Adverse selection analysis for profit and loss sharing contracts

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Abstract

Purpose – The purpose of this paper is to determine the optimal profit-and-loss sharing (PLS)-based contract when market frictions occur.

Design/methodology/approach – This paper opts for an adverse selection analysis and Monte Carlo simulation to assess the less risky contract for the principal and the agent when musharakah, mudarabah and venture capital financings are used in imperfect markets. Furthermore, this framework enables us to capture the level of market frictions that the principal can bear and the level of audit that he/she may undertake to mitigate bankruptcy.

Findings – The simulation results reveal that Musharakah is the less risky contract for the principal compared to Mudarabah and venture capital when the shock is low and high. Furthermore, our findings indicate that the increase of market frictions engender higher audit cost and profit-sharing ratios. The increase of the safety index in the case of high shock is most likely attributed to the increase of the audit parameter for all contracts to mitigate the selfish behavior of the agent. Accordingly, the principal tends to require a higher profit-sharing ratio to compensate for the severer information asymmetry.

Research limitations/implications – This paper has two main limits. First, the results were not compared to real data because the latter are not available. Second, this paper is a general framework to determine the less risky contract for the principal and does not consider the firm and sectoral characteristics. However, it can be extended in various ways where stress can be put on conflicts of interest between the principal and the agent with the aim to determine the contract that aligns their interests. In addition, the examination of firm dynamics in the case of equity and debt financing can provide further arguments for economic agents regarding the value of the firm, the growth rate and the lifetime of the project when information is asymmetrically distributed.

Practical implications – The findings shed some light on the necessity of the Islamic finance experts to rethink of the promotion of Musharakah because it dominates the two other contracts when market frictions occur.

Social implications – Although Maghrabi and Mirakhor (2015), Alanzi and Lone (2015) and Lone and Ahmad (2017) among others showed that profit and loss sharing can ensure economic growth, findings may motivate economic players to consider Musharakah financing with the aim to reach financial inclusion and social, which is in line with Shari’ah requirements and Islamic values.

Originality/value – Although several papers highlighted the financial contracting theory from Shari’ah perspective, they ignored the financial issues that are associated to adverse selection. This paper provides theoretical evidence regarding the selection of the less risky financing mode in case of equity financing using Monte Carlo simulation.

Keywords Monte Carlo simulation, Adverse selection analysis, Market frictions, PLS contracts

Paper type Research paper

1. Introduction

Several papers dealt with the financial contracting theory to examine the financing decision of economic agents with the aim to handle the risk of bankruptcy. Aghion and Bolton (1997),
Aghion et al. (1992) and Hart (1995) have studied this issue using the bargaining power to mitigate conflicts of interest and handle the risk of default. Nevertheless, these studies have made two main assumptions:

1. There is an ex-ante symmetric information; and
2. Risk neutrality of the principal.

While providing simplicity to the issue, these assumptions are not realistic when dealing with banks. Although the incomplete contract theory provides several insights regarding contract arrangements, the negligence of the adverse selection analysis constitutes a serious issue. Accordingly, this paper puts stress on the adverse selection analysis in case of profit-and-loss sharing (PLS) agreements with the aim to find out whether venture capital can substitute Islamic PLS contracts, as stated by Al-souwailem (1998). More precisely, we undertake an adverse selection analysis to determine the less risky contract for the principal and assess whether venture capital can be a potential model of Musharakah.

Based on the study by Ahmed (2002), we calculate a safety index that defines the principal's subjective perception of the default risk when equity financing is used with the aim to determine the less risky contract. To build this index, we consider several relevant variables that affect the principal's financing decision, such as the profit share, the profit generated by the firm, the set-up investment, the risk-free rate, the market frictions defining moral hazard and information asymmetry, the audit parameter and the industrial/technology shocks as in Ahmed (2002). Strictly speaking, this paper enables us to find out the financial contract that maximizes the safety index for the principal subject to the safety constraint by taking into account market frictions and the audit parameter. In addition, it allows us to identify the level of market frictions that the principal may bear and the level of audit that he/she can consider to handle bankruptcy.

The simulation findings show that Musharakah is the most preferred for the principal because it generates the significant safety index in both cases of low and high shocks compared to the other contracts. Similarly, the results indicate that venture capital can be the second choice for the principal, whereas Mudarabah is the least contract to be adopted. In the same context, we found that the increase of market frictions' level in case of high shock have not had a negative impact on the safety index value. More precisely, the optimal level of audit and the increase of the principal's profit share in case of high shock enabled her to mitigate the selfish behavior of the agents who hided relevant information about the project to satisfy her interests.

To achieve this purpose, Section 2 defines the theoretical foundation of our research. In Section 3, the general model and equations linked to this analysis are highlighted. In Section 4, the calibration of parameters is discussed, whereas the simulation process is illustrated in Section 5. Finally, the simulation results, the discussion and the conclusion are highlighted in Sections 5, 6 and 7, respectively.

2. Literature review
The principal-agent problem has been widely examined in the conventional literature, starting from Smith (1776) who determined some incentive issues linked to sharecropping contracts, which are PLS sharing, monitoring process and adverse selection in human cooperation. To deal with these issues, Coase (1937) came up with a new theory to assess the performance of the firm by putting stress on the role of technology and return to scale, as important determinants of the size of the firm and the optimal production. However, he considered the firm as a black box and completely ignored incentive problems within it. To
this extent, Hart (1995) claimed that the theory of the firm has nothing to say about the internal organization of firms. This theory was later extended by Williamson (1979) and Jensen and Meckling (1976) and became known as the economics of organization. The authors established the agency theory that considers the effect of the manager and the selfish behaviour of agents to assess the contractual relationship when moral hazard and asymmetric information exist. Nevertheless, the agency approach falls foul of the same criticism while the authors did not say much about the internal organization of the firm, as stated by Hart (1995).

