

# From defining as assertion to defining as explaining meaning: teachers' learning through theory-informed lesson study

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## Abstract

**Purpose** – The aim is the introduction of lesson study (LS) in geometry in Malawi secondary schools supported by a teaching framework that includes a focus on language responsive teaching.

**Design/methodology/approach** – The study reports an LS on geometry for professional development (PD) of secondary teachers. Data analysed includes lesson plans, transcripts of lessons, reflective discussions. The analytical approach is qualitative content analysis.

**Findings** – Teachers' lexicalisation of an exterior angle of a triangle evolved as a function of a teaching framework that guided their participation in planning, teaching and reflecting through LS cycle, and that was derived from networking between theories.

**Research limitations/implications** – This is both a small-scale study, and a limited content focus in the lesson, a function of LS being a new practice, and teachers simultaneously learning ideas about geometry teaching, those embedded in the framework and doing LS.

**Practical implications** – The paper includes a description of how LS might contribute to teachers' learning of language responsive teaching, and so is useful for others working on LS and language practices.

**Originality/value** – This paper fulfils an identified need to learn more about how networking theories to inform and support LS can create learning opportunities for teachers, particularly about language responsive teaching, an interest and concern worldwide.

**Keywords** Malawi, Geometry, Lesson study, Explanatory communication, Language responsive teaching, Networking theories

**Paper type** Research paper



## Introduction

This paper shares some results of the introduction of lesson study (LS) with experienced secondary mathematics teachers in Malawi. We aimed to provide a professional development (PD) experience to enhance the teaching and learning of geometry which has been recognised as a significant problem in the country (MoEST, 2020). Geometry teaching and learning was also a concern for the teachers invited to participate in the LS. They welcomed an opportunity for collaborative PD. The teachers shared records of Grade 8 and 9 students' work that indicated students could identify and use some properties related to lines, angles, and triangles (e.g. "adjacent angles on a straight line add up to 180°") when given numerical measures in simple geometric figures like two intersecting lines, or a single triangle. However, properties were often incorrectly applied when figures were more complex, or when measures were expressed algebraically, suggesting fragility in students' understanding of these geometric objects, their definitions and properties. This "problem" served as the starting point of collaborative LS – a practice new to the teachers.

We are aware of research advocating theory-informed [1] approaches to LS (Huang *et al.*, 2019a). We recall Adler and Alshwaikh's (2019) study supporting explicit theory-informed planning, observation and reflection when LS was a new practice for teachers. Specifically, teachers were introduced to a conceptual framework, derived from networking theories (Prediger *et al.*, 2008), called the Mathematics Teaching Framework (MTF) that was used to guide planning, teaching and reflection. In the LS we report here teachers were introduced to the MTF as a tool for reflecting on teaching. In addition, the teachers engaged in geometry activities (themselves theory-informed) designed to illustrate and mediate aspects of the framework. As LS was a new practice, teachers were introduced to the purpose of LS with its cycle of planning, observing, reflecting and replanning a research lesson, guided by the MTF.

We focus on the participation of teachers from one school over one LS cycle, and a critical point that emerged in the first lesson enactment – defining and explaining the meaning of an exterior angle of a triangle. The research questions pursued are (1) what can we learn about teachers' learning of the practices of language responsive teaching in geometry (a key focus of the MTF framework) through their participation in a LS cycle? And (2) what is the role of a conceptual framework, derived from networking theories, in teachers' learning?

