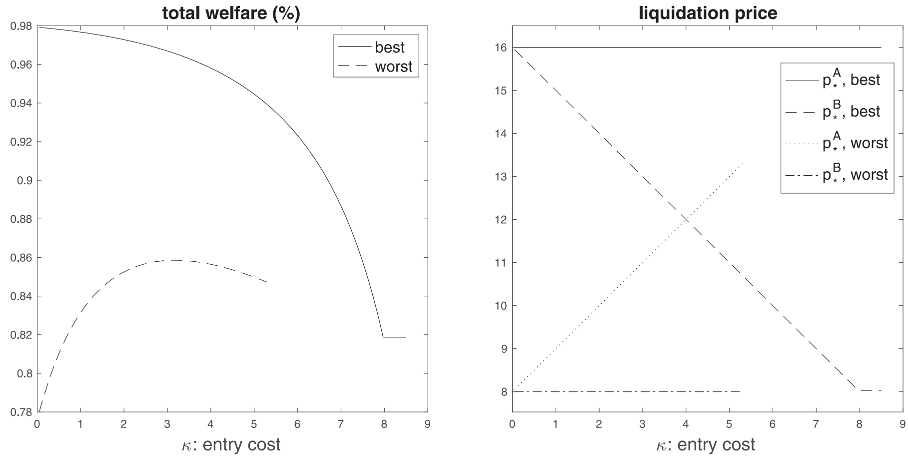
5.2 The effects of the entry cost

This section analyzes how the entry cost affects total welfare. Throughout the section, the economy in Example 3 is mainly considered, because the effects of this policy were studied in depth for the market conditions in Examples 1 and 2. Recall that in Example 3, three equilibria arise; thus, the policy implications will depend on which equilibrium is selected.

First, suppose the best equilibrium is selected when the entry cost is reduced from ∞ to a finite number $\kappa < \frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho}$. Then, as seen before, the price in market A remains at the highest level $\frac{\alpha_h}{\rho}$ and the price in market B increases. In other words, the economy in this equilibrium behaves as if market A is sufficiently liquid as in Example 2. The markets will respond in the same way when the entry cost is reduced continuously. Thus, we can repeat the arguments used in Example 2 to show that policies of reducing the entry cost increase total welfare in the present example as well. The solid curve in the left panel in Figure 5 exhibits this positive effect numerically. Specifically, in this figure, f^A and f^B are chosen to be 0.035 and 0.004, respectively, to ensure f^A is slightly larger than $q\left(\frac{\alpha_l}{\rho}\right)$ and f^B is slightly less than $q\left(\frac{\alpha_h}{\rho}\right)$. The baseline parameter values are used for the other parameters. In addition, the solid and dotted lines in the right panel confirm that the price in market A remains the same, whereas the price in market B increases, as the entry cost is reduced. To clarify, when the entry cost is larger than $\frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho} = 8$, the economy behaves as if the markets are fully segmented.

Risk-free rate	$r = 4\%$
Debt maturity	$m = 1$
Death rate	$\phi = 4\%$
Asset growth rate	$\mu = 3\%$
Asset volatility	$\sigma = 20\%$
Principal payment	$P = 100$
Coupon size	$c = 9$
High productivity	$\alpha_h = 80\%$
Low productivity	$\alpha_l = 40\%$
Asset size of a new firm	$x_N = 7.4$

Table 1.
Baseline parameter values



Note(s): The solid curve corresponds to the best equilibrium and the dashed curve corresponds to the worst equilibrium. When $\kappa > 5.5$, the worst equilibrium does not exist. The right panel plots the effects of a change in κ on the liquidation prices. The solid curve represents p_*^A in the best equilibrium. The dashed line represents p_*^B in the best equilibrium. The dotted line represents p_*^A in the worst equilibrium. The dash-dotted line represents p_*^B in the worst equilibrium. This figure uses $f^A = 0.035$, $f^B = 0.004$, and the baseline values for the other parameters

Figure 5. The left panel plots how a change in κ affects total welfare expressed in the percentage term

Second, suppose the worst equilibrium is selected when the entry cost is reduced from ∞ to a finite number $\kappa < \frac{\alpha_h}{\rho} - \frac{\alpha_l}{\rho}$. Then, the price in market A decreases, but the price in market B stays at the lowest level $\frac{\alpha_l}{\rho}$, as seen before. That is, the economy now behaves as if market B is sufficiently illiquid as in Example 1. The markets will exhibit the same patterns when the entry cost is reduced continuously. Hence, we can again repeat the arguments used in Example 1 to show that total welfare may decrease when the entry cost is lowered. More concretely, as stated in Theorem 2.1, (1) total welfare for $\kappa = 0$ is lower than that for $\kappa = \infty$ and (2) total welfare increases in κ, especially when κ itself is small. The dashed curve in the left panel in Figure 5 illustrates both the results numerically. The dashed and dash-dotted lines in the right panel also show that the price in market A decreases but remains the same in market B when the entry cost is lowered.