An alternative approach to address the issues in incomplete contract proposed that the cornerstone of the incomplete contract theory is the allocation of decision rights, which later was extended to include the roles of bargaining power to align the interests of agents. Hart (1989), Hart (2003), Hart (2017) and Hart and Moore (1994) claim that careful allocation of decision rights can substitute the contractually specified rewards. Accordingly, this approach has been developed based on important conditions:

- the principal is risk-neutral;
- there is only one principal and one agent; and
- the agents have symmetric information ex-ante.

The assumption that agents do not face asymmetric information problem ex-ante is a rather strong assumption to make, which agrees with Ross (1973), Arrow (1971), Jensen and Meckling (1976), Akerlof (1970) and Tirole (1999). Consequently, it would be difficult to admit the first and third conditions in our study because bankers cannot be risk-neutral in real practice. In addition, information asymmetry represents the main determinant of any investment decision. To summarize, we shall argue that financial contracting theory and incomplete contract approach have brought relevant solutions for decisions and rights control with further insights and procedures that must be implemented regarding adverse selection issue between agents.

The financial contracting theory has also been explored from the Islamic perspective, with particular focus given to the notion of PLS and risk sharing, in addition to moral hazard and asymmetric information problems. This aims at determining the optimal contracts and incentive system compatible to the Islamic law. However, the literature was divided into two mainstreams where the first justifies the marginalization of PLS-based contracts, whereas the second encourages their adoption.

Among those who justified the marginalization of PLS contracts, Dar and Presley (2000), Farooq (2007) and Ebrahim and Sheikh (2016) assumed that an imbalance between management and control rights is attributed as a major cause of lack of PLS in the practice of Islamic finance. Given this imbalance, the agency problem becomes severer, which renders the PLS principle less attractive vis-à-vis other modes of financing. Farooq and Ahmad (2013) found various reasons for the slow growth of Musharakah financing in Pakistan, namely, lack of interest of the bank management in Musharakah financing, lack of expertise and lack of government support, which is in line with the studies by Muhammad (2014) and Lone and Quadir (2017).

To this extent, Al-souwailem (1998) stated that venture capital can be a potential model of musharakah. However, the author did not provide relevant evidences about the relationship between venture capital’s model and the Islamic model of partnership regarding moral hazard and asymmetric information problem. The recent study by Mehri et al. (2017) proposed a theory of profit-sharing ratio (PSR) with information asymmetry and considered the negotiated PSR as a screening device in their framework. Although this theoretical
framework constitutes a new tool for the screening managers’ type, the authors found that adverse selection can be captured when the PSR accepted by the manager exceeds a given threshold value, which represents the maximum payoff to the venture capitalist.

Among those who encouraged the adoption of PLS agreement, Muda and Ismail (2010) and Sapuan (2016) proposed optimal conditions to minimize the problem of asymmetric information such as providing incentives for entrepreneurs in case of profit and the establishment of monitoring device for Musharakah. In the same context, Ernawati (2016) analysed the risk of PLS financing in Indonesian Islamic banking and argued that it is more secure for Islamic banks to allocate funds in Musharakah contracts instead of Mudarabah.

In line with the aforementioned studies, Nabi (2012) examined the effect of PLS contract on the evolution of the income inequality with capital accumulation process based on the study by Aghion and Bolton (1997). He examined the problem of wealth inequality between two investors with different wealth classes. He found that the wealth inequality between the two classes of investors decreases over time, which proves that the profit-sharing contract changes the dynamic of wealth toward income inequality. This evidence indicates that the entrepreneurship allows the latter to catch-up the initially wealth class, which is in line with the study by Maghrebi and Mirakhor (2015).

Based on the agency issues related to equity-based contracts, Mansour et al. (2015-b) proposed a new equity-based instrument through a three-tier partnership by including a new contracting party defined as the risk moderator to absorb the underlying risk of default and adjust the annual revenue to a predetermined annual cost. Interestingly, the simulation results show that immunization against premature default through the involvement of the risk moderator to absorb any potential loss is indicative of an incentive factor for the project’s survival and business continuity. Al-Souwailem (2003) examined the optimal sharing contracts by comparing the PLS contract to the standard debt contract (involving riba) under the cases of symmetric and asymmetric information. It is found that the aggregate expected profits from the sharing contract exceed those of the debt contract under both symmetric and asymmetric information. Moreover, for a certain range of the opportunity cost, both the financier and the agent are better off when they get involved in a sharing contract instead of debt contract. Ahmed (2002) came up with a theoretical framework for PLS financing contracts based on the study by Gale and Hellwing (1985) with the aim of determining the incentive-compatible contracts. While banks do not mostly have the incentive to enforce PLS contracts, Ahmed (2002) provided several incentives to bankers and entrepreneurs to proceed with this financing contract. The author asserted that the specification of the profit share, the adverse selection analysis, the auditing rule and the reward/punishment rules are fundamental to build a strong partnership in imperfect markets.