## Locating our LS in the wider literature on mathematics LS

LS, a practice-based longstanding professional learning process in Asia (Huang *et al.*, 2019b), has been implemented in many countries, and inevitably shaped by specific cultural and contextual conditions (e.g. Bussi *et al.*, 2017; Fujii, 2016; Huang *et al.*, 2019). In LS, teachers develop joint goals to improve their teaching and students' learning through collaborative planning, implementing, reflecting and replanning a "research lesson" (e.g. Lewis *et al.*, 2006). Due to the widely reported benefits of collaboration (e.g. Xu and Pedder, 2014; Willems and Van den Bossch, 2019), LS has captured the attention of mathematics teacher PD and research in many countries, including South Africa and Malawi. Focused studies have reported a range of opportunities for teacher learning, insights on the functioning of LS, and the varying role of the knowledgeable other through a LS cycle, including when LS is a new practice (e.g. Fujii, 2016; Adler and Alshwaikh, 2019; Fauskanger *et al.*, 2021). As with any social practice, challenges have been reported across contexts (e.g. Fujii, 2014, 2016; Wood, 2018). Fujii's studies are particularly pertinent as they report how key aspects of LS, like lesson planning in relation to an identified research problem, have not been understood well by novice teachers in some countries outside Japan, including Malawi.

Cautions have also been raised about overclaiming the benefits of LS, particularly as most studies have been small scale (Xu and Pedder, 2014). Willems and Van den Bossch (2019) identified five studies on the effectiveness of LS for teachers' professional learning that used

well-controlled designs, where results show positive changes. [Willems and Van den Bossch \(2019\)](#) showed further that where LS was combined with theoretically derived conceptual frameworks, these were effective in mapping teachers' professional learning. There is increasing evidence of theory-informed LS particularly with an interest in teachers' learning. Theories range from psychological to sociocultural theory, cultural-history activity theory and the theory of didactical situations ([Huang et al., 2019](#)). The studies indicate take-up of LS practice across the USA, UK, Australia, and some European and African countries where LS has been introduced ([Bussi et al., 2017](#); [Adler and Alshwaikh, 2019](#)). Two pertinent issues emerge. Firstly, there are few studies reporting LS research in Africa (also noted by [Fauskanger et al., 2021](#)), hence the importance of the study reported here. Secondly, the wide range of theories and related LS foci point to possible advantages of networking theories ([Prediger et al., 2008](#)) to inform LS.

Recent studies in South Africa and Malawi are informative. [Adler and Alshwaikh \(2019\)](#) reported positive impacts on teacher learning from a case of LS in South Africa. The LS model was adapted to suit conditions of teachers' work, with after school meetings once a week. In line with advice on theory-informed LS, MTF structured planning and reflection, with the researcher-teacher educator (the researcher is also leading the PD), in addition to her participation with the teachers over the LS cycle, playing a key role in mediating the framework with the teachers and so as a particular kind of knowledgeable other. Significantly, as already noted and as we will show, the framework emerged from networking theories. [Adler and Alshwaikh \(2019\)](#) argued that it was both the ideas in the framework and *their mediation* that enabled teachers' learning.

Recently, after successful LS with primary mathematics teacher educators in Malawi, [Fauskanger et al. \(2021\)](#) introduced LS to primary teachers. The goal was to challenge and support teachers to shift from traditional to more learner-centred, participative teaching using pedagogical theories, curriculum materials and alternative activities on counting numbers. Shifts away from traditional views and student activities were observed in the teachers' written views of teaching as well as in the activities on number in lesson plans. Teachers also reported on the benefits of working with their colleagues in the LS. [Fauskanger et al. \(2021\)](#) thus show that LS, notwithstanding substantial adaptation to suit contextual conditions, can enable primary teachers to shift their ideas and practices. Two observations of this LS are pertinent. First, use of resources indicated a theory-informed approach. Theories of learning as participation (specifically [Sfard, 1998](#); [Lave and Wenger, 1991](#)) were supported by theory-informed curriculum materials ([Franke et al., 2018](#)) (all referred in [Fauskanger et al., 2021](#)). Second, teachers' interpretations of these ideas, as expressed in their initial lesson plans and lesson reflections, were explicitly mediated by knowledgeable others. We agree with [Fauskanger et al. \(2021\)](#) that challenging teachers' knowledge and views through collaborative LS are key to influencing teachers' instruction. We add that challenging teachers entails theoretical and practical resources (supporting calls for theory-informed LS) *and* their mediation by the knowledgeable others in the LS enactment.