Lastly, when the intermediate equilibrium is selected, the above two results will be mechanically mixed together. That is, reducing the entry cost can still either increase or decrease total welfare. Figure 5 omits to include this case to avoid plotting unnecessarily many graphs.

5.3 The effects of liquidity injection

This section examines whether injecting liquidity into the secondary markets can alternatively improve total welfare. That is, we analyze the effects of a change in f^A and f^B. But, we focus on the parameter f^A because the result about f^B will be similar.

The key mechanism underlying this policy can be explained using a single fully isolated market only. To see the details, look at the left panel in Figure 6 which describes a single isolated market that produces three equilibria. Those three equilibria are respectively called the best, intermediate, worst equilibrium as in the above.

First, suppose f^i increases by Δ as in the middle panel of the figure, where Δ is large enough. Then, regardless of which equilibrium was selected before, the new liquidation price is given by $\frac{\alpha_h}{\rho}$, which is the highest possible level. Therefore, total welfare must increase. Of course, both price and total welfare remain unchanged if the economy originally stayed in the best equilibrium.

However, if only a moderate amount of capital is injected as in the third panel, equilibrium multiplicity is not eliminated. In this case, the effects of liquidity injection depend on which equilibrium is selected. But, for simplicity, assume that the economy selects at least the same type of equilibrium before and after liquidity is injected.

If the best equilibrium is selected, such a policy does not make any differences as seen before. Yet, if the intermediate equilibrium is selected, total welfare decreases. To see why, note that in this equilibrium, the market has just enough high-type buyers who can absorb all failed assets. As such, if only a small number of new high-type buyers are introduced, the liquidation price rather goes down, as shown in the third panel. Thus, the asset quantity liquidated increases by Δ, and therefore, total welfare decreases by $(1-\alpha_h)\Delta$. Intuitively, this result can be explained via the self-fulfilling beliefs again. That is, imagine that the market participants believe the liquidation price will go down when the government injects new capital. Equityholders then default earlier to avoid enlarged rollover risk, causing more asset liquidation. But, because the government has already put in some new capital, the demand can actually meet the supply, justifying the self-conjectured low price can be sustained as an equilibrium.

This result contrasts with the results of Benmelech and Bergman (2012) and Bleck and Liu (2018). Those papers show that an excessive amount of liquidity injection causes either negative effects or limited positive effects. However, the present model says that when the government adds an insufficient amount of capital, total welfare may go down, whereas the opposite outcome occurs when enough liquidity is provided.

Finally, if the worst equilibrium is selected, total welfare increases, although the asset price remains the same at $\frac{\alpha_l}{\rho}$. The reason is that Δ units of the asset are now liquidated to the additionally introduced high-type buyers rather than to low-type buyers. This improved asset allocation increases total welfare by $(\alpha_h-\alpha_l)\Delta$.

When the markets are partially segmented, the same intuition applies. In other words, the liquidity injection policy affects the economy in a similar way regardless of whether the markets are segmented or not. A systematic analysis is omitted here to avoid plotting unnecessarily complicated figures. Instead, the numerical results are presented in Figure 7. The dashed line in the left panel shows that total welfare indeed decreases as f^A increases if the intermediate equilibrium is selected. The right panel confirms that the liquidation prices in both markets decline as f^A increases. But, when f^A further increases beyond 0.049, total welfare jumps up because only the best equilibrium exists for those values of f^A. However, the solid line in the left panel shows that the same policy does not affect total welfare if the best equilibrium is selected. Also, as the dash-dotted line in that panel shows, such a policy

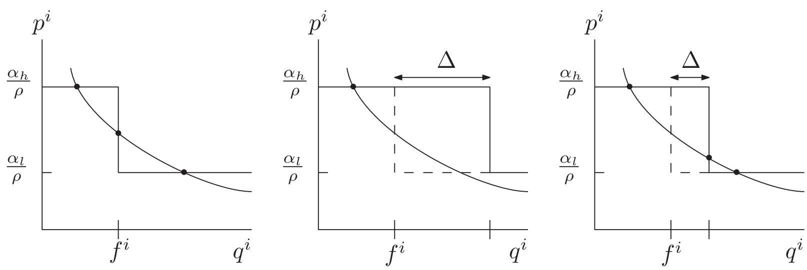


Figure 6.
The effects of an increment in f^A on equilibrium in a single isolated market