Based on these studies, it has been noticed that there is a lack of evidence regarding the selection of the less risky contract for the principal among Musharakah, Mudarabah and venture capital. In practice, banks undertake a given project when the risk-adjusted expected rate of return exceeds the return from the risk-free return. While the first component of the risk-adjusted expected rate of return is composed of a random parameter defining the safety index and the expected profit generated, the second component represents the return from the risk-free return, which is the conventional benchmark.

In this study, we will determine the safety index that corresponds to each financing contract for the principal to provide theoretical evidence regarding market frictions. This adverse selection analysis enables us to determine the less risky contract for the principal and identify the level of audit that he/she may undertake to maximize his/her safety index for every financing mode.
3. Model design

The primary purpose of our approach is to determine the less risky financing instrument for the principal among several contracts, namely, Musharakah, Mudarabah and venture capital. Our study aims to identify the contract that maximizes the safety index’s value for the principal subject to the safety constraint, regarding market frictions, the audit factor and the industrial shocks that can be high or low. According to Tauchen (1986) and Adda and Cooper (2002), these shocks follow a first-order Markov process, which means the value of the high shock depends on the low shock value. Similarly, this analysis allows us to capture the level of market frictions that the principal may encounter and the level of audit that he/she may undertake to mitigate bankruptcy when equity financing is used. Because our approach is inspired from the study by Ahmed (2002), the author considers that the repayment function defines the transfer promised to the principal (the bank) by the agent (the firm) as a function of profit. In particular, the repayment function depicts the ratio at which profit will be shared between the firm and the bank. Ahmed (2002, p. 46) argues that “since a profit-sharing arrangement is a partnership, we assume both parties share ex ante the auditing cost A equally”. Although A ranges between zero and the unity, agents must share the audit cost, indicating that each value of A has to be shared equally between the bank and the firm. Based on Ahmed (2002), the PSR S (the share that goes to the principal) is derived from the following expression:

\[ S_t(\sigma_t \pi_t - 0.5A) = r_f F_t \]

where, \( S \), \( r_f \) and \( F_t \) are the profit share, the risk-free rate and the total funds invested, respectively. \( 0 < \sigma_t < 1 \) represents the safety index, \( \pi_t \in [-F, +\infty) \) is the profit generated by the firm and \( A \) defines the auditing cost. From the aforementioned equation, Ahmed (2002) determines the expression for the PSR as follows:

\[ S_t = r_f F_t / (\sigma_t \pi_t - 0.5A_t) \]

Consequently, the general form of the safety index equation is given as follows:

\[ \sigma_t = \left[ \frac{r_f F_t}{S_t} + (0.5A_t) \right] \frac{1}{\pi_t} \]  

(1)

After determining the general form of the safety index value for the principal, it is convenient to select the contract providing the highest safety index’s value with the aim to avoid bankruptcy in imperfect markets. Nevertheless, the literature asserted that agents may compare their expected income with the risk-free rate, which is the benchmark adopted by bankers to assess the feasibility of a given project. In this regard, Ahmed (2002) indicates that the bank will be willing to finance such project only when the risk-adjusted expected income exceeds the risk-free income, which is illustrated as follows:

\[ \sigma_t Y_t > r_f F_t \]

where \( Y_t = S_t \pi_t \) is defined as the income received by the principal.

Based on the previous inequality, it is possible to capture the level of safety index that the principal may consider to avoid bankruptcy when financing by Musharakah, Mudarabah, and venture capital is used. Accordingly, the safety constraint for the principal is given by the following equation:
At this level, the principal may choose the contract that yields the highest safety index using Equation (1), subject to the safety constraint defined by Equation (2). We shall notice that the higher the risk, the closer $\sigma_t$ is to zero, as stated by Ahmed (2002). By examining the inequality given by Equation (2), the safety threshold depends on the invested funds, the risk-free rate and the principal’s profit share. Accordingly, the principal’s maximization program is given as follows:

$$\max \sigma_t = \left( \frac{r_f F_t}{S_t} + 0.5A \right) \left( \frac{1}{\pi_t} \right)$$

subject to:

$$\sigma_t > \frac{r_f F_t}{Y_t}$$

### 3.1 Profit equations for the principal

This sub-section aims at determining the profit equations for the principal when PLS contracts are used. Cooley et al. (2004) assumed that it is convenient to define the discounted expected profit generated by the firm before determining the profits equations related to all contracts. Accordingly, the profit equation form is given as follows:

$$\pi_i(F_t, l_t, \omega_t, Z) = -F_t + \left( \frac{1}{1+r_f} \right) \left[ bF_t + (1-\alpha) [(1-\delta)F_t + f_i(Z, F_t, \alpha) - \omega_t l_t] \right]$$

The production functions can take two different forms:

$$f_i(z_L, F_t, \alpha) = z_L F_t (1-\alpha) \quad \text{in case of low shock}$$

$$f_i(z_H, F_t, \alpha) = z_H F_t (1-\alpha) \quad \text{in case of high shock}$$

where the parameter $0 < b < 1$ is the probability of liquidation that stems from the event of losing the project because of the agent’s death or any other unexpected events, as stated by Cooley et al. (2004). The function $f_i$ depends on the industrial shock $Z = (z_L, z_H)$ and the invested funds. The parameter $0 < \alpha < 1$ measures market frictions. The parameter $0 < \delta < 1$ is a random variable defining the depreciation rate. Finally, the parameters $\omega_t$ and $l_t$ represent the wage and the labor, respectively.