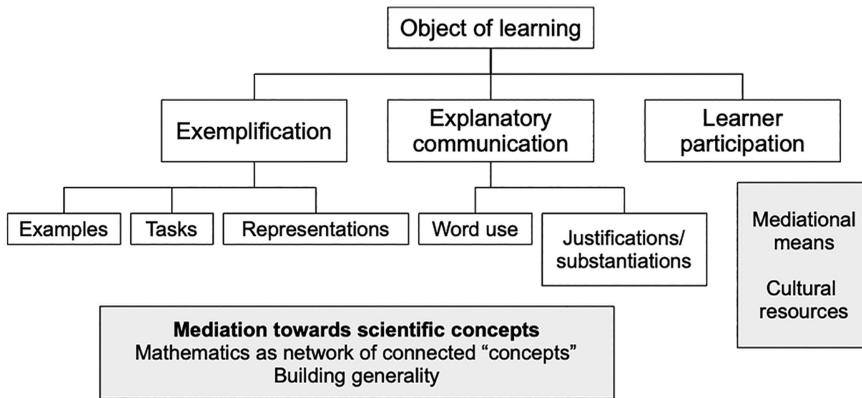
The research on lesson study suggests that explicit theoretical resources and their mediation are enabling of both the design and practice of LS. This is particularly so in relatively under-researched contexts where teachers and/or researcher-teacher educators as knowledgeable others are engaging in LS for the first time, as is the case in this paper. It served as motivation for our geometry LS with secondary mathematics teachers in Malawi. Our design integrates aspects of both the SA and Malawi LS models; and draws on ideas from research on teaching and learning geometry, networking these into aspects of the MTF.

### **Theorising teaching and a conceptual frame for LS in geometry**

Teaching is a complex social and cultural practice ([Stigler and Hiebert, 1999](#)), filled with dilemmas as teachers go about their day-to-day work to support the learning of diverse

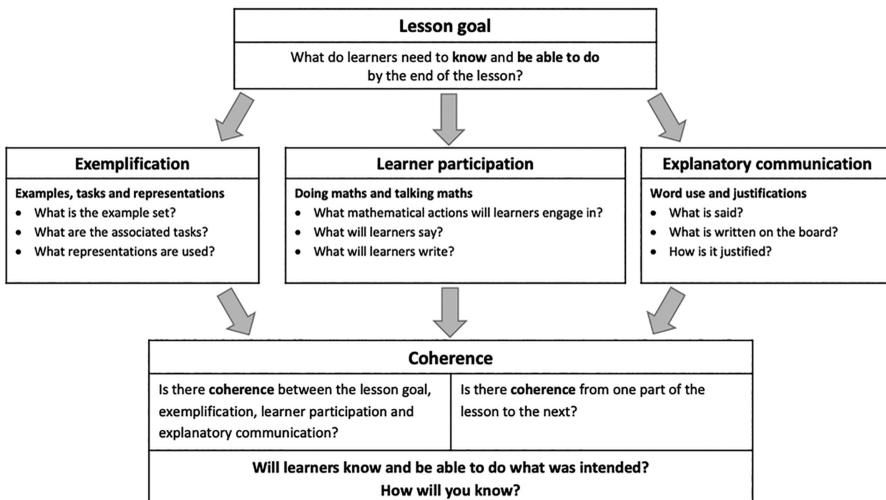
students in their specific educational context (Adler, 2001). Notwithstanding complexity and diversity, there are also regularities, shaped by socio-cultural norms that enable teaching practices to be described. Descriptions offer a shared language, enhancing possibilities for teachers' collaborative inquiry.

