

Dynamic risk-based optimization on cryptocurrencies

Bayu Adi Nugroho^{ID}

Sekolah Tinggi Ilmu Ekonomi YKPN, Yogyakarta, Indonesia

Received 13 January 2021
Revised 17 February 2021
Accepted 25 February 2021

Abstract

Purpose – It is crucial to find a better portfolio optimization strategy, considering the cryptocurrencies' asymmetric volatilities. Hence, this research aimed to present dynamic optimization on minimum variance (MVP), equal risk contribution (ERC) and most diversified portfolio (MDP).

Design/methodology/approach – This study applied dynamic covariances from multivariate GARCH(1,1) with Student's *t*-distribution. This research also constructed static optimization from the conventional MVP, ERC and MDP as comparison. Moreover, the optimization involved transaction cost and out-of-sample analysis from the rolling windows method. The sample consisted of ten significant cryptocurrencies.

Findings – Dynamic optimization enhanced risk-adjusted return. Moreover, dynamic MDP and ERC could win the naive strategy (1/N) under various estimation windows, and forecast lengths when the transaction cost ranging from 10 bps to 50 bps. The researcher also used another researcher's sample as a robustness test. Findings showed that dynamic optimization (MDP and ERC) outperformed the benchmark.

Practical implications – Sophisticated investors may use the dynamic ERC and MDP to optimize cryptocurrencies portfolio.

Originality/value – To the best of the author's knowledge, this is the first paper that studies the dynamic optimization on MVP, ERC and MDP using DCC and ADCC-GARCH with multivariate *t*-distribution and rolling windows method.

Keywords Cryptocurrencies, Minimum variance, Equal risk contribution, Most diversified portfolio, Multivariate GARCH

Paper type Research paper

1. Introduction

The popularity of cryptocurrencies has attracted investors to add cryptocurrencies into their portfolios. The purpose of adding cryptocurrencies is to gain a diversification advantage (Kajtazi and Moro, 2019; Urquhart and Zhang, 2019; Bouri *et al.*, 2020). However, the increasing risk of cryptocurrencies has raised some concerns (Palamalai *et al.*, 2020). Moreover, cryptocurrencies tend to have asymmetric volatility (Baur and Dimpfl, 2018). Therefore, finding portfolio optimization techniques that result in minimal estimation errors is challenging.

Several studies investigated the performance of cryptocurrencies portfolio under different optimization techniques (Platanakis *et al.*, 2018; Symitsi and Chalvatzis, 2018; Brauneis and Mestel, 2019; Guesmi *et al.*, 2019; Kajtazi and Moro, 2019; Liu, 2019; Platanakis and Urquhart, 2019; Schellinger, 2020; Susilo *et al.*, 2020). However, Markowitz's approach (Markowitz, 1952) has two noticeable drawbacks based on theoretical perspectives (Kaucic *et al.*, 2019). First, Markowitz's approach precipitately disregards variables with negatively skewed distribution. Second, investors are more anxious concerning downside risk.

Risk-based strategies can minimize Markowitz's drawbacks. Some of the most popular approaches are equal risk Contribution (ERC), minimum variance (MVP) and most diversified

JEL Classification — F30, G11, G15

© Bayu Adi Nugroho. Published in *Journal of Capital Markets Studies*. Published by Emerald Publishing Limited. This article is published under the Creative Commons Attribution (CC BY 4.0) license. Anyone may reproduce, distribute, translate and create derivative works of this article (for both commercial and non-commercial purposes), subject to full attribution to the original publication and authors. The full terms of this license may be seen at <http://creativecommons.org/licenses/by/4.0/legalcode>



portfolio (MDP). Chouiefaty and Coignard (2008) implied that holding assets that are not perfectly correlated could lead to diversification. Further, MDP was exceptionally well regarding relative performance (Chouiefaty *et al.*, 2013). Also, ERC defines that a weight vector can attain diversification. The weight is obtained from a diversified portfolio concerning its constituents' risk allocations (Qian, 2006, 2011). Notably, there is one similarity in the original ERC and MDP: the approaches do not use time-varying covariances or correlations. Therefore, this study uses dynamic parameters from GARCH estimations to create a dynamic portfolio.

The present study attempted to answer the following question: Do risk-based portfolios using multivariate GARCH enhance portfolio performance compared with the conventional approaches and the naïve approach (1/N)? Hence, this research applied dynamic optimization on MVP, ERC and MDP. Moreover, this study used dynamic covariances from multivariate GARCH with the Student's *t*-distribution. Also, this research applied the method of rolling windows with various GARH refits.

Further, this study varies in many respects from previous literature. Some research explored the diversification advantage through multivariate GARCH based on bivariate portfolios (Basher and Sadorsky, 2016; Ahmad *et al.*, 2018; Jalkh *et al.*, 2020; Yousaf and Ali, 2020a, 2020b), while this study applies GARCH estimations on ten risky assets. Second, previous studies extensively used dynamic hedge ratios and optimal weights strategy (Pal and Mitra, 2019; Antonakakis *et al.*, 2020; Bouri *et al.*, 2020; Yousaf and Ali, 2020b). This study implements three risk-based approaches. Third, while the conventional ERC, MVP and MDP optimization do not use time-varying covariances, this study applies dynamic parameters from multivariate GARCH. Fourth, this study uses multivariate Student's *t* to account for skewed distribution (Antonakakis *et al.*, 2020). Fifth, this study captures extreme volatilities into account in the period of COVID-19 pandemic and the years 2017 and 2018. Thus, this research mimics real investment. Kajtazi and Moro (2019) opted to disregard the extreme volatilities.

There are two consequences, one investor-oriented and one that activates future study. For investment managers, the importance of finding models that can minimize estimation error in portfolio optimization is significant. Put differently, improving estimation error in covariances can enhance portfolio diversification gains, which is hugely significant, considering the crypto market's stylized facts. The findings of this research can improve investors' understanding of concerning crypto market.

The second implication is that this research paves the way for future research-other risk-based methods, such as inverse volatility, efficient risk portfolio and maximum-decorrelation. Moreover, the application of other GARCH specifications such as Copula GARCH is left for future research.

This paper is structured as follows. Section 2 shows the method and data used in this study. Section 3 discusses the findings. Section 4 exhibits a robustness check. The last part is the conclusion.

2. Methodology

This research applied risk-based strategies. Portfolio construction was referring to static and dynamic optimization. The covariances used in static optimization were not time-varying, while dynamic optimization implemented time-varying covariances from GARCH modeling. The covariance of multivariate GARCH was obtained from the rolling windows method to create an out-of-sample analysis (Basher and Sadorsky, 2016; Ahmad *et al.*, 2018). Moreover, the researcher conducted all calculations in "R" data analysis software, and some of the packages used in this research were RiskPortfolios (Ardia *et al.*, 2017b), rmgarch (Ghalanos, 2019), FRAPO (Pfaff, 2016) and fPortfolio (Wuertz *et al.*, 2017). This study also only had one portfolio constraint, which was long-only, to provide a detailed comparison between static and dynamic optimization.

2.1 Data

This paper is exploratory research. In line with previous studies, this research used the majority of the sample from other researchers (Antonakakis *et al.*, 2019; Liu, 2019). This research used ten major cryptocurrencies in portfolio construction. The cryptos were Bitcoin (BTC), Ethereum (ETH), Ripple (XRP), Litecoin (LTC), Stellar (XLM), Monero (XMR), Dash (DASH), Tether (USDT), Nem (XEM) and Dogecoin (DOGE). The researcher obtained daily prices of the cryptocurrencies (CC) from www.coinmarketcap.com. Following previous literature (Liu, 2019; Schellinger, 2020), this research took into account the cryptocurrency bubbles from mid-2017 until the beginning of 2018. This approach reflects a realistic portrait of the cryptos, and it gives a more informed investment strategy. Furthermore, the sample period was started from August 8, 2015, to October 20, 2020, indicating 1900 observations.

2.2 Risk-based portfolios

This paper applied risk-based strategies (Ardia *et al.*, 2017a): MV, MDP and ERC portfolio. Besides, this research also created the hard-to-beat naïve strategy (EWP) or 1/N (DeMiguel *et al.*, 2007).

MVP strategy:

$$\text{minimize} \rightarrow \sigma_p^2 = \sum_{j=1}^N \sum_{j=1}^N \sigma_{a,b} w_a w_b \quad (1)$$

where σ_p^2 is the variance of p portfolio, and $\sigma_{i,j}$ is the covariance of asset a and asset b .

MDP is referring to Chouiefaty and Coignard's (2008) findings. If \sum represents the variance of the covariance matrix of N assets, the diversification ratio (DR) for a weight vector ω in a portfolio Ω is defined as

$$DR_{\omega \in \Omega} = \frac{\omega' \sigma}{\sqrt{\omega' \Sigma \omega}} \quad (2)$$

The denominator is portfolio standard deviation, while the numerator is the weighted average of asset volatility. The decomposition of Eqn 2 is

$$DR_{\omega \in \Omega} = \frac{1}{\sqrt{(\delta + CR)} - \delta CR} \quad (3)$$

where δ is the volatility-weighted average correlation and CR is the volatility-weighted concentration ratio. Highly correlated assets are poorly diversified. Chouiefaty *et al.* (2013) then showed the following formula for MDP strategy

$$P_{MDP} = \arg \max_{\omega \in \Omega} DR \quad (4)$$

Minimizing $\omega' C \omega$ leads to maximum DR, where C is the correlation matrix. This treatment is similar to MV optimization, but rather than using a covariance matrix, MDP uses a correlation matrix.