The profit function by Cooley et al. (2004) considers several variables that may affect the production function and the survival of the firm in imperfect markets. They considered that the industrial shocks, the probability of liquidation and the wage and labour are fundamental to assess the profit generated by the firm. If the firm is liquidated, which happens when $b$ reaches the unity, the firm’s value is equal to $\frac{-r_f F_t}{(1+r_f)} < 0$. Nevertheless, if the firm does not face any risk of liquidation, i.e. $b = 0$, the production takes place and the firm’s value is $-F_t + \left( \frac{1}{1+r_f} \right) [(1-\delta)F_t + f_i(Z, F_t, b) - \omega_t l_t]$. The disutility from working is defined by Cooley et al. (2004) as:
where $B$ is a factor that captures the amount of time spent on working and $\varepsilon$ corresponds to the elasticity of labor.

With the properties of the disutility function can be given by the following partial derivatives with respect to labour: $\varrho(0) > 0$; $\varrho(l_t) > 0$; and $\varrho''(l_t) > 0$. Cooley et al. (2004) claimed that the wage factor is the first derivative of the disutility from working $\omega = \varrho(l_t)$, indicating that $\varrho'(l_t) = \omega = B \frac{1+\varepsilon}{\varepsilon} \left(\frac{l^{1+\varepsilon}}{(1+\varepsilon)}\right)^{-1}$.

3.1.1 Profit function for Musharakah contract. While Musharakah is based on the PLS principle, losses are borne by both the principal and agent according to their respective contribution. However, the profit shared between them is based on a pre-determined rate. We denote by $\lambda_t$ the profit share of the agent, which means that the principal receives $(1 - \lambda_t)$. Let $F_t = F_t^e + F_t^i$ be the total funds invested by the firm, where the first component corresponds the internal funds invested by the agent and the second component corresponds to the external funds. The profit equation of the principal in case of Musharakah contract is defined as follows:

$$Y_{musharakah}^t = (1 - \lambda_t) \pi_t(F_t, l_t, \omega_t, Z).$$

In case of profit, the agent receives $\lambda_t \pi_t(F_t, l_t, \omega_t, Z)$, where $\lambda_t$ ranges between zero and the unity. In the opposite case, losses will be divided between the principal and the agent according to their respective contribution. In the same context, the Musharakah contract has an explicit cost which is linked to the principal’s participation.

3.1.2 Profit function for Mudarabah. Considering Mudarabah as the second type of Islamic PLS-based contracts, the profit should be shared according to a pre-determined agreement. However, losses will be borne by the principal alone. More precisely, the principal finances the project and the agent brings only her managerial expertise and knowledge. Consequently, her share will be lower than the share of the principal while the investment is totally financed by external funds. Consider the variable $\gamma_t$ as the parameter defining the share of the agent, the profit equation for the principal is illustrated as follows:

$$Y_{mudarabah}^t = (1 - \gamma_t) \pi_t(F_t^e, l_t, \omega_t, Z)$$

where $(1 - \gamma_t)$ is the profit received by the principal when the project does not go bankrupt. Nevertheless, in case of default, she receives nothing and she will be recommended to bear the losses.

3.1.3 Profit function for venture capital. In this case, the venture capitalist provides the assets and his/her managerial expertise for the firm in exchange of a considerable equity share until the end of the contract. In other words, the profit share of the venture capitalist will be greater than the share of the agent. Consider $\eta_t$ as the profit share of the agent. Let $F_t = F_t^e + F_t^i$ be the total funds invested. The profit share of the principal will be defined as below:

$$Y_{venture capital}^t = (1 - \eta_t) \pi_t(F_t^e, l_t, \omega_t, Z)$$

where $(1 - \eta_t)$ is greater than $\eta_t$ because the principal (venture capitalist) provides the assets and all her managerial skills. In case the project does not fail, the profit will be divided according to this parameter.
3.2 Determination of the two levels shocks

Based on Adda and Cooper (2002) and Tauchen (1986), the two levels of shocks are determined by the following first-order autoregressive process, AR(1):

\[ z_{t+1} = \rho z_t + \varepsilon_{t+1}, \quad \text{var}(\varepsilon_{t+1}) = \sigma^2, \quad \text{where } |\rho| < 1. \]  

(11)

where \( \varepsilon_{t+1} \) is defined as the white noise and is distributed with mean zero and unit variance \( \sigma^2 \). The parameter \( \rho \) is the slope coefficient of the AR(1) process, which represents the persistence of the shock. According to Adda and Cooper (2002) and Stokey and Lucas (1989), the quality of the approximation remains good except when the parameter \( \rho \) is very close to the unity. Even though, Tauchen (1986) argued that the parameter \( \rho \) must be less than 0.9 for high persistence of the shock. Experimentations showed that when it is close to 0.9, the gap between consecutive shocks becomes very low.