The LS discussed in Adler and Alshwaikh (2019) was part of the Wits Maths Connect Secondary (WMCS) project, a South African research-informed mathematics PD project. WMCS developed a conceptual framework called Mathematics Discourse in Instruction (MDI), illustrated in Figure 1a below, to describe and analyse mathematics teaching and opportunities made available to learn (Adler and Ronda, 2015). The MTF, in Figure 1b, is a teaching, planning and reflection version of MDI. The MTF, developed and used to work collaboratively with teachers on their teaching practices, mirrors the structure of MDI and



(a)

Source(s): Adapted from Adler & Ronda, 2015



(b)

Source(s): Adler, 2021a

Figure 1.  
(a) Constitutive elements of MDI and  
(b) MTF

reflects its theoretical underpinnings, as illustrated in [Figure 1](#). The purpose of MTF was its representation of a shared language on teaching practices for use in development work with teachers. For example, the object of learning in MTF is redescribed as the lesson goal pointing to what is to be mediated and thus the goal-directed activity of the teacher.

[Prediger et al. \(2008\)](#) argue for the value of networking of different theoretical resources for studying the complexity of teaching and learning mathematics; and the dynamic view of theory that underpins this argument. This view of theory and engagement with the complexity of teaching and learning are evident in the development of MDI as a living framework, flexing to meet both research and practice goals, and evolving as it is used in different contexts and with different content ([Adler, 2021](#)).

MDI derives from and is structured by a broad philosophical orientation to knowledge, teaching and learning as inherently social, and rooted in key tenets of sociocultural theory. Mathematics is viewed as an interconnected network of scientific concepts ([Vygotsky, 1978](#)), and mathematics teaching therefore as goal directed, with mediation of learning towards sophisticated ways of thinking in a discipline. The orientation to teaching/learning underlying the MDI is that it is always about “something”, an “object” of learning. In Vygotskian terms, and as elaborated by [Wertsch \(2007\)](#), what mediates the object and the student in a mathematics lesson are (1) tools/artefacts that in secondary mathematics are typically symbolic mediational means such as *examples, their representational forms and the tasks they are embedded in*, and (2) human communication and so, in school mathematics, *how words are used, and justifications made*. These work together with student participation – what students do and say – with these means, to exemplify and elaborate/explain the object. As illustrated in [Figure 1a](#), all mediation is towards the building of scientific concepts, and so towards increasing generality and more specialized ways of thinking, doing and speaking mathematically.

It cannot be taken for granted that students will detect symbolic relations, no matter how obvious they might seem to the teacher. This assumption is key in socio-cultural theory:

Symbolic tools (e.g. letters, codes, mathematical signs) have no meaning whatsoever outside the cultural convention that infuses them with meaning and purpose. ([Kozulin, 2003](#), p. 26)

Elaborating further, [Adler \(2021\)](#) argues that symbolic mediational means like examples, representations and the ways in which we use words in school mathematics are culturally shaped by educational practices. If they are to be productively used as resources for mathematical learning, they need to be “transparent” to the student – visible so that they can be used to extend ideas, and simultaneously invisible so that the mathematics they represent can be “seen”. This requires explicit mediation. So too cultural resources in PD.

To give further and more practice-oriented meaning these mediational means, and in ways that drew on relevant research in mathematics education, additional theoretical resources were drawn from (1) Variation Theory (VT) (e.g. [Marton and Tsui, 2004](#)) and its wide application in research on examples in the field (e.g. [Watson and Mason, 2006](#)); and from (2) socio-linguistic theory ([Planas, 2018](#)) and selected tools to investigate and work with explanatory communication. In [Prediger et al. \(2008\)](#) terms, the networking strategy connecting these theories was (local) co-ordination into an overarching Vygotskian framing. They use the word “*co-ordinating* when a conceptual framework is built by well fitting elements from different theories” (p. 172), as discussed below.

Previous work, including aspects of our recent LS work in South Africa and Malawi, has elaborated extensively on the mediational means of exemplification and how LS can promote teachers’ learning of this practice ([Adler and Alshwaikh, 2019](#); [Mwadzaangati et al., 2022](#)). We have explained how constructs from VT (e.g. variance amidst invariance within and across examples sets in building generality) fitted well with our overarching framing, notwithstanding its phenomenographic underpinnings.