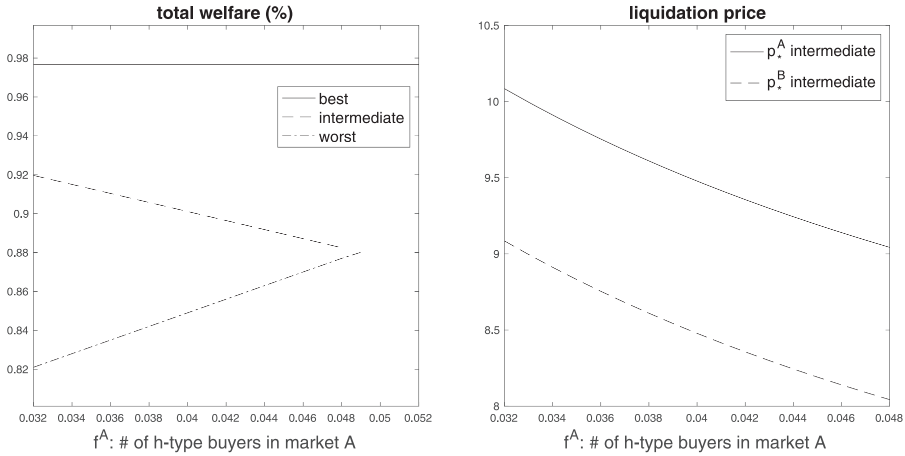


Figure 7. The left panel plots how a change in f^A affects total welfare expressed in the percentage term

Note(s): The solid line corresponds to the best equilibrium, the dashed line the intermediate equilibrium, and the dash-dotted line the worst equilibrium. The right panel plots the effects of a change in f^A on the liquidation prices only for the intermediate equilibrium. The solid (resp. dashed) line denotes p_*^A (resp. p_*^B) in the intermediate equilibrium. This figure uses $f^B = 0.004$, $\kappa = 1$ and the baseline values for the other parameters

increases total welfare if the worst equilibrium is selected. The right panel in Figure 7 shows the effects on the liquidation prices, focusing on the outcomes in the intermediate equilibrium. For the same reason discussed above, the liquidation prices in both markets decrease as f^A increases.

6. Conclusion

This paper developed a short-term debt model with two sectors, in which secondary asset markets are partially segmented. The paper showed that reducing market barriers between the secondary markets may decrease total welfare. When the market barriers are lowered, assets in default are more efficiently allocated, because potential asset buyers can move across the markets more flexibly. However, this expedited capital flow shrinks the liquidation price gap between the markets, causing an unusual general equilibrium effect. That is, after the reduction in the entry barriers, equityholders in a relatively liquid market that experiences a price drop choose to default earlier to protect themselves against enlarged rollover risk. As a result, more of the assets will be liquidated in the relatively liquid market. In other words, the supply curves in the secondary markets are downward sloping. When this negative contagion effect is sufficiently large, even total welfare can decrease. The paper rigorously showed that when the relatively liquid market is not sufficiently liquid, reducing the market barriers can indeed decrease total welfare. For future research, studying other mechanisms through which market integration can adversely affect the aggregate economy will be interesting.

Notes

1. Although all the results of this paper hold for firms issuing finite-maturity debt, we mainly focus on firms issuing short-term debt as in He and Xiong (2012b) because firms issuing short-term debt are more concerned about rollover risk, compared to firms issuing long-term debt.

2. Here, we can more generally assume that when a buyer does not buy any assets today, she can buy two units of the asset at some point in time in the future. But, in steady-state equilibrium, a buyer who decides not to buy any assets today will not buy any assets in the future, either. Thus, this alternative assumption does not make any differences in the model. Moreover, a potential buyer is allowed to resell her asset afterward. But, she has no strong incentives to do so, because the asset prices remain constant in steady-state equilibrium.