(*ERC* is characterized by a minimum asset allocation concerning the risk contribution to its portfolio (Qian, 2006, 2011; Maillard *et al.*, 2010). The definition of risk contribution is

$$C_a M_{\omega \in \Omega} = \omega_a \frac{\partial M_{\omega \in \Omega}}{\partial \omega_a} \quad (5)$$

where ω_a is the weight of a asset and $M_{\omega \in \Omega}$ is the portfolio's standard deviation. Hence, the optimization problem of ERC is

$$P_{\text{ERC}}: \omega_a \left(\sum \omega \right)_a = \omega_b \left(\sum \omega \right)_b \quad \forall a, b, \quad (6)$$

$$\left. \begin{aligned} 0 \leq \omega_a \leq 1 \text{ for } a = 1, \dots, N, \\ \omega' a = 1 \end{aligned} \right\}$$

where a is a vector ($N \times 1$) of 1s. The optimization's goal is to minimize the risk contributions. Assets with high volatilities get low weights.

While the previous study used around 360 days of estimation windows for covariance creation (Liu, 2019; Schellinger, 2020), this study used different optimization days (see Figures 1 and 2).

Further, the covariance in the static approach is as follows:

$$\sigma_{a,b} = \text{Cov}(a, b) = \frac{1}{M-1} \sum_{t=1}^M (r_{at} - \bar{r}_a) \cdot (r_{bt} - \bar{r}_b), \quad a, b = 1, \dots, N \quad (7)$$

where \bar{r}_a is the mean returns of asset a of N assets in M periods. The variance is when $a = b$. Eqn 7 is called the sample variance-covariance matrix.

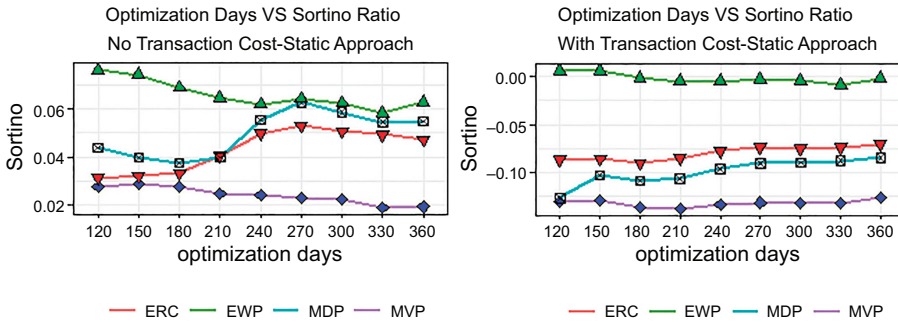


Figure 1. Optimization days vs Sortino ratio (static approach)

Note(s): These figures exhibit Sortino Ratio across optimization days. The transaction cost was 50 basis points

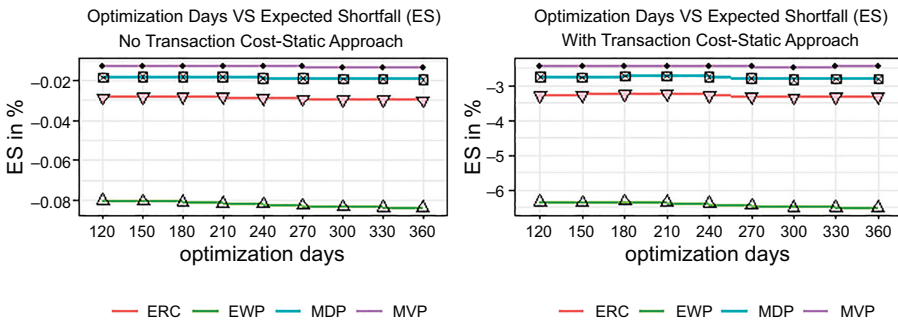


Figure 2. Optimization days vs expected shortfall (static approach)

Note(s): These figures exhibit Expected Shortfall (ES) across optimization days. ES used a 95% confidence level. The transaction cost was 50 basis points

Further, this research applied out-of-sample evaluation. The estimation windows or optimization days for portfolio construction ranged from 120 to 360 days. For instance, the log-returns data set ranged from August 8, 2015 to October 20, 2020, and suppose the optimization days were 360 days, meaning that this research used the date from August 8, 2015 to August 2, 2016 to obtain the weights and applied the weights on August 3, 2016. The next optimization days were from August 9, 2015 to August 3, 2016, and it resulted in optimal weights that applied the weights on August 4, 2016. The researcher repeated the process for each new window created from the remaining sample. The researcher used daily sliding windows in order to be comparable with the time-varying approach of GARCH. Also, the researcher argued that frequent rebalancing could minimize estimation error.

Moreover, this research applied the dynamic conditional correlation (DCC)-GARCH model of [Engle \(2002\)](#) and asymmetric DCC (ADCC) of [Cappiello et al. \(2006\)](#). The dynamic variance-covariance matrix:

$$V_t = D_t R_t D_t \quad (8)$$

Where R_t is a matrix of conditional correlation and D_t is a diagonal matrix for conditional standard deviation

$$D_t = \text{diag}\left(v_{n,t}^{\frac{1}{2}}, v_{x,t}^{\frac{1}{2}}\right) \quad (9)$$

$$R_t = \text{diag}(J_t)^{-1/2} J_t \text{diag}(J_t)^{-1/2} \quad (10)$$

The dynamics of J in the DCC process is

$$J_t = (1 - \delta_1 - \delta_2) \bar{J} + \delta_1 z_{t-1} z'_{t-1} + \delta_2 J_{t-1} \quad (11)$$

Where \bar{J} is the unconditional variance-covariance matrix of standardized residuals, δ_1 is a shock parameter and δ_2 is the persistency variable. [Cappiello et al. \(2006\)](#) modify the DCC by adding an asymmetric term

$$J_t = (\bar{J} - P' \bar{J} P - Q' \bar{J} Q - R' \bar{J} R) + P' z_{t-1} z'_{t-1} P + Q' J_{t-1} Q + R' z_{t-1} z'_{t-1} R \quad (12)$$

A rolling window analysis created one-step-ahead dynamic covariances. The forecast length was fixed at 1750 observations. However, the researcher also analysed the results from various forecast lengths (see [Figure 3](#)). The GARCH models were refit starting at 120 observations. Every GARCH refit defines how many times the model is recalculated and the forecast duration is actually measured. For example, for a forecast length of 1,500 and refit every 120 days, for a total actual forecast length of 1,500, there are 10 windows of 120 periods each.

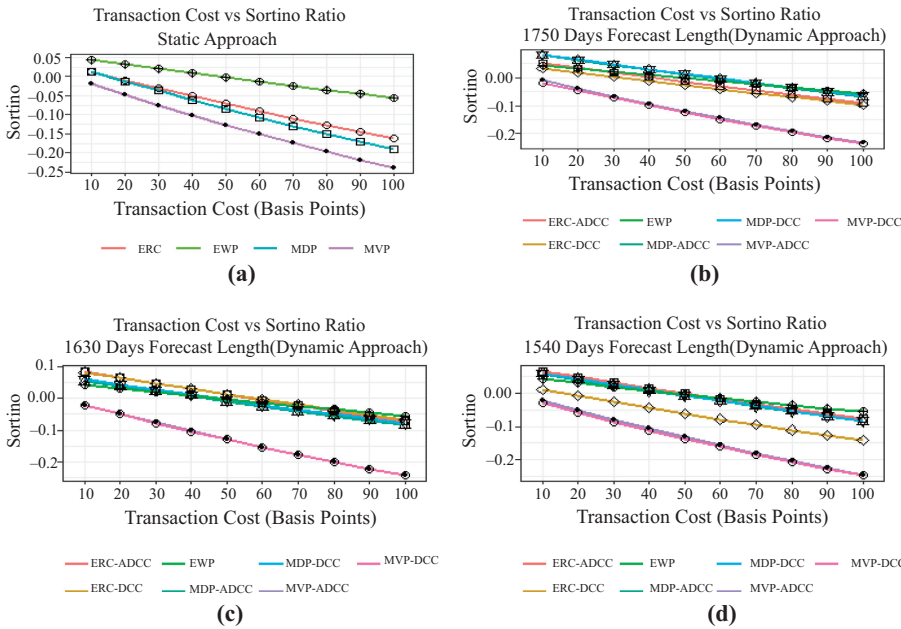
Moreover, this research also imposed 50 basis points of transaction cost (TCost). TCost is the cost of purchasing or selling securities in order to construct a portfolio ([Chavalle and Chavez-Bedoya, 2019](#)). The formula of TCost is ([Platanakis et al., 2018](#)):

$$TCost_t = \sum_{a=1}^N T_a \left(|w_{a,t} - w_{a,t-1}^+| \right) \quad (13)$$

Where $w_{a,t-1}^+$ represents the weight of the a th asset at the end of the period $t - 1$, and T_a is the a th asset's proportionate transaction expense.