To discretize the AR(1) process, Tauchen (1986) assumed that the process stays within a bounded interval to be able to solve the problem. Specifically, he considered that the shock can be approximated by a two-state Markov chain such that \( Z \) can take on two values, namely, \( z_L \) and \( z_H \) (\( z_L < z_H \)). Adda and Cooper (2002) assumed that the probability of the realization of the shocks can be determined by the following symmetric transition matrix:

\[ \Pi = \left( \begin{array}{cc} q & 1-q \\ 1-q & q \end{array} \right) \]

The variables \( z_L, z_H \) and \( q \) are selected by Adda and Cooper (2002) such that the process reproduces the conditional first and second order moments of the AR(1) process as follows:

First-order moment:

\[ qz_L + (1-q)z_H = \rho z_L \]

\[ (1-q)z_L + qz_H = \rho z_H \]

Second-order moment:

\[ qz_L^2 + (1-q)z_H^2 - (\rho z_L)^2 = \sigma^2 \]

\[ (1-q)z_L^2 + qz_H^2 - (\rho z_H)^2 = \sigma^2 \]

From the two equations of the first-order moment, we obtain \( z_L = -z \), and \( q = \frac{1+\rho}{2} \). Inserting these two results into the two equations of the second-order moment generates the following:

\[ z_L = \sqrt{\frac{\sigma^2}{1-\rho^2}} \]  

(12)

\[ z_H = -z_L = -\sqrt{\frac{\sigma^2}{1-\rho^2}} \]  

(13)

However, one practical concern for the above approach is how to deal with negative values of the shock. More precisely, this means that the firm’s technology produces negative...
output, which does not hold from an economic perspective. To prevent this situation, it is required to transform the shock by taking its exponential form to ensure that all values of the shock are positive.

3.3 Assumptions

Assumption 1: The contract is safe when it generates a greater impact on the safety index for the principal subject to the safety constraint. The higher the risk the closer is the safety index to zero, as stated by Ahmed (2002).

Assumption 2: The principal can observe the information related to the firm only in case of bankruptcy. While it has always been a difference between declared and non-declared profit, the moral hazard problem occurs. Thus, problems of information asymmetry and moral hazard still exist and cannot be ignored, as mentioned by Cooley et al. (2004).

Assumption 3: There is only one principal and one entrepreneur. Hart (1995) considered only one principal and one entrepreneur for contract arrangement because, in case of multiple agents and principles, it will be difficult to satisfy the incentives of compatible contracts and the optimality of the transaction.

Assumption 4: The principal is rational. The literature indicates that the principal is rational because he/she continuously aims to maximize her profit and minimize agency costs.

3.4 Model determination based on contracts

3.4.1 Musharakah contract. While Musharakah is a PLS-based agreement, losses will be borne by both contracting parties according to their contributions. Following Ahmed (2002), we assume that the auditing cost is shared equally between the principal and the agent. Hence, the safety index’s maximization program of the principal for Musharakah is defined as follows:

$$\max s_{musharakah} = \left[ \left( \frac{r_f F_t}{1 - \lambda} + (0.5A) \right) \frac{1}{\pi_t} \right]$$ (14)

subject to

$$s_{musharakah} > \frac{r_f F_t}{Y_{musharakah}}$$ (15)

Equation (14) represents the safety index of the bank for Musharakah, and $Y_{musharakah}$ and $(1 - \lambda)$ are the expected discounted profit generated and the profit share for the principal, respectively. Equation (15) defines the safety constraint for the bank where the first component defines the safety index of the project and the second component corresponds to the safety threshold.

3.4.2 Mudarabah contract. When mudarabah financing is used, the project is completely financed by external funds and the auditing parameter is equally divided between parties. Hence, the safety index of the principal is illustrated as follows:

$$\max s_{mudarabah} = \left[ \left( \frac{r_f F_t^*}{1 - \gamma} + (0.5A) \right) \frac{1}{\pi_t} \right]$$ (16)

subject to
where equations (16) and (17) represent the maximization program of the safety index and the safety constraint for the bank in case of mudarabah, respectively.

3.4.3 Venture capital contract. The maximization program is given as follows:

$$\max \sigma^\text{venture capital} = \left[ \left( \frac{r_F Y_t}{1 - \eta_t} + (0.5A) \right) \frac{1}{\pi_t} \right]$$

subject to

$$\sigma^\text{venture capital} > \frac{r_F Y_t}{Y^\text{venture capital}}$$

Equations (18) and (19) represent the maximization program of the safety index of the principal and the safety constraint, respectively, when financing is done by a venture capital.

4. Calibration

Tables I and II show the calibration of the state and control variables. While a control variable corresponds to a variable that can be parameterized, a state variable is random and cannot be controlled.

### Table I. Calibration of control variables

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Label</th>
<th>Value</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.0579</td>
<td>(Cooley et al., 2004)</td>
</tr>
<tr>
<td>$r_F$</td>
<td>Risk-free rate</td>
<td>0.0400</td>
<td>(Cooley et al., 2004; Ahmed, 2002)</td>
</tr>
<tr>
<td>$b$</td>
<td>Probability of liquidation</td>
<td>0.0500</td>
<td>(Cooley et al., 2004)</td>
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<tr>
<td>$l$</td>
<td>Labor factor</td>
<td>0.3300</td>
<td>(Cooley et al., 2004; Evans, 1987; Atkeson and Kehoe, 2007)</td>
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<tr>
<td>$B$</td>
<td>Disutility from working</td>
<td>0.001</td>
<td>(Cooley et al., 2004)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>The elasticity of labor</td>
<td>1.000</td>
<td>(Cooley et al., 2004)</td>
</tr>
<tr>
<td>$F$</td>
<td>Invested funds</td>
<td>100</td>
<td>Adda and Cooper (2002)</td>
</tr>
<tr>
<td>$F'$</td>
<td>External funds</td>
<td>50</td>
<td>Hasan (1985)</td>
</tr>
<tr>
<td>$F''$</td>
<td>Internal funds</td>
<td>50</td>
<td>Hasan (1985)</td>
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### Table II. Calibration of state variables