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Appendix

A.1. Proof of Theorem 2.1

In this proof, we only need to show $W'(\kappa) > 0$ when κ is small enough, because we have already proved the other parts of the theorem. To this aim, first, suppose that the entry cost is reduced from ∞ to κ_1. Let Δ_1 denote the increment in the asset quantity liquidated in market A after this reduction in the entry cost. Also, denote by g_1 the total number of high-type buyers in market A, who buy assets from B, when the entry cost is κ_1. Now imagine that the entry cost is reduced from ∞ to κ_2, where $\kappa_1 > \kappa_2$. Again, let Δ_2 denote the increment in the asset quantity liquidated in market A after this reduction in the entry cost. Similarly, denote by g_2 the total number of high-type buyers in market A, who buy assets from B, when the entry cost is κ_2. Then, the formula in (3) implies that when the entry cost decreases from κ_1 to κ_2, total welfare changes by:

$$\begin{aligned} W(\kappa_2) - W(\kappa_1) &= W(\kappa_2) - W(\infty) - [W(\kappa_1) - W(\infty)] \\ &= -\kappa_2 g_2 + \kappa_1 g_1 + (\alpha_h - \alpha_l)(g_2 - g_1) - (1 - \alpha_h)(\Delta_2 - \Delta_1). \end{aligned} \tag{A1}$$

Then, using the fact that $\Delta_1 + g_1 = f^A - q^A_\infty = \Delta_2 + g_2$, the above net change can be rewritten as:

$$W(\kappa_2) - W(\kappa_1) = (\kappa_1 - \kappa_2)g_1 - (\alpha_h - \alpha_l - \kappa_2)\delta - (1 - \alpha_h)\delta,$$

Where $\delta := \Delta_2 - \Delta_1 = g_1 - g_2 > 0$. But, the first term $(\kappa_1 - \kappa_2)g_1$ is negligible because both $\kappa_1 - \kappa_2$ and g_1 are sufficiently small. Hence, $W(\kappa_2) - W(\kappa_1)$ must be negative, which proves the theorem.

A.2. Equity value

The HJB equation in (7) has the following closed-form solution:

$$E(x) = \frac{x}{r + \phi - \mu} + Ax^\eta - \frac{c + \lambda P}{r + \lambda + \phi} - \left(p^i x^i_D - \frac{c + \lambda P}{r + \lambda + \phi}\right)\left(\frac{x}{x^i_D}\right)^\xi,$$

where η is given by (9). The coefficient A and the default threshold x^i_D must satisfy the following value-matching and smooth-pasting conditions:

$$\frac{x^i_D}{r + \phi - \mu} + A\left(x^i_D\right)^\eta = p^i x^i_D, \qquad \frac{x^i_D}{r + \phi - \mu} + A\eta\left(x^i_D\right)^\eta = \xi\left(p^i x^i_D - \frac{c + \lambda P}{r + \lambda + \phi}\right).$$

These two conditions lead to:

$$x^i_D = \frac{-\xi(r + \phi - \mu)(c + \lambda P)(r + \phi)}{\left(1 - \eta + (\eta - \xi)(r + \phi - \mu)p^i\right)(r + \phi)(r + \lambda + \phi)}.$$

We can then compute A as well from the above conditions.

A.3. Supply curve

This section first provides a closed-form solution for $m(x; p)$. Let $x_D = x_D(p)$ be an optimal default threshold for any given p. The solution to equation (10) is then given by:

$$m(x) = \begin{cases} m^1(x) = A_1 x^{\eta_1} + A_2 x^{\eta_2}, & \text{if } x \in [x_D, x_N] \\ m^2(x) = A_3 x^{\eta_2}, & \text{if } x \in [x_N, \infty), \end{cases}$$

where

$$\eta_1 = \frac{\mu - \dfrac{3\sigma^2}{2} + \sqrt{\left(\mu - \dfrac{\sigma^2}{2}\right)^2 + 2\sigma^2\phi}}{\sigma^2} > -1,$$

$$\eta_2 = \frac{\mu - \dfrac{3\sigma^2}{2} - \sqrt{\left(\mu - \dfrac{\sigma^2}{2}\right)^2 + 2\sigma^2\phi}}{\sigma^2} < -1.$$

The coefficients $\{A_1, A_2, A_3\}$ satisfy the following conditions:

$$m^1(x_D) = 0, \quad m^1(x_N) = m^2(x_N), \quad \int_{x_D}^{\infty} xm(x)dx + \frac{\sigma^2}{2} x_D^2 m_x(x_D) F(x_D) = 1.$$