3. Empirical results and discussions

This segment displays the effects of the creation of the portfolio. The first part of this section concerns the analysis of results from stylized facts and the efficient frontier. The second part



Note(s): These figures display portfolio performance concerning the Sortino ratio across different transaction costs and forecast lengths of dynamic approaches. These portfolios applied 360 days of optimization (static approach) and GARCH refits (dynamic method)

Figure 3. Sortino ratio vs transaction costs

discusses the static method results, while the dynamic optimization results are in the last part of this section.

3.1 Stylized fact

Table 1 presents descriptive statistics. All log-returns did not conform with normality distribution, which was verified by the Shapiro-Wilk test. Interestingly, the returns were positively skewed besides BTC, implying that buying assets with positive skewness could lead to a sizeable positive return (Eraker and Wu, 2017). XEM had the highest mean returns, and this finding is different from Antonakakis et al. (2019), who found that ETH had the highest mean returns.

Also, XEM had the highest standard deviation, and this finding is different from Liu (2019). XEM was the most risky crypto because it had the worst ES and VaR, while USDT was the least risky crypto. Further, XEM had the best risk-adjusted return. Moreover, Phillips and Perron (PP), augmented Dickey-Fuller (ADF) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests showed that all returns were stationary. Also, all series were autocorrelated. Hence, GARCH estimation is feasible.

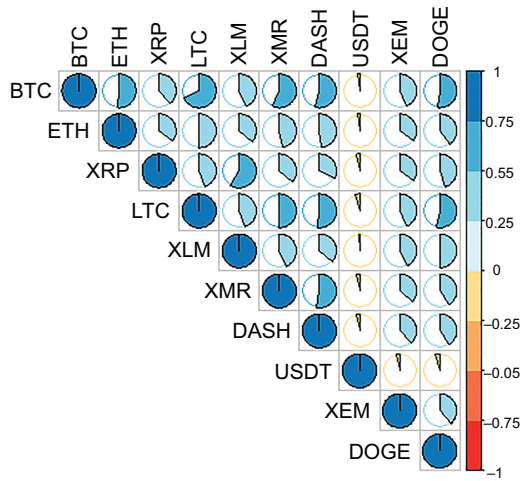
Figure 4 shows the correlogram of daily log returns. Blue color shows a positive correlation, while a red color reveals a negative correlation. The darker the color, the greater the level of the correlations. The color becomes washed out when the correlations are near zero. Findings show that most of the assets had positive correlations, consistent with Liu (2019). Interestingly, USDT had a weak and negative correlation with other cryptos.

Also, Figure 5 presents an efficient frontier from a long-only portfolio. USDT and XEM were located on the efficient frontier, indicating the lowest risk and the best-expected return

Table 1.
Descriptive statistics

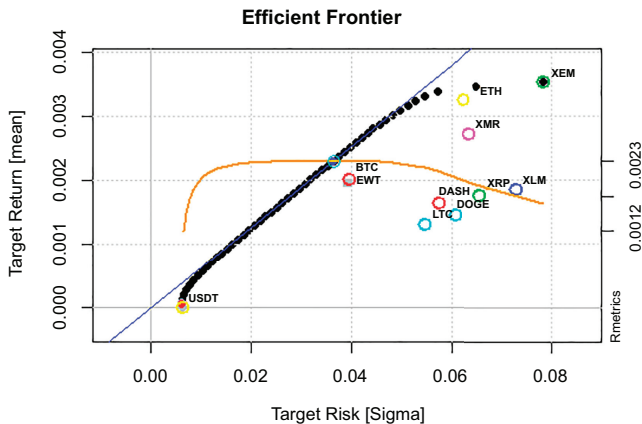
	BTC	ETH	XRP	LTC	XLM	XMR	DASH	USDT	XEM	DOGE
Obs	1901	1901	1901	1901	1901	1901	1901	1901	1901	1901
Min	-0.464	-0.551	-0.616	-0.449	-0.410	-0.494	-0.459	-0.049	-0.362	-0.493
Median(%)	0.206	0.000	-0.230	0.000	-0.200	0.000	-0.080	0.000	0.000	0.000
Max	0.225	0.412	1.027	0.510	0.723	0.585	0.438	0.057	0.996	0.518
Mean(%)	0.201	0.326	0.179	0.127	0.184	0.267	0.162	0.000	0.349	0.144
Std	0.039	0.062	0.066	0.055	0.073	0.064	0.057	0.006	0.078	0.061
Skewness	-0.096	0.079	2.960	0.764	2.000	0.678	0.636	0.288	1.974	1.034
Kurtosis	14.133	8.147	46.686	13.256	18.957	9.839	8.964	16.772	19.808	15.181
ES	-0.079	-0.125	-0.133	-0.111	-0.148	-0.128	-0.116	-0.013	-0.158	-0.123
VaR	-0.062	-0.099	-0.106	-0.088	-0.118	-0.102	-0.092	-0.010	-0.125	-0.098
Drawdown	0.886	0.976	0.988	0.976	0.990	0.983	0.993	0.115	0.996	0.973
Omega	1.180	1.133	1.107	1.079	1.087	1.133	1.090	1.000	1.153	1.085
Sortino	0.071	0.051	0.047	0.036	0.042	0.064	0.043	0.000	0.075	0.037
Sharpe	0.051	0.037	0.027	0.023	0.025	0.042	0.028	0.000	0.045	0.024
KPSS	0.063	0.081	0.092	0.108	0.096	0.093	0.099	0.007	0.106	0.046
PP	-44.86 ^{***}	-43.28 ^{***}	-45.86 ^{***}	-44.20 ^{***}	-41.61 ^{***}	-46.19 ^{***}	-45.00 ^{***}	-82.99 ^{***}	-47.10 ^{***}	-41.64 ^{***}
ADF	-30.33 ^{***}	-28.88 ^{***}	-27.45 ^{***}	-30.62 ^{***}	-30.02 ^{***}	-31.43 ^{***}	-31.57 ^{***}	-39.86 ^{***}	-34.30 ^{***}	-28.61 ^{***}
Shapiro-W	0.88 ^{***}	0.90 ^{***}	0.71 ^{***}	0.85 ^{***}	0.82 ^{***}	0.90 ^{***}	0.89 ^{***}	0.66 ^{***}	0.85 ^{***}	0.81 ^{***}

Note(s): This table presents descriptive stats of the log returns of the cryptos. Expected shortfall (ES) and value at risk (VaR) used 95% confidence level. ADF (PP) is augmented Dickey-Fuller (Phillips and Perron) statistic. KPSS is Kwiatkowski, Phillips, Schmidt and Shin statistic. The Shapiro-Wilk is to test normality. ^{***} Statistically sig at 1 % level. The risk-free rate was assumed to be zero for the calculation of Sharpe and minimum acceptance return (MAR) of Sortino. A target threshold of zero was also used in omega ratio



Note(s): This figure shows correlogram matrix of the assets. The blue colour presents a positive correlation while red colour shows a negative correlation. The darker the colour, the greater the level of the correlation

Figure 4. Correlogram



Note(s): This figure presents efficient frontier from short-selling constrained mean-variance portfolio. The x and y-axis are in percentage. This figure consists of efficient frontier, tangency line and EWP which is the naïve strategy. The line of Sharpe ratio (yellow) which coincides with tangency point of the portfolio is also shown. The right hand side axis of the figure represents the range of Sharpe ratio

Figure 5. Efficient frontier

among other cryptos. Moreover, Figure 5 displays in-sample analysis, implying that the covariance matrix was estimated based on the entire sample, which was not a realistic perspective. In-sample analysis is mainly used as a theoretical perspective rather than a practical perspective (Liu, 2019; Schellinger, 2020). Therefore, this paper focused on out-of-sample analysis using the rolling windows method.

3.2 Static approach

This section shows MVP, MDP and ERC portfolios' optimization results based on a static covariance matrix. Figure 6 shows optimal weights from static portfolios. The highest proportion of assets in the portfolio was USDT for all optimization methods. The average proportions of USDT for MVP, ERC and MDP portfolio were 0.971, 0.697 and 0.832, respectively. Interestingly, the lowest proportion of assets in the portfolio was XMR for MVP and MDP portfolios. It was XLM for the ERC approach.

Further, Table 2 shows the performance evaluation of MDP, ERC and MVP portfolios. Panel A exhibits the performance without transaction cost, while Panel B shows the performance with transaction cost (50 basis points). Without transaction costs, the level of risk (ES and VaR) of MDP, ERC and MVP portfolios was significantly lower than individual crypto (see Table 1). Interestingly, the level of risk was still reduced considerably after imposing transaction cost to the portfolio. Moreover, the portfolio's risk-adjusted return was not significantly greater than individual cryptos, indicating that diversification across cryptos did not significantly enhance return. This finding is different from Liu (2019). Note that Table 2 only shows the performance of static MVP, ERC and MDP portfolio under 120, 150 and 180 estimation days. One should refer to Figures 1 and 2 for various estimation days.