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<th>Variable</th>
<th>Label</th>
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<th>References</th>
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<td>$\lambda$</td>
<td>Profit share of the firm for Musharakah</td>
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<td>Hasan (1985)</td>
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<td></td>
<td></td>
<td></td>
<td>Shari'ah Advisory Council (SAC Bank Negara Malaysia)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Profit share of the firm for Mudarabah</td>
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<td>$\eta$</td>
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<td>Hasan (1985)</td>
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<td>$\alpha$</td>
<td>Market frictions' parameter</td>
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<td>Ahmed (2002)</td>
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<tr>
<td>$A$</td>
<td>Audit' parameter</td>
<td>[0.1]</td>
<td>Ahmed (2002)</td>
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</tbody>
</table>
Table I shows all control variables used in our study. The same risk-free interest rate, \( r_f \), was considered for Islamic and conventional PLS contracts because the Islamic Inter-bank Rate (IIBR) and London Inter-bank Offered Rate (LIBOR) are significantly dependent (Ben Amar (2018)). Although “Islamic banks pricing practices are likely to converge towards conventional ones” Ben Amar (2018, p. 7), the risk-free interest value calibrated by Cooley et al. (2004) is considered in this study for Musharakah, Mudarabah and venture capital.

The probability of liquidation is set to \( b = 0.05 \). According to Cooley et al. (2004), this is consistent with the numbers reported in industry dynamics studies such as Evans (1987).

The elasticity of labor is set to \( e = 1 \), which is the value often used in business cycle studies, as stated by Cooley et al. (2004). The parameter \( B \) is chosen by the authors such that one third of available time is spent on working.

Although this study examines equity-based contracts, it is recommended to provide theoretical evidences regarding the calibration of the state variables (Table II). The literature asserts that there are two types of opinion about equity-like instruments among the jurists. Hasan (1985) claimed that the Hanafis and Hanbalis with other few Shafiis[3] scholars assume that suppliers of capital have to bear losses in proportion to their contribution. However, they are free to negotiate the PSRs. In the same way, the Malikis and most of Shafiis pretend that in Musharakah, the profits and losses must be shared according to the same ratio, which is \( \frac{P_t}{P} \) for the principal and \( \frac{P_t}{P} \) for the agent.

If the principal and the agent provide the same contribution, they are supposed to have the same profit share. However, in practice, this profit share is not identical because the principal brings only funds, whereas the agent provides her managerial skills, expertise and a share of the capital, which is equal to the principal’s contribution in our study. According to Hasan (1985), Islamic economists argue that profit is the result of the combined effort of capital and expertise. Hence, keeping the PLS equal for the financier would be unfair to the firm. It will discriminate in favour of the dormant supplier of funds.

Based on the aforementioned proposition, the firm may receive a profit share equal or greater than 50 per cent when the invested funds are identical in case of Musharakah. Admitting that the lower bound of the profit share is equal to 50 per cent, the question is related to the determination of the upper bound, which defines the maximum profit share. Although the gap between the profit shares received by the principal and the agent should be significant, the upper bound of the agent is set to 65 per cent, according to the Shari’ah Advisory Council of Bank Negara Malaysia. Consequently, we set the profit share of the agent and the financier to \( \lambda_1 = [0.5; 0.65] \) and \( 1 - \lambda_1 = [0.35; 0.5] \), respectively.

Considering venture capital as the conventional form of partnership, the profit distribution is not identical to Musharakah, while the venture capitalist provides external funds, in addition to his managerial expertise, and the agent (the firm) brings only the initial funds. Comparing to Musharakah, the agent cannot receive more than (50 per cent) when venture capital financing is used, which is considered as the upper bound. Based on the contribution of the venture capitalist, the agent finds himself/herself in the opposite situation of Musharakah financing. As a consequence, the firm is hypothetically supposed to receive \( \eta_t = [0.35; 0.5] \) and the principal receives \( 1 - \eta_t = [0.5; 0.65] \).

The Shari’ah Advisory Council of Bank Negara Malaysia argues that Mudarabah’s profit is shared between the capital provider and the agent according to a mutually agreed-upon PSR. However, losses are borne solely by the capital provider if such loss is not caused by the agent’s negligence or violation of the pre-specified conditions.

In this context, the Shari’ah Advisory Council of Bank Negara Malaysia discussed several illustrations regarding the PSR in case of Mudarabah financing with the aim to determine an approximated rate. Although the investment is fully financed by external
funds and losses are borne by the capital provider, the agent receives a small remuneration that could range between 10 and 20 per cent; thus, we set \( \gamma_l = [0.1; 0.2] \).

5. Simulation and results

In this section, the objective is to determine the less risky financial contract for the principal by considering market frictions, the invested capital invested, the two levels of shocks, the profit share and the audit parameter. For this purpose, the simulation process is illustrated as follows:

- First, it is required to calculate the two levels of shocks using the equations (11), (12) and (13) based on the study by Adda and Cooper (2002), Tauchen (1986) and Stokey and Lucas (1989).
- Second, we write the script of the objective function (equation (3)) after calculating the profit generated by the firm using equations (5), (6) and (7).
- Third, we build the code of the safety constraint (equation (4)) in a separate file.
- Fourth, we use the optimization toolbox in Matlab to generate our results by identifying the objective function, the constraint and the lower and upper bounds of the state variables.