In this paper we zoom in on the mathematics teaching practice of explanatory communication. Elsewhere it is referred to as language responsive mathematics teaching (e.g. Prediger, 2019), where language includes word use, ways of reasoning, and producing justifications, all cultural tools or resources in a practice. This part of the MDI/MTF conceptual framework is informed by research on language as resource (Adler, 2001), specifically the work of Planas (2018, 2021). Planas builds theoretically from Halliday (1985) and through empirical study to argue how and why lexicalisation – naming and sentence formation – is critical in language responsive teaching. Words and phrases are always polysemic and not simply vocabulary. Their mathematical meaning needs to be elaborated so that specialised ways of talking, necessary for progress in mathematics are made available to learn. Simply, the words and encompassing sentences teachers use to talk about mathematical ideas, and how they support students' talk matters. As a relevant example here, while there is a formal lexicon for communicating the meaning of the exterior angle of a triangle, students' colloquial meanings would need to be engaged. Moreover, as a geometric concept, this discursive elaboration would need to be accompanied by a diagrammatic representation. There is substantial literature in geometry education (e.g. Fischbein, 1993) that theorises the complex relationship between the visual or diagrammatic and the discursive, and points to the difficulties students face when moving between these. Research in South Africa has evidenced the salience of linking careful selection of words with accompanying diagrams (Chiphambo and Feza, 2020). MTF draws attention to the different functions of colloquial/informal and formal word use as we name and elaborate mathematical objects, processes and procedures in teaching and learning. Given broad philosophical coherence, lexicalisation in sociolinguistics, in Prediger *et al.* (2008)'s terms, can be viewed as (local) co-ordination into an overarching Vygotskian frame.

How we use words in the teaching and learning of mathematical ideas is only part of what is entailed in explanatory communication. Language responsive teaching needs to attend critically to how mathematical arguments are made, and the rich discursive tasks that can provoke what it means to reason mathematically and build robust mathematical justifications for ideas, procedures and practices (e.g. Prediger, 2019). For the concept of an exterior angle of a triangle, meaningful explanatory communication in a lesson would include why only some of the angles formed at a vertex of a triangle when sides are extended are considered "exterior angles". In mathematics, an "exterior" angle of a triangle means something quite specific, and different from the everyday use of related words (outside, external). This requires clarification both visually and discursively, with justification coming through the way we use words to name and produce mathematical objects. Introducing the exterior angle of a triangle itself to students is thus not a trivial task.

What is and is not an "exterior angle" could simply be asserted by the teacher, or explained at an appropriate level, in relation to the importance of definitions in mathematics. Of course, more is needed for reasoning and justifying the relationship between an exterior angle of a triangle and the interior opposite angles, again discursively and visually. Prediger (2019) shows that for teachers participating in PD and their students, developing lexical elaborations, particularly of procedures – of what to do – is easier than the more demanding discourse of justifying conceptual relations, pointing to our research question on what teachers learn about language responsive teaching – explanatory communication – through participation in LS.

## Methodology

For our design of the geometry focused LS, we built on the theory-informed LS in South Africa discussed above, and its case study research methods. We incorporated aspects of the Malawi LS design also discussed above, and thus the LS cycle included PD sessions set up to

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introduce LS to the teachers, engage in geometry teaching/learning activities, and to introduce the MTF framework. The process included teachers working together in their schools for initial planning and reflecting on their teaching of the lessons, and other, substantial time in PD workshops on the University campus. This model differs substantively, for contextual reasons, from the typical LS cycle described elsewhere, both in terms of a networked framework guiding LS, and the multiple roles of the knowledgeable other.

The participants in our wider study were experienced secondary mathematics teachers from two schools (five teachers from each school), supported by researcher-teacher educators at the University of Malawi, from here-on referred to as the KOs. In this paper, we focus what we can learn about teachers' learning the practice of language responsive teaching through LS and the role of the theoretical resource (MTF) in this learning. In MTF, explanatory communication is defined using language responsive teaching practices related to word use and justifying.