Figures 1 and 2 display portfolio performance (Sortino and ES) across different optimization days [1]. Without transaction costs, the MDP portfolio was the best strategy in terms of the Sortino ratio. After considering transaction costs, EWP outperformed other portfolios regarding the Sortino ratio. In the level of risk or expected shortfall (Figure 2), EWP was the worst performer (with and without transaction cost), while static MVP was the least risky strategy.

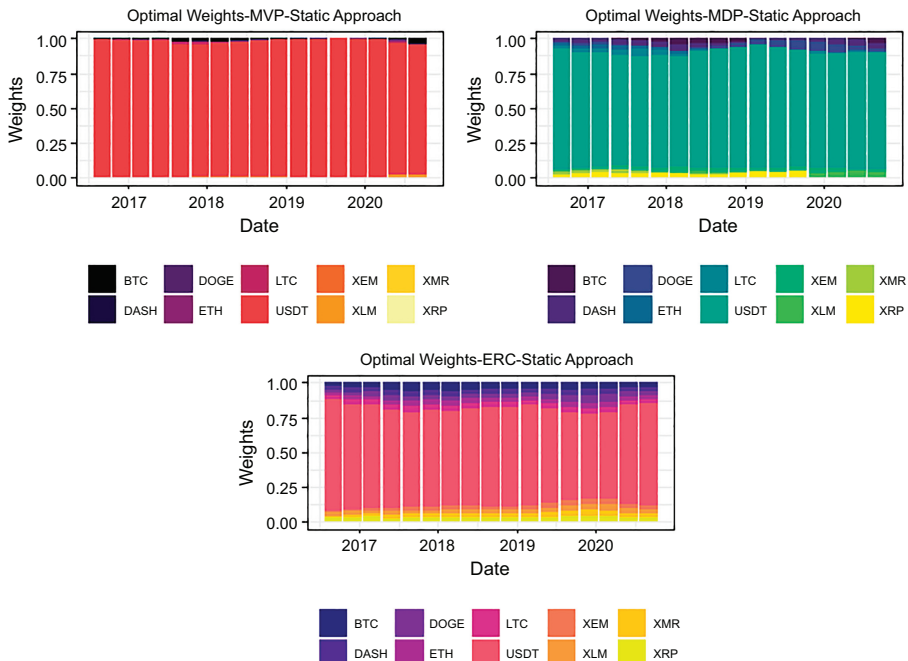


Figure 6. Optimal weights of static approach

Note(s): These figures exhibit optimal weights from MVP, MDP, and ERC portfolio. The optimization days were 360 days with short-selling constraints

	Optimization 120 days		Optimization 150 days		Optimization 180 days	
	MVP	ERC	MDP	MVP	ERC	MDP
<i>Panel A</i>						
ES (%)	-1.276	-2.846	-0.792	-1.284	-2.840	-1.847
VaR (%)	-1.015	-2.263	-0.628	-1.021	-2.258	-1.468
Drawdown (%)	12.700	82.400	7.970	14.400	80.680	46.830
Omega	1.077	1.077	1.258	1.079	1.077	1.091
Sortino (%)	2.761	3.124	7.487	2.889	3.212	3.979
Sharpe (%)	1.887	2.219	4.430	1.970	2.283	2.690
<i>Panel B</i>						
ES (%)	-2.466	-3.293	-2.118	-2.478	-3.279	-2.777
VaR (%)	-1.990	-2.646	-1.708	-1.999	-2.635	-2.235
Drawdown (%)	88.815	86.581	38.767	88.961	86.908	86.118
Omega	0.583	0.780	0.684	0.585	0.782	0.704
Sortino (%)	-13.087	-8.639	-12.701	-12.930	-8.610	-10.311
Sharpe (%)	-10.040	-6.355	-9.179	-9.915	-6.363	-7.622

Note(s): This table exhibits performance measurement of MVP, ERC and MDP portfolio with different estimation windows (120, 150 and 180 days). Panel A shows the performance without transaction cost, while Panel B indicates the performance with transaction cost (50 basis points). The risk-free rate was assumed to be zero for the calculation of Sharpe and minimum acceptance return (MAR) of Sortino. A target threshold of zero was also used in omega ratio

Table 2.
Performance
evaluation-static
approach

Unlike previous studies, this research computed cumulative returns. Figure 7 presents the cumulative returns of the portfolios (static approach). Based on the statistics, EWP was less affected by transaction costs, and this finding is similar to Liu (2019). EWP had positive average of net cumulative returns, while MVP, MDP and ERC had negative average of net cumulative returns.

3.3 Dynamic approach

Before applying the optimization, the researcher implemented diagnostic tests. Table 3 displays results of diagnostic tests. This paper used GARCH (1,1). Table 3 shows three coefficients from the variance equation. The omega is the intercept, while alpha (1) and beta (1) are the first lag of squared returns and conditional variance, respectively. The sum of α (1) and β (1) was less than one and significant, indicating that the series was mean-reverting. Moreover, the Ljung-Box tests exhibit that the GARCH estimation could obtain all returns volatility since autocorrelation did not exist in the standardized residuals. Hence, the GARCH (1,1) in this research is fit.

Further, Figure 8 displays the optimal weights of the dynamic approach. The models' settings were the rolling windows method with 1,750 days of forecast length and 360 days of GARCH refits. Similar to the static process, USDT had immense weight in the portfolio. However, the average of USDT's weight in the dynamic ERC and MDP portfolio is noticeably different from the static portfolio. For instance, the average weight of USDT in ERC-ADCC and ERC-DCC portfolios were 40 percent and 33 percent, respectively. The mean weight of USDT in MDP-ADCC and MDP-DCC

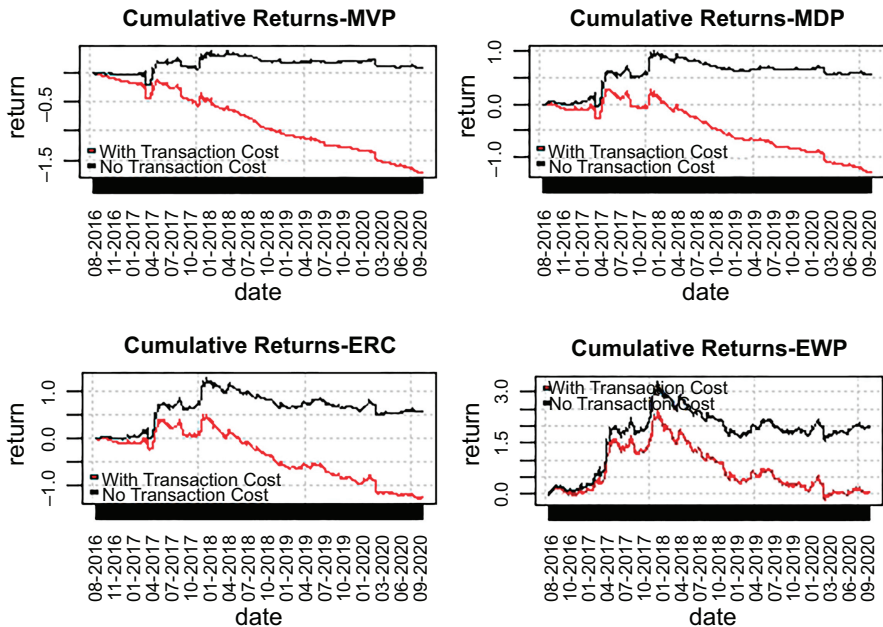


Figure 7.
Cumulative returns – static approach

Note(s): These figures exhibit cumulative returns of MVP, ERC, MDP, and EWP portfolio. The black line represents a portfolio without transaction cost, while the red line indicates a portfolio with transaction cost (50 bps). The optimization days were 360

	Estimate	t-value
Ω	0.011	3.783 ^{***}
α (1)	0.154	5.796 ^{***}
β (1)	0.805	23.892 ^{***}
<i>Information criterion stats</i>		
AIC	1.124	
BIC	1.133	
SIC	1.124	
HQIC	1.128	
Log-likelihood	-1106.876	
<i>Standardised residuals tests</i>		
	Statistic	p-value
Ljung-Box test (Q10)	9.988	0.442
Ljung-Box test (Q15)	16.888	0.326
Ljung-Box test (Q10)-squared	9.159	0.517
Ljung-Box test (Q15)-squared	16.336	0.360
LM arch test	9.884	0.626

Note(s): This table exhibits diagnostic test of GARCH(1,1). The coefficients in the variance equation are listed, Ω , α (1) and β (1). ^{***} Statistically sig at 1%

Table 3.
Diagnostic tests

portfolios were 32% and 33%, respectively. Interestingly, BTC had the second-largest weight in the dynamic ERC portfolio, while XEM had the second-largest weight in the dynamic MDP portfolio.

Furthermore, [Table 4](#) indicates the performance evaluation of dynamic MVP, ERC and MDP portfolio. Panel A shows the performance without transaction cost, while Panel B indicates the performance with transaction cost (50 basis points). Without transaction cost, dynamic MVP had the lowest portfolio risk level, followed by a dynamic MDP portfolio (based on ES approach). Moreover, the ERC strategy was the most risky approach. With transaction cost, dynamic MDP was the best performer concerning the Sortino and Omega ratio.