The adverse selection analysis aims at the determination of the contract that maximizes the safety index’s value for the principal subject to the safety threshold in case of low and high shocks. To this extent, the optimization problem generates three plots, namely, the current point (indicating the simulated state variables), the current function (indicating the optimal value of the safety index) and the first-order optimality (indicating the safety index constraint violation) for each contract. We have to mention that Matlab’s output generates negative values for the second and third plots. However, we interpret such simulated values as positive numbers (Table III).

Figures 1 and 2 show the optimal values of Musharakah contract’s safety index when the shock is low and high, after considering the market frictions and the audit factor. The first plots of Figures 1 and 2 show the optimal values of the parameters related to market frictions, the profit share and the audit factor. We notice that the simulated values of the parameters \( \alpha \) and \( A \) have been changed for the three contracts when moving from the low shock to the high shock. Indeed, the market frictions and the audit’s parameters are optimally equally to 0.2 and 0.3 in case of low shock and 0.7 and 0.5 in case of high shock, respectively. This indicates that in a riskier environment, i.e. when moving from the low shock.
Figure 1. Musharakah (low shock)

Figure 2. Musharakah (high shock)
shock to the high shock, the severity of market frictions tend to increase and the audit becomes more costly.

In case of low shock, the profit share, \( \lambda \), generated is equal to 0.65, which is the optimal share received by the agent. Accordingly, the principal receives 0.35 of the profit generated by the firm. In addition, the optimal values of the safety index for the principal subject to the safety constraint is equal to 0.2977 and 0.61112 for the low and high shocks, respectively. As for the first-order optimality, the optimal values are equal to 0.0038 and 0.1677 for the low and high shocks, respectively.

These simulation results indicate that when the shock is low the optimal level of audit allows the principal to handle the increase of market frictions, enabling him/her to get a safety index value around 0.2977. In case of high shock, the simulated values of the state variables have shown a significant change explained by the increase of market frictions' parameter and the audit factor for the three contracts. The profit share for the agent decreased; consequently, the optimal value of the safety index for the principal subject to the safety constraint increased to 0.6111, whereas the constraint violation became equal to 0.1677.

For a given level of market frictions at \( \alpha = 0.7 \), the increase of the optimal value from 0.2977 (in case of low shock) to 0.6111 (in case of high shock) indicates that the principal is compensated for this riskier situation (higher risk of default) by having a higher profit sharing rate. In case of high shock, the level of market frictions increases to reach 0.7, indicating that the agent is more likely to cheat and hide significant information about the project to satisfy his/her interest. However, the increase of the audit parameter to 0.5 allows the principal to handle this higher level of information asymmetry.

Accordingly, the principal observes that his/her maximized value function increases as a consequence of a higher value of the shock. It is for this reason that the simulated parameters regarding the market frictions, the profit share and the audit have been changed after moving from low shock to high shock. Similarly, it is noticed that the increase of the shock alters the behavior of the agent, which can be expressed in terms of more pronounced moral hazard. As a consequence, the audit’s parameter tends to increase because in a riskier situation the agency costs get bigger. The Musharakah contract seems obviously to be more attractive in the case of high shock because the maximized safety index is higher.

As for the Mudarabah and venture capital contracts, similar findings are found. Indeed, Table III and Figures 3-6 show that the market frictions and audit’s simulated parameters are optimally equal to 0.2 and 0.3 in the case of low shock, and 0.7 and 0.5 in the case of high shock, respectively. These simulated parameters correspond to the same values in the case of Musharakah contract, which can be attributed to the common PLS-based financial arrangements to the three contracts. Indeed, the principal considers that the agent can exhibit the same level of information asymmetry in the three contracts, which requires the same audit parameter for all of them. In the same manner, we notice that the principal’s PSRs increase when moving from the case of low shock to the case of high shock. This is attributed to the fact that the principal observes an increase in the severity of information asymmetry (i.e. higher \( \alpha \)) and he/she judges relevant to increase his/her profit share. This applies to the three contracts.

Regarding the optimized function value, it is clear that it increases in the case of high shock as a response to increased risk. Indeed, we notice that the optimal function value increased from 0.1324 to 0.3478 for Mudarabah in the cases of low and high shocks, respectively, and from 0.2096 to 0.5114 for venture capital in the cases of low and high shocks, respectively. The comparison between the three contracts on the basis of the function value shows that the Musharakah contract has the highest simulated value in both cases of low and high shocks.
Figure 3.
Mudarabah (low shock)

Figure 4.
Mudarabah (high shock)
This is indicative that the Musharakah contract dominates the Mudarabah and venture capital contracts from the perspective of the principal who aims at maximizing her safety function. For a higher value of market frictions’ parameter, the principal considers that she is safer when getting engaged in a Musharakah contract because:
The examination of the simulated values that correspond to the first-order optimality indicates that, for the three contracts, they tend to increase from a lower value (in case of a low shock) to a higher value (in case of a high shock). This is explained in terms of the safety constraint that corresponds to the principal’s maximization program. Furthermore, it is noticeable that the simulated values of the state variables have been changed for the three contracts. A possible explanation is related to the selfish behavior of the agent who is more likely to cheat and hide a significant amount of information about the project to maximize her profit when economic conditions do not improve.

6. Discussion
Ahmed (2002) argued that the adverse selection analysis aims at the determination of the contract that maximizes the safety index value for the principal subject to the safety constraint in case of low and high shocks. Assuming that the financing decision is derived from the principal’s perspective, the adverse selection analysis provides additional insights regarding market frictions and audit factor to determine the contract that allows the principal to mitigate bankruptcy. In other words, the contract providing the highest safety index is considered as the most attractive financing mode for the principal.