The LS PD began with a two-day workshop (PDW1) at the University, where all activities were developed and run by the KOs. Sessions in the workshop focused on all the mediational means within the MTF (setting up a lesson goal; tasks, examples and representations for exemplifying geometric ideas; word use and justification for explanatory communication; and student participation) and illustrated with geometry activities. For example, activities related to exemplification drew attention to orientation and complexity of diagrams and to the visual and discursive demands in student tasks. Discussion on explanatory communication included the role of board work in teaching, distinguishing between colloquial and mathematical language, communicating the meaning of definitions and properties, and promoting reasoning in mathematics. In the final PDW1 session, teachers began discussing a research problem and the related lesson. The problem identified was current – students were having difficulties applying the property of the exterior angle of a triangle being equal to the sum of its interior opposite angles. The teachers began to collaboratively discuss and write up the first lesson plan with its stated goal, teacher and student activities, completing this after the workshop, in their schools.

One KO (KO1) was present to video-record each stage of LS cycle and to provide support. We deliberately planned that KO1 would not participate in the initial lesson planning (LP1A). Our aim was to identify how aspects of the MTF were visible in the independently planned lesson, anticipating that further mediation of new ideas might be needed. KOs provided feedback on LP1A to the teachers through KO1 in the form of suggestions. She suggested, in line with the MTF, that the teachers consider defining the exterior angle more clearly, as well as tasks and examples for illustrating an exterior angle to guide the teaching. The teachers revised their lesson plan (to LP1B) before it was taught (T1) by one of the teachers and the other four teachers observed. Teachers met after the lesson to reflect on the lesson plan and the first enactment (R1). This was followed by PDW2 at the University where teachers from the two schools shared their reflections from teaching and then revised their lesson (LP2) in light of the MTF. They continued revising LP2 back at their schools and then taught (T2) the lesson in a different class. In PDW3, the teachers met to share their reflections on T2 and complete the LS cycle. At the end of PDW3, focused group interviews were conducted to understand what teachers learnt about the use of the theoretical resource in LS.

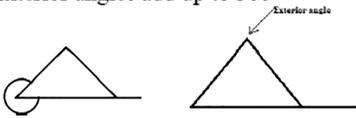
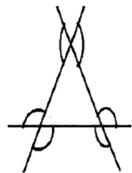
The transcripts of video data were read several times for familiarisation, and then content analysis was done. We drew on data from: (1) PDW1 sessions, (2) lesson plans LP1A, LP1B, LP2 together with their respective planning sessions, (3) T1 and T2 (one episode from each on defining exterior angle), (4) lesson reflections (R1 and R2), and (5) focus group discussions conducted at the end of the LS cycle. For the sake of anonymity, in our reporting we refer to the teachers in the school as T1, 2 . . . 5.

## Findings

Given our understanding of embarking on LS in South Africa, we appreciated that learning about the three sets of ideas – on geometry, those encoded into the MTF and how to do LS – would evolve, in interacting ways, over the course of LS cycles. This is indeed what transpired, and there are many aspects of this learning process, beyond the scope of the research questions informing this paper. We zoom in on the planning, teaching and reflection on defining the exterior angle of a triangle, the first, but not the only critical issue to emerge.

### *Lesson planning, teaching and reflection on the definition of exterior angle*

As the teachers were concerned with a property of the exterior angle of a triangle, the planned lesson steered towards this. The plan included asking students to discuss and then share what they thought was an exterior angle of a triangle, and that they expected students to use word like “outside” or “additional”, and that the “learning point” of this was “Meaning of external angle in a triangle” [2]. LP1A was shared with the KOs. KO1 met with the teachers to provide feedback and then to participate with them in the revised plan. LP1B in [Figure 2](#)