Interestingly, the transaction cost could decrease the dynamic portfolio's risk under dynamic ERC and MDP models. This finding implied that frequent rebalancing might result in higher mean returns without significantly increasing risk, and this finding is consistent with a previous study ([Brauneis and Mestel, 2019](#)). Note that [Table 4](#) only shows the performance of dynamic MVP, ERC and MDP portfolio under 120, 150 and 180 GARCH refit days. One should refer to [Figure 9](#) for various GARCH refit days.

Moreover, [Figure 10](#) shows cumulative returns from dynamic portfolios. With and without transaction cost, dynamic MDP had the highest mean of cumulative returns, followed by dynamic ERC. The average cumulative returns from the dynamic portfolios were higher than the average from the static portfolios. However, the average cumulative return from the EWP portfolio (see [Figure 7](#)) was slightly lower than the dynamic MDP, which had the highest mean of cumulative returns under dynamic strategies.

Further, [Figure 9](#) presents the Sortino ratio's comparison between static and dynamic MVP, ERC and MDP optimization. The results of [Figure 9](#) included 50 basis points of transaction costs. Regarding the Sortino ratio, dynamic MDP ([Figure 9C](#)) was the best strategy. Intriguingly, dynamic MDP could win over naïve system in most of GARCH refit days. The second-best strategy was dynamic ERC. Dynamic ERC ([Figure 9B](#)) could also beat the 1/N method, although it was inconsistent under various

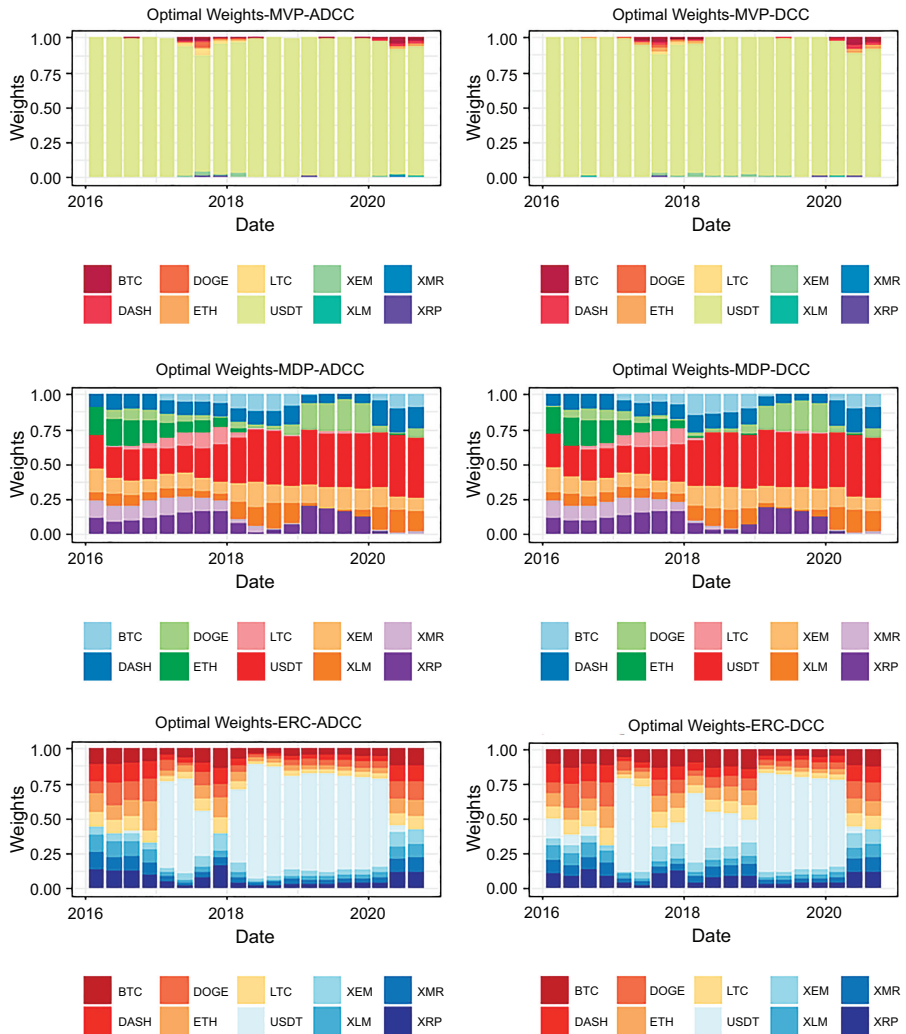


Figure 8.
Optimal weights of
dynamic approach

Note(s): These figures show optimal weights from MVP, MDP, and ERC portfolio based on DCC-GARCH (1,1) and ADCC-GARCH (1,1) with multivariate Student'-t-distribution and rolling windows method with 1750 days of forecast length. The GARCH models were refit every 360 days

refit days. However, [Figure 9](#) only shows the performance of dynamic MVP, ERC and MDP portfolio under 1750 days of forecast length. One should refer to [Figure 3](#) for various forecast lengths.

Moreover, [Figure 9](#) also shows the expected shortfall's (ES) comparison between static and dynamic MVP, ERC and MDP optimization. Concerning ES, none of the dynamic strategies could beat static strategies. The static MVP ([Figure 9D](#)) was the least risky strategy. Interestingly, the 1/N approach was the most risky strategy. The

	Refit days	ES	VaR	Drawdown	Omega	Sortino	Sharpe
<i>Panel A</i>							
MVP-ADCC	120	-0.014	-0.011	0.357	1.080	0.028	0.019
MVP-DCC		-0.014	-0.011	0.371	1.122	0.043	0.028
ERC-ADCC		-0.052	-0.041	0.724	1.198	0.068	0.051
ERC-DCC		-0.063	-0.050	0.871	1.145	0.051	0.038
MDP-ADCC		-0.061	-0.049	0.978	1.217	0.098	0.066
MDP-DCC		-0.061	-0.049	0.959	1.213	0.096	0.065
MVP-ADCC	150	-0.014	-0.011	0.456	1.091	0.032	0.021
MVP-DCC		-0.015	-0.012	0.377	1.140	0.050	0.031
ERC-ADCC		-0.064	-0.051	0.783	1.109	0.039	0.029
ERC-DCC		-0.067	-0.053	0.853	1.141	0.052	0.038
MDP-ADCC		-0.061	-0.048	0.942	1.203	0.092	0.062
MDP-DCC		-0.062	-0.049	0.927	1.206	0.093	0.063
MVP-ADCC	180	-0.015	-0.012	0.326	1.150	0.051	0.033
MVP-DCC		-0.013	-0.011	0.374	1.174	0.062	0.039
ERC-ADCC		-0.061	-0.048	0.684	1.240	0.087	0.062
ERC-DCC		-0.066	-0.053	0.775	1.174	0.064	0.047
MDP-ADCC		-0.062	-0.049	0.975	1.214	0.097	0.065
MDP-DCC		-0.061	-0.048	0.988	1.211	0.096	0.065
<i>Panel B</i>							
MVP-ADCC	120	-0.027	-0.022	0.864	0.662	-0.108	-0.079
MVP-DCC		-0.027	-0.022	0.877	0.644	-0.115	-0.085
ERC-ADCC		-0.050	-0.040	0.894	0.953	-0.018	-0.013
ERC-DCC		-0.057	-0.046	0.944	0.956	-0.017	-0.012
MDP-ADCC		-0.048	-0.038	0.895	1.000	0.000	0.000
MDP-DCC		-0.048	-0.038	0.886	1.004	0.002	0.001
MVP-ADCC	150	-0.026	-0.021	0.871	0.638	-0.115	-0.085
MVP-DCC		-0.027	-0.022	0.865	0.658	-0.110	-0.110
ERC-ADCC		-0.058	-0.047	0.966	0.880	-0.048	-0.036
ERC-DCC		-0.061	-0.049	0.964	0.940	-0.024	-0.018
MDP-ADCC		-0.049	-0.039	0.892	1.004	0.002	0.001
MDP-DCC		-0.049	-0.039	0.889	1.008	0.003	0.002
MVP-ADCC	180	-0.026	-0.021	0.887	0.619	-0.120	-0.091
MVP-DCC		-0.026	-0.021	0.894	0.604	-0.126	-0.126
ERC-ADCC		-0.056	-0.044	0.903	1.002	0.001	0.000
ERC-DCC		-0.060	-0.048	0.944	0.982	-0.007	-0.005
MDP-ADCC		-0.050	-0.040	0.889	1.022	0.010	0.006
MDP-DCC		-0.051	-0.041	0.884	1.029	0.012	0.008

Note(s): This table exhibits performance measurement of dynamic MVP, ERC and MDP portfolio with different number of GARCH refits (120, 150 and 180 days). Panel A shows the performance without transaction cost, while Panel B indicates the performance with transaction cost (10 basis points). The time-varying covariances were obtained from DCC-GARCH (1,1) and ADCC-GARCH (1,1) with multivariate Student's *t*-distribution. Rolling windows method with 1,750 days of forecast length was employed. The risk-free rate was assumed to be zero for the calculation of Sharpe and minimum acceptance return (MAR) of Sortino. A target threshold of zero was also used in omega ratio

Table 4. Performance evaluation-dynamic approach

static MDP (Figure 9F) was the second-least risky approach. Moreover, the findings were also in line with previous literature that stated $\sigma_{MVP} \leq \sigma_{ERC} \leq \sigma_{EWP}$ (Maillard *et al.*, 2010).