Based on our simulation results, it is obviously clear that the market frictions have not shown a significant negative impact on the safety index value in the case of high shock because of the optimal level of audit undertaken by the principal. Although the state variables have increased in the case of high shock, the increase of the market frictions’ parameter indicates an adverse selection problem, where relevant information about the project might be hidden by the agent. However, such adverse selection problem did not become more pronounced because the safety index value experienced a significant increase. The principal can still handle the risk of default when the shock is high by the optimal auditing rule undertaken under the inaccessibility to all types of information. As a result, the principal gauges that the selfish behavior of the agent would occur in this case. Consequently, it is possible to maintain a higher level of safety in case of high shock because of an increased level of audit for all contracts, alongside with higher profit-sharing rates.

Although the main purpose of this paper is to find out the less risky contract for the principal among equity-based contracts, our simulation findings reveal that Musharakah is the most preferred for the principal because it generates the highest safety index in both cases of low and high shocks in comparison to the other contracts. While the findings indicate that venture capital can be the second choice for the principal, Mudarabah is the least contract to be adopted.

7. Conclusion
In contract arrangements, an excessive degree of asymmetric information problem can cause bankruptcy. The studies by Nabi (2012), Nabi (2012), and Al-souwailem (2003) have shown that agents are more likely to undertake Musharakah instead of standard debt to ensure financial inclusion and alleviate income inequality. In the same context, the recent paper of Mehri et al. (2017) highlighted the adverse selection problem in PLS arrangements from an interesting angle where they adopted the PSR as a screening device to capture agents’ risk aversion.

Although Al-souwailem (1998) suggested that venture capital can be a potential model of Islamic PLS-based arrangement, this paper has contributed differently to the literature by...
examining the adverse selection analysis to determine the less risky financing contract for
the principal among Musharakah, Mudarabah and venture capital to alleviate the risk of
bankruptcy. In comparison to the existing studies in this context, our paper contributes to
the literature by determining the contract that maximizes the safety value for the principal
subject to the safety constraint. Our paper shows three important results. First, Musharakah
is the safest financing tool for the principal. Second, Mudarabah and venture capital
contracts cannot be preferred to musharakah. Thirdly, it would be effective to undertake
equity-based contracts (Musharakah in particular) when the industrial shock is high.

Our findings shed some light on the necessity of the Islamic finance experts to re-think of
the promotion of Musharakah because it dominates the two other contracts when market
frictions occur. Although the PLS-based agreements can ensure economic growth (Maghrebi
and Mirakhor (2015), Alanzi and Lone (2015), and Lone and Ahmad (2017)), our findings
may motivate economic players and policy makers to consider Musharakah financing with
the aim to ensure financial inclusion and social welfare, which is in line with Shari’ah
requirements and Islamic values.

Our paper has two main limitations. First, our results were not compared to real data
because the latter are not available. Second, our paper is a general framework to determine
the less risky contract for the principal and does not consider the firm and sectoral
characteristics. However, it can be extended in various ways where stress can be put on
conflicts of interest between the principal and the agent with the aim to determine the
contract that aligns their interests. In addition, the examination of firm dynamics in the case
of equity and debt financing can provide further arguments for economic agents regarding
the value of the firm, the growth rate and the lifetime of the project when information is
asymmetrically distributed.

Notes
1. See Chichti and Mansour (2010a, 2010b, 2012) and Mansour (2014) for a theoretical background
   on information asymmetry.
2. See Majdoub and Mansour (2014), Majdoub et al. (2016, 2018), Bedoui and Mansour (2015), and
   Mansour et al. (2015-a) for an examination of the theoretical foundation of equity-based contracts.
3. The Hanafi, Hanbali, Maliki and Shafii are the four Sunni Islamic Schools of jurisprudence. The
   Hanafi school is named after the scholar Abu Hanifa An-Nu’man (d. 767), whereas the Maliki,
   Hanbali and Shafii schools were founded by Malik Ibn Anas in the 8th century, Ahmad Ibn
   Hanbal (d.855), and Muhammad Ibn Idris Al-shafii in the 9th century respectively.
   Basically, all schools of Islamic thoughts derive Sharia (Islamic law) from the Quran,
   the Hadiths (sayings and customs of Muhammad), and the views of Sahabah (Muhammad’s
   companions). However, in cases where there is no clear answer in sacred texts of Islam, Some
   differences have been found between the aforementioned Islamic schools. For instance,
   the Hanbalis do not accept jurist discretion or customs of a community as a sound basis to derive
   Islamic law, a method that Hanafis and Malikis accept. Where passages of Quran and Hadiths are
   ambiguous, Shafis first seek religious law guidance from the consensus of Sahabah
   (Muhammad’s companions). If there was no consensus, Shafii school relies on individual opinion
   (Ijtihad) of the companions of Muhammad, followed by analogy. Hanafi School considers the
   consensus of the companions of Muhammad, then individual’s opinion from the Sahabah (the
   companions of Muhammad), Qiyas (analogy), Istihsan (juristic preference), and finally
   local Urf (local custom of people). One of the many differences between the Shafis and Hanafis
   schools is that the Shafii school does not consider Istihsan (judicial discretion by suitably
   qualified legal scholars) as an acceptable source of religious law because it amounts to “human
   legislation” of Islamic law.
References


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Further reading


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