Plan	Teacher’s activities	Student’s Activities	Learning points
1b	<p><b>Introduction</b></p> <p>Ask learners to be in pairs and brainstorm the meaning of exterior angles of a triangle</p> <p>Ask learners to demonstrate/identify exterior angles and come up with the number of exterior angle</p>	<p><b>Expected answers</b></p> <p>External angle is an outside angle in a triangle; External angle is an additional angle; External angle is an angle adjacent to one of the interior angle</p>	<p>Meaning of external angle in a triangle: an angle formed after extending any of the interior angles</p>
2	<p>Ask learners to be in pairs and brainstorm the meaning of exterior angles of a triangle</p> <p>Ask learners to demonstrate/identify exterior angles of a triangle</p> <p>Consolidating the meaning of exterior angle and interior angle of a triangle</p>	<p><b>Predictions</b></p> <p>Outside angle in a triangle An addition angle outside a triangle Exterior angles add up to <math>360^0</math></p>  <p><b>Expected answer</b></p> <p>An adjacent angle formed outside a triangle after extending any of the side of a triangle</p> 	<p>Meaning of exterior angle in a triangle: an adjacent angle formed outside a triangle after extending any of the side of a triangle</p>

**Figure 2.**  
Extracts from teachers’  
LP1B and LP2

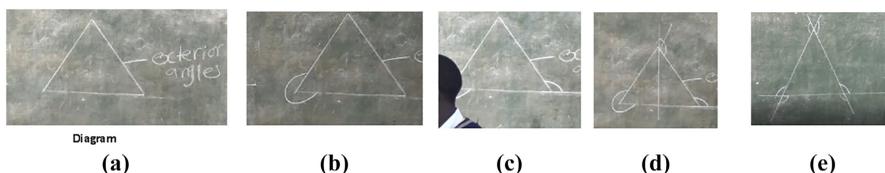
below is the result of this collaborative planning and included the definition of an exterior angle as “an angle formed after extending any of the interior angles of a triangle”, and a task where teachers would ask students to indicate on the chalkboard, examples of exterior angles of a triangle and the number of exterior angles. It was only after the lesson was taught, and through their reflection on this and replanning, that a fuller, and more meaningful definition was articulated by the teachers in LP2, including a change to what is extended, i.e. the side of the triangle.

The evolution of the definition and related teacher learning was thus a function of mediation by the KO and the teachers’ collaborative discussion. While discussing and analysing examples that they could use to define and show examples of exterior angles, the teachers gained some mathematical insight – that there were six exterior angles of a triangle, two formed at each vertex and that these pairs were vertically opposite angles.

During teaching 1 (Teacher 3), after brief recall of the properties of interior angles of a triangle, he asked the students to state what was an exterior angle of a triangle. As anticipated, the responses included words such as “outside a triangle”. Accepting this description, and revoicing that interior angles were “inside”, and exterior angles “outside” the teacher asked a student (S1) to “come to the board to draw a triangle and indicate exterior angles”. S1 drew diagram (see Figure 3a), and the teacher asked if other students could “add to this”. S2 and S3 drew triangles (b) and (c) respectively. The teacher continued as in the transcript below.

What was discursively defined as an exterior angle in the lesson remained as in the bold statements, with gestures pointing to the angle formed by an extended side, and an assertion by the teacher of the one he was “interested in” and supported visually.

In the discussion of this episode during lesson 1 reflection, the teachers realised that their definition of an exterior angle in LP1B as formed by extending a side of a triangle was not sufficiently clear as there was also a straight angle formed. We noticed them browsing the web on their phones, and after further discussion they agreed to extend the definition to an adjacent angle formed outside a triangle after extending one of the sides. They reiterated that the missing words were “adjacent angle”.



T: ... So, ... Are we noticing the difference, the one we’ve been given on this vertex (one drawn by 2<sup>nd</sup> student), and the one that we’ve been given on this vertex (one drawn by the 3<sup>rd</sup> student)? **We can observe the difference, eh? ... We have got an extended line here, right? (line drawn by the 3<sup>rd</sup> student), and here (angle drawn by the 2<sup>nd</sup> student), we don’t have an extended line.** Are you able to notice that?... But both of them we are saying they are exterior angles. ... Why? **Because they are outside what, the triangle?**

T: Now, uhm, **we have this extended line.** Can someone come and add more lines on the vertices so that we have more exterior angles. **I am interested on this one (extended line drawn by the 3<sup>rd</sup> student).**