Figure 3 exhibits the Sortino ratio across different transaction costs and forecast lengths. Note that the results displayed in Figure 3 used 360 days of estimation windows and GARCH refit. Figure 3A indicates that EWP outperformed all static portfolios. Also, static MVP was the worst performer regarding the Sortino ratio across different

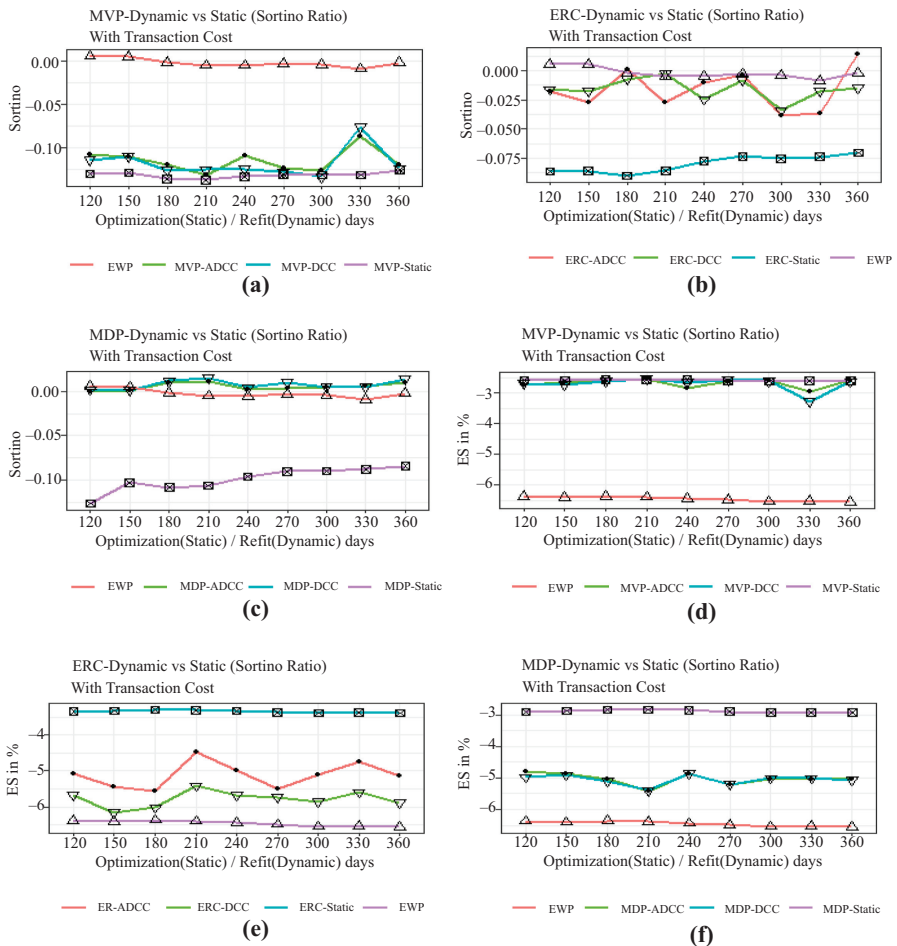
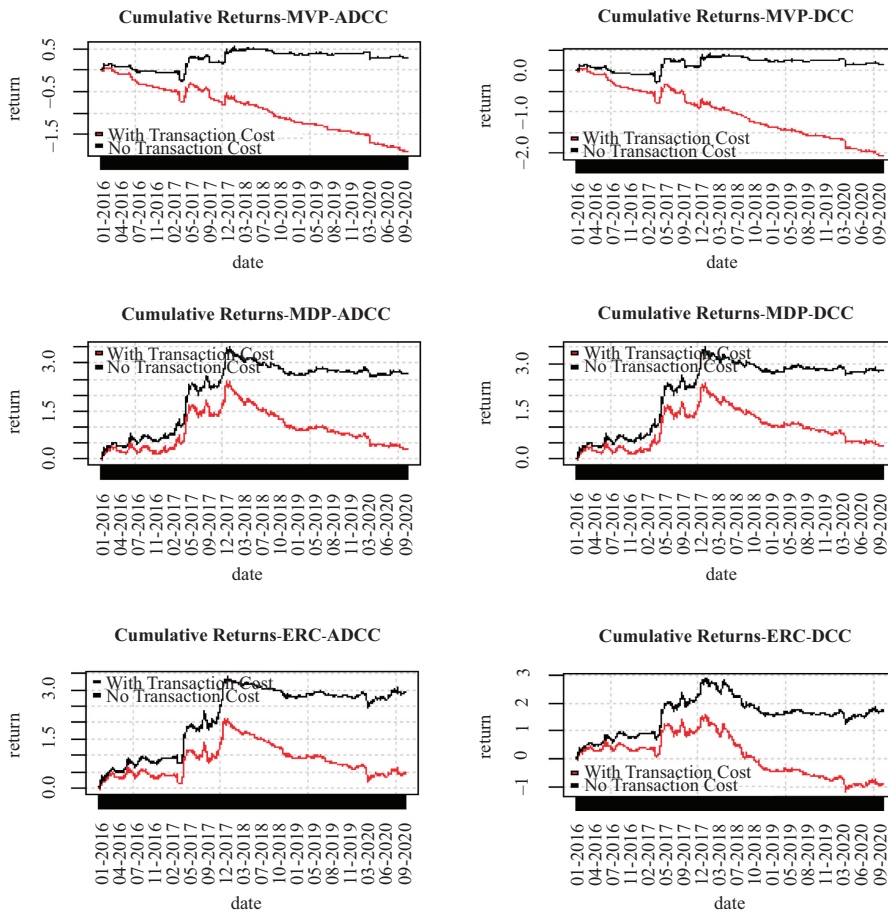


Figure 9. Dynamic vs static approach

Note(s): These figures exhibit a comparison between static and dynamic MVP, MDP, and ERC portfolio. The static approach used sample covariance or correlation matrix while the dynamic system used covariances from DCC-GARCH (1,1) and ADCC-GARCH (1,1) with multivariate Student's t distribution. The transaction cost was 50 bps. The forecast length for dynamic models was 1750 days

transaction costs. These findings are consistent with Liu (2019). Moreover, there are two interesting findings in Figure 3B. Firstly, dynamic MVP was still the worst performer for the Sortino ratio across different transaction costs. Secondly, only dynamic ERC and MDP could beat the naïve strategy, although it was not consistent across many transaction costs. Dynamic MDP could win the naïve approach when the transaction costs ranged from 10 to 50 bps. However, Figures 3C and 3D display different results under different forecast lengths. Figure 3C shows that dynamic ERC could win the naïve approach when the transaction costs ranged from 10 to 60 bps, while Figure 3D indicates that only ERC-ADCC and dynamic MDP could win the naïve approach when the transaction costs ranged from 10 to 40 bps. Dynamic MVP had the worst risk-adjusted performance in Figures 3C and 3D.



Note(s): These figures exhibit the cumulative return of MVP, MDP, and ERC portfolio using DCC-GARCH (1,1) and ADCC-GARCH (1,1) with multivariate Student’s t distribution. The black line represents a portfolio without transaction cost, while the red line indicates a portfolio with transaction cost (50 bps). GARCH refits days were 360

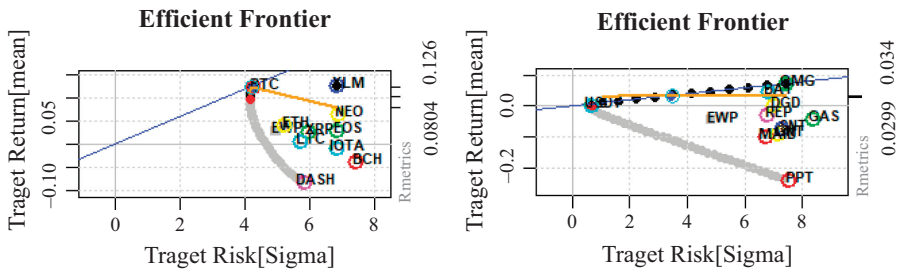
Figure 10. Cumulative return-dynamic approach

4. Robustness tests

The robustness test is the last part of the empirical results section of this research. There were two types of robustness tests in this study. First, the researcher used another researcher’s sample (Schellinger, 2020). Second, the researcher applied extension models of DCC and ADCC which were VAR-DCC and VAR-ADCC (Yousaf and Ali, 2020b). As stated earlier, this research used ten blended cryptocurrencies. A previous study (Schellinger, 2020) divided the cryptos into coins (BTC, IOTA, ETH, EOS, XRP, NEO, LTC, BCH, XLM, DASH) and tokens (MAID, USDT, GNT, GAS, OMG, DGD, BAT, PPT, SNT, REP). Global minimum variance (GMV) strategy was the best performer in regards to risk-adjusted return. Hence, this research’s robustness check used the sample and study period of the previous study and set GMV as the benchmark to beat. The sampling started from August 1, 2017, to May 31, 2019.