That is,

$$A_1 x_D^{\eta_1} + A_2 x_D^{\eta_2} = 0, \tag{A2}$$

$$A_1 x_N^{\eta_1} + A_2 x_N^{\eta_2} = A_3 x_N^{\eta_2}, \tag{A3}$$

$$\frac{A_1 x_N^{2+\eta_1}}{2+\eta_1} + \frac{A_2 x_N^{2+\eta_2}}{2+\eta_2} - \frac{A_1 x_D^{2+\eta_1}}{2+\eta_1} - \frac{A_2 x_D^{2+\eta_2}}{2+\eta_2} - \frac{A_3 x_N^{2+\eta_2}}{2+\eta_2} + \frac{\sigma^2 \left(A_1 \eta_1 x_D^{2+\eta_1} + A_2 \eta_2 x_D^{2+\eta_2} \right)}{2(r+\phi-\mu)} = 1. \tag{A4}$$

Using this system of linear equations, A_1, A_2 and A_3 can be pinned down explicitly.

Now we show that:

$$q(p) = \frac{\sigma^2}{2} x_D^3 m_x(x_D)$$

decreases in p, where $x_D = x_D(p)$ as in the above. But, we only need to show that $x_D^3 m_x(x_D)$ increases in x_D because x_D decreases in p. Note that the conditions in (A2) and (A3) imply that:

$$A_2 = -A_1 x_D^{\eta_1-\eta_2} \quad \text{and} \quad A_3 = A_1 x_N^{\eta_1-\eta_2} - A_1 x_D^{\eta_1-\eta_2}, \tag{A5}$$

which leads to:

$$x_D^3 m_x(x_D) = A_1 x_D^{2+\eta_1}(\eta_1 - \eta_2).$$

Thus, to show the above claim, it suffices to show $A_1 x_D^{2+\eta_1}$ increases in x_D. Now, plugging the conditions in (A5) into (A4), we have:

$$A_1 x_D^{2+\eta_1} \left[\frac{(\eta_2 - \eta_1)}{(2+\eta_1)(2+\eta_2)} \left(\left(\frac{x_N}{x_D} \right)^{2+\eta_1} - 1 \right) + \frac{\sigma^2(\eta_1 - \eta_2)}{2\rho} \right] = 1.$$

Then, the conditions such that $\eta_1 > -1$, $\eta_2 < -2$ and $x_D < x_N$ imply $A_1 > 0$. But then, because:

$$\frac{(\eta_2 - \eta_1)}{(2+\eta_1)(2+\eta_2)} \left(\left(\frac{x_N}{x_D} \right)^{2+\eta_1} - 1 \right) + \frac{\sigma^2(\eta_1 - \eta_2)}{2\rho}$$

decreases in x_D, $A_1 x_D^{2+\eta_1}$ must increase in x_D, which completes the proof.

A.4. Existence of equilibrium

This section proves that this economy has at least one equilibrium. Without loss of generality, we can assume $f^A \geq f^B$. Let $q_h = q\left(\frac{\alpha_h}{\rho}\right)$ and $q_l = q\left(\frac{\alpha_l}{\rho}\right)$. Then, consider the following three cases: (1) $q_h \leq f^B$, (2) $f^A \leq q_l$ and (3) $f^B \leq q_h < q_l \leq f^A$, which may not be exclusive from each other. In case (1), we can find one equilibrium such that $(p_*^A, p_*^B) = \left(\frac{\alpha_h}{\rho}, \frac{\alpha_h}{\rho}\right)$ and $g = 0$. In case (2), we can find one equilibrium such that $(p_*^A, p_*^B) = \left(\frac{\alpha_l}{\rho}, \frac{\alpha_l}{\rho}\right)$ and $g = 0$. In case (3), we further split this case into the following two cases: (a) $q_l - f^B \leq f^A - q_h$ and (b) $q_l - f^B > f^A - q_h$. In case (a), we can find one equilibrium such that $(p_*^A, p_*^B) = \left(\frac{\alpha_h}{\rho}, \frac{\alpha_h}{\rho} - \kappa\right)$ and $g = q\left(\frac{\alpha_h}{\rho} - \kappa\right) - f^B$. Indeed, the collection of (p_*^A, p_*^B, g) can be an equilibrium because $q_h \leq f_B + g \leq q_l$ and $q_h \leq f^A - g$. In case (b), we can find one equilibrium such that $(p_*^A, p_*^B) = \left(\frac{\alpha_l}{\rho} + \kappa, \frac{\alpha_l}{\rho}\right)$ $g = f^A - q\left(\frac{\alpha_l}{\rho} + \kappa\right)$. Indeed, the collection of (p_*^A, p_*^B, g) can be an equilibrium because $q_h \leq f^A - g \leq q_l$ and $f^B + g \leq q_l$, which completes the proof.

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