Before applying dynamic optimization, the researcher created efficient frontiers of long-only constrained mean-variance portfolio from token and coin-type cryptos (Figure 11). The left-hand side of the figure shows an efficient frontier from coin-type cryptocurrency. BTC and DASH are located on the efficient frontier indicating the lowest risk and the best-expected return among other cryptos. Moreover, the right-hand side of the figure shows an efficient frontier from token-type cryptocurrency. BAT and USDT are located on the efficient frontier indicating the lowest risk and the best-expected return among other cryptos.

Table 5 shows the results of the robustness test. For coin-type cryptos, all dynamic strategies outperformed the benchmark. While GMV's Sharpe annualized was marginally better than Sharpe annualized from the dynamic process, the Sortino and omega ratios are better measurements due to their ability to capture a downward deviation (Sortino and Price, 1994; Keating and Shadwick, 2002). More interestingly, dynamic ERC and MDP yielded a



Note(s): These figures present efficient frontiers from long-only constrained mean-variance portfolios. The x and y-axis are in percentages. These figures consist of the efficient frontier, tangency line, and EWP, which is the naïve strategy. The Sharpe ratio line (yellow), which coincides with the portfolio's tangency point, is also in the figure. The right-hand side axis of the model represents the range of the Sharpe ratio. The left-hand side of the figure shows an efficient frontier from coin-type cryptocurrencies. The right-hand side of the figure offers an efficient frontier from token-type cryptocurrencies

Figure 11.
Efficient frontiers-robustness test

Strategies	Coin-type cryptos			Token-type cryptos		
	Sortino	Omega	Sharpe.Annualized	Sortino	Omega	Sharpe.Annualized
GMV	0.0086	1.0215	-0.4956	-0.0483	0.9400	-0.6755
MVP-ADCC	0.0085	1.0215	-0.4957	-0.0169	0.9907	-3.2830
VAR-MVP-ADCC	-0.0059	0.9916	-0.4969	-0.0199	0.9832	-3.2585
MVP-DCC	-0.0059	0.9916	-0.4969	-0.0125	1.0013	-3.2588
VAR-MVP-DCC	0.0095	1.0230	-0.4486	-0.0199	0.9832	-3.2585
ERC-ADCC	0.0094	1.0228	-0.4487	-0.0279	0.9424	-0.7384
VAR-ERC-ADCC	0.0094	1.0228	-0.4487	-0.0263	0.9462	-0.7862
ERC-DCC	0.0095	1.0230	-0.4486	-0.0451	0.9048	-0.7344
VAR-ERC-DCC	0.0048	0.9934	-0.4437	-0.0383	0.9190	-0.7695
MDP-ADCC	0.0048	0.9935	-0.4436	-0.0190	0.9637	-0.6397
VAR-MDP-ADCC	0.0051	0.9928	-0.4409	-0.0250	0.9508	-0.6459
MDP-DCC	0.0051	0.9928	-0.4409	-0.0190	0.9637	-0.6432
VAR-MDP-DCC	0.0086	1.0215	-0.4956	-0.0244	0.9520	-0.6463

Note(s): This table exhibits performance evaluations of dynamic MVP, ERC and MDP portfolio against GMV (Schellinger, 2020). The models applied DCC and ADCC-GARCH(1,1) with multivariate Student t and 40 days of GARCH refits. Following Schellinger (2020), the risk-free rate was 2.28% p.a. The rate was for calculating Sharpe and minimum acceptance return (MAR) of Sortino, while the Omega ratio's target threshold used zero value

Table 5.
Performance evaluation-robustness test

positive Sortino ratio compared with other strategies. For token-type cryptos, all dynamic system also outperformed the benchmark. Moreover, VAR-based models did not provide significant portfolio risk-adjusted performance compared with non-VAR models. However, VAR-based models could also win the benchmark. Overall, the dynamic MDP and ERC were the best strategies. Note that the robustness test applied 40 days of GARCH refits in the optimization [2]. The results of this study support the finding of a research by [Inci and Lagasse \(2019\)](#) who used an earlier time-series sample that Bitcoin is feasible for cryptocurrencies diversification.

5. Conclusion

This research aimed to present dynamic optimization on MVP, ERC and MDP. This study applied dynamic covariances from multivariate GARCH (1,1) with the Student's *t*-distribution. This research also constructed static optimization from the conventional MVP, ERC and MDP as the comparison. Moreover, this research applied the rolling windows method with different GARCH refits and forecast lengths to ensure out of sample analysis.

Findings showed that diversification across cryptos could lower the risk under expected shortfall and VaR. MVP was the least risky strategy under static and dynamic approaches. However, none of the portfolios under the static process could beat the naïve system when the transaction cost more than 60 bps was imposed. Although dynamic portfolios outperformed static portfolios concerning Sortino and omega ratios, dynamic portfolios were riskier than the static approach.

Moreover, this research also created simulation regarding portfolio performance across different transaction costs and forecast lengths. None of the portfolio strategies could consistently outperform the 1/N approach under different schemes: net cumulative returns, various estimation windows and forecast lengths. To further validate the findings, this study used another researcher's sample as a robustness check. Results showed that the dynamic approach could beat another researcher's best strategy: GMV. Notably, dynamic MDP and ERC were the most consistent methods of outperforming 1/N system and the GMV under certain schemes.

Although this paper has some interesting findings, still this paper is not without its drawbacks. First, this paper only focuses on three risk-based portfolios, while there are still other risks based methods such as inverse volatility, risk-efficient portfolios and maximum decorrelation. Second, this paper uses dynamic covariances for ERC and MVP approaches while applying dynamic correlation may result in better performance. Third, GARCH estimations have been extensively researched. The validity of the findings under alternative methods is left for future research. Lastly, the models seem too complicated to retail investors.

This study also paves the way for future research. For instance, a dynamic approach can be applied to other, less risky assets such as stocks and bonds. Since this study only used one portfolio constraint, future study can apply more than one portfolio constraint and objective. Also, future studies can make comparisons between complicated method (e.g. GARCH-based portfolio) and simple method (e.g. momentum-based portfolio).

ORCID iDs

Bayu Adi Nugroho  <http://orcid.org/0000-0001-6113-9362>

Notes

1. The same conclusion is obtained from omega ratio and VaR.
2. Since the sample period is shorter, shorter GARCH refits days is required.

References

- Ahmad, W., Sadorsky, P. and Sharma, A. (2018), "Optimal hedge ratios for clean energy equities", *Economic Modelling*, Vol. 72, pp. 278-295, doi: [10.1016/j.econmod.2018.02.008](https://doi.org/10.1016/j.econmod.2018.02.008).
- Antonakakis, N., Chatziantoniou, I. and Gabauer, D. (2019), "Cryptocurrency market contagion: market uncertainty, market complexity, and dynamic portfolios", *Journal of International Financial Markets, Institutions and Money*, Vol. 61, pp. 37-51, doi: [10.1016/j.intfin.2019.02.003](https://doi.org/10.1016/j.intfin.2019.02.003).
- Antonakakis, N., Cunado, J., Filis, G., Gabauer, G. and de Gracia, F.P. (2020), "Oil and asset classes implied volatilities: investment strategies and hedging effectiveness", *Energy Economics*, Vol. 91, 104762, doi: [10.1016/j.eneco.2020.104762](https://doi.org/10.1016/j.eneco.2020.104762).
- Ardia, D., Bolliger, G., Boudt, K. and Gagnon-Fleury, J.-P. (2017a), "The impact of covariance misspecification in risk-based portfolios", *Annals of Operations Research*, Vol. 254 No. 1, pp. 1-16, doi: [10.1007/s10479-017-2474-7](https://doi.org/10.1007/s10479-017-2474-7).
- Ardia, D., Boudt, K. and Gagnon-Fleury, J.-P. (2017b), "RiskPortfolios: computation of risk-based portfolios in R", *The Journal of Open Source Software*, Vol. 2 No. 10, p. 171, doi: [10.21105/joss.00171](https://doi.org/10.21105/joss.00171).
- Basher, S.A. and Sadorsky, P. (2016), "Hedging emerging market stock prices with oil, gold, VIX, and bonds: a comparison between DCC, ADCC and GO-GARCH", *Energy Economics*, Vol. 54, pp. 235-247, doi: [10.1016/j.eneco.2015.11.022](https://doi.org/10.1016/j.eneco.2015.11.022).
- Baur, D.G. and Dimpfl, T. (2018), "Asymmetric volatility in cryptocurrencies", *Economics Letters*, Vol. 173, pp. 148-151, doi: [10.1016/j.econlet.2018.10.008](https://doi.org/10.1016/j.econlet.2018.10.008).
- Bouri, E., Lucey, B. and Roubaud, D. (2020), "Cryptocurrencies and the downside risk in equity investments", *Finance Research Letters*, Vol. 33, doi: [10.1016/j.frl.2019.06.009](https://doi.org/10.1016/j.frl.2019.06.009).
- Brauneis, A. and Mestel, R. (2019), "Cryptocurrency-portfolios in a mean-variance framework", *Finance Research Letters*, Vol. 28, pp. 259-264, doi: [10.1016/j.frl.2018.05.008](https://doi.org/10.1016/j.frl.2018.05.008).
- Cappiello, L., Engle, R. and Sheppard, K. (2006), "Asymmetric dynamics in the correlations of global equity and bond returns", *Journal of Financial Econometrics*, Vol. 4 No. 4, pp. 537-572, doi: [10.1093/jfinc/nbl005](https://doi.org/10.1093/jfinc/nbl005).
- Chavalle, L. and Chavez-Bedoya, L. (2019), "The impact of transaction costs in portfolio optimization: a comparative analysis between the cost of trading in Peru and the United States", *Journal of Economics, Finance and Administrative Science*, Vol. 24 No. 48, pp. 288-311, doi: [10.1108/JEFAS-12-2017-0126](https://doi.org/10.1108/JEFAS-12-2017-0126).
- Choueifaty, Y. and Coignard, Y. (2008), "Toward maximum diversification", *The Journal of Portfolio Management*, Vol. 35 No. 1, pp. 40-51, doi: [10.3905/JPM.2008.35.1.40](https://doi.org/10.3905/JPM.2008.35.1.40).
- Choueifaty, Y., Froidure, T. and Reynier, J. (2013), "Properties of the most diversified portfolio", *Journal of Investment Strategies*, Vol. 2 No. 2, pp. 1-22.
- DeMiguel, V., Garlappi, L. and Uppal, R. (2007), "Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy?", *The Review of Financial Studies*, Vol. 22 No. 5, pp. 1915-1953, doi: [10.1093/rfs/hhm075](https://doi.org/10.1093/rfs/hhm075).
- Engle, R. (2002), "Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models", *Journal of Business and Economic Statistics*, Vol. 20 No. 3, pp. 339-350, doi: [10.1198/073500102288618487](https://doi.org/10.1198/073500102288618487).
- Eraker, B. and Wu, Y. (2017), "Explaining the negative returns to volatility claims: an equilibrium approach", *Journal of Financial Economics*, Vol. 125 No. 1, pp. 72-98, doi: [10.1016/j.jfineco.2017.04.007](https://doi.org/10.1016/j.jfineco.2017.04.007).
- Ghalanos, A. (2019), *rmgarch: Multivariate GARCH Models*, R package version 1.3-7.
- Guesmi, K., Saadi, S., Abid, I. and Ftiti, Z. (2019), "Portfolio diversification with virtual currency: evidence from bitcoin", *International Review of Financial Analysis*, Vol. 63, pp. 431-437, doi: [10.1016/j.irfa.2018.03.004](https://doi.org/10.1016/j.irfa.2018.03.004).

- Inci, A.C. and Lagasse, R. (2019), "Cryptocurrencies: applications and investment opportunities", *Journal of Capital Markets Studies*, Vol. 3 No. 2, pp. 98-112, doi: [10.1108/jcms-05-2019-0032](https://doi.org/10.1108/jcms-05-2019-0032).
- Jalkh, N., Bouri, E., Vo, X.V. and Dutta, A. (2020), "Hedging the risk of travel and leisure stocks: the role of crude oil", *Tourism Economics*. doi: [10.1177/1354816620922625](https://doi.org/10.1177/1354816620922625).
- Kajtazi, A. and Moro, A. (2019), "The role of bitcoin in well diversified portfolios: a comparative global study", *International Review of Financial Analysis*, Vol. 61, pp. 143-157, doi: [10.1016/j.irfa.2018.10.003](https://doi.org/10.1016/j.irfa.2018.10.003).
- Kaucic, M., Moradi, M. and Mirzazadeh, M. (2019), "Portfolio optimization by improved NSGA-II and SPEA 2 based on different risk measures", *Financial Innovation*, Vol. 5 No. 1, p. 26, doi: [10.1186/s40854-019-0140-6](https://doi.org/10.1186/s40854-019-0140-6).
- Keating, C. and Shadwick, W. (2002), "A universal performance measure", *Journal of Performance Measurement*, Vol. 6 No. 3, pp. 59-84, available at: www.researchgate.net/publication/0D228550687_A_Universal_Performance_Measure.
- Liu, W. (2019), "Portfolio diversification across cryptocurrencies", *Finance Research Letters*, Vol. 29, pp. 200-205, doi: [10.1016/j.fr.2018.07.010](https://doi.org/10.1016/j.fr.2018.07.010).
- Maillard, S., Roncalli, T. and Teiletche, J. (2010), "The properties of equally weighted risk contribution portfolios", *The Journal of Portfolio Management*, Vol. 36 No. 4, pp. 60-70, doi: [10.3905/jpm.2010.36.4.060](https://doi.org/10.3905/jpm.2010.36.4.060).
- Markowitz, H. (1952), "Portfolio selection", *The Journal of Finance*, Vol. 7 No. 1, pp. 77-91, doi: [10.2307/2975974](https://doi.org/10.2307/2975974).
- Pal, D. and Mitra, S.K. (2019), "Hedging bitcoin with other financial assets", *Finance Research Letters*, Vol. 30, pp. 30-36, doi: [10.1016/j.fr.2019.03.034](https://doi.org/10.1016/j.fr.2019.03.034).
- Palamalai, S., Kumar, K.K. and Maity, B. (2020), "Testing the random walk hypothesis for leading cryptocurrencies", *Borsa Istanbul Review*, In Press. doi: [10.1016/j.bir.2020.10.006](https://doi.org/10.1016/j.bir.2020.10.006).
- Pfaff, B. (2016), *Financial Risk Modelling and Portfolio Optimisation with R*, 2nd Ed., John Wiley & Sons, London.
- Platanakis, E. and Urquhart, A. (2019), "Portfolio management with cryptocurrencies: the role of estimation risk", *Economics Letters*, Vol. 177, pp. 76-80, doi: [10.1016/j.econlet.2019.01.019](https://doi.org/10.1016/j.econlet.2019.01.019).
- Platanakis, E., Sutcliffe, C. and Urquhart, A. (2018), "Optimal vs naïve diversification in cryptocurrencies", *Economics Letters*, Vol. 171, pp. 93-96, doi: [10.1016/j.econlet.2018.07.020](https://doi.org/10.1016/j.econlet.2018.07.020).
- Qian, E. (2006), "On the financial interpretation of risk contribution: risk budgets do add up", *Journal of Investment Management*, Vol. 4 No. 4, pp. 41-51.
- Qian, E. (2011), "Risk parity and diversification", *The Journal of Investing*, Vol. 20 No. 1, pp. 119-127, doi: [10.3905/joi.2011.20.1.119](https://doi.org/10.3905/joi.2011.20.1.119).
- Schellinger, B. (2020), "Optimization of special cryptocurrency portfolios", *The Journal of Risk Finance*, Vol. 21 No. 2, pp. 127-157, doi: [10.1108/JRF-11-2019-0221](https://doi.org/10.1108/JRF-11-2019-0221).
- Sortino, F.A. and Price, L.N. (1994), "Performance measurement in a downside risk framework", *The Journal of Investing*, Vol. 3 No. 3, pp. 59-64, doi: [10.3905/joi.3.3.59](https://doi.org/10.3905/joi.3.3.59).
- Susilo, D., Wahyudi, S., Pangestuti, I.R.D., Nugroho, B.A. and Robiyanto, R. (2020), "Cryptocurrencies: hedging opportunities from domestic perspectives in Southeast Asia emerging markets", *SAGE Open*, Vol. 10 No. 4, doi: [10.1177/2158244020971609](https://doi.org/10.1177/2158244020971609).
- Symitsi, E. and Chalvatzis, K.J. (2018), "Return, volatility and shock spillovers of Bitcoin with energy and technology companies", *Economics Letters*, Vol. 170, pp. 127-130, doi: [10.1016/j.econlet.2018.06.012](https://doi.org/10.1016/j.econlet.2018.06.012).
- Urquhart, A. and Zhang, H. (2019), "Is Bitcoin a hedge or safe haven for currencies? An intraday analysis", *International Review of Financial Analysis*, Vol. 63, pp. 49-57, doi: [10.1016/j.irfa.2019.02.009](https://doi.org/10.1016/j.irfa.2019.02.009).
- Wuertz, D., Setz, T., Chalabi, Y. and William, C. (2017), "Rmetrics - portfolio selection and optimization", available at: www.rmetrics.org.

Yousaf, I. and Ali, S. (2020a), "Discovering interlinkages between major cryptocurrencies using high-frequency data: new evidence from COVID-19 pandemic", *Financial Innovation*, Vol. 6 No. 45, doi: [10.1186/s40854-020-00213-1](https://doi.org/10.1186/s40854-020-00213-1).

Yousaf, I. and Ali, S. (2020b), "The COVID-19 outbreak and high frequency information transmission between major cryptocurrencies: evidence from the VAR-DCC-GARCH approach", *Borsa Istanbul Review*, In Press. doi: [10.1016/j.bir.2020.10.003](https://doi.org/10.1016/j.bir.2020.10.003).

Corresponding author

Bayu Adi Nugroho can be contacted at: bayunugrohomito@gmail.com