S4: (Draws lines and angle as shown in diagram d)

T: Okay, we’ve been given different representations of exterior angles, right?... But now when it comes to this part the definition that we have, we are saying it’s an angle that is found outside a triangle. That was an explanation that came from one of us?... but after extending one of the lines of a triangle. (Draws another exterior angle). ... We are creating an angle here. **That’s the angle which we refer to as an exterior angle...**

**Figure 3.**  
From introduction in  
Lesson 1

This extended definition informed their revised lesson plan. LP2 was developed during PDW2 reflecting on the interaction of simultaneously learning to do LS, learning about their students' thinking and learning about defining in geometry. One of the foci related to doing LS in PDW1 was the importance of including predictions of students' responses, including possible errors. Extract 2 below reflects some of this discussion, including the input from a KO. Predictions appear in LP2 suggesting they were informed by what they had seen in Teaching 1. The extract below reveals that hearing and seeing students' mathematical thinking was surprising for the teacher, which suggests that inviting students to share meanings of constructs (as opposed to the procedures done) was a new practice.

T5: (Laughs) I saw him shocked. (Referring to the teacher who taught the lesson)

T3: Yeah, I was shocked, because now, . . . it means they were hinting, somehow, . . . we couldn't tell the student . . . you're wrong, no; that one was an outside angle of a triangle (they all laugh). Yeah, so, on the learning point, of course we want to talk za [about] modification. We need to modify the learning, eh?

And after further discussion . . .

T3: Okay, aaah yes! Okay, why can't we say "adjacent angle" formed outside a triangle, after extending any of the sides of the triangle? Adjacent, okay, . . . an adjacent angle outside a triangle after extending any of the sides of the triangle.

The comments by Teacher 3 are revealing, first of his surprise at what students offered, and then also his concern to not say "you're wrong", but that there needed to be "modification". The modification was built into LP2, as we see in [Figure 2](#) above. The teachers' learning about students' spontaneous ideas of an exterior angle appear in the predictions column, together with the extended definition for the exterior angle. This now included not only the extending of a side, but the critically important words, "adjacent angle".

This collaborative interwoven learning about doing LS, student thinking and defining was taken up by the teacher enacting LP2. In the introduction in Lesson 2, Teacher 3 was summarising what the students offered about interior angles of a triangle, and said:

T: . . . So, at first, we were saying that we're looking at the interior angles, right? That means we have angles inside the figure. . . . As someone just says, there's also an exterior angle outside the figure. Who can . . . show us where the exterior angle is?

S1: (marks reflex angles as exterior angles)

T: Do we have anyone with a different idea? Yes!

S2: (marks three exterior angles, one on each vertex)

T: Do you agree to this one? (many students respond "yes")

T: So, which one is the exterior angle?

S3: Both

S4: The second one.

T: Aren't they all exterior angles? The definition of exterior angle is not just saying that any outside angle, no, right? . . . when you want to define, use this part (points at an angle drawn on one of the extended lines). That means we may say that an adjacent angle formed after extending one of these sides, right?

Teacher 3 emphasised that the definition of exterior angle includes adjacent angle, and not just any angle outside the triangle. This clarifies, to some extent, what is or is not included in the definition. Further elaboration on the precision of definitions might have helped students

in understanding the importance of justification. That said, in the lesson 2 reflection, all the teachers agreed that defining and clarifying the meaning of an exterior angle as the adjacent angle formed, “helped learners to understand what really the exterior angle is”.

This learning by the teachers, indicating shifts in how they used words and an accompanying diagram, was followed by focus group discussions. The teachers were asked what they had learned over the lesson study cycle. Referring to this episode Teacher 2 said:

The mathematical language should be clear, explanatory communication, like the definitions. Remember what we did in explaining the exterior angle? Now, we should explain to them what really exterior angle is. Is it the exterior angle in English or in mathematics? (Teacher 2)

Other teachers concurred. They thus explicitly articulated appreciation of aspects of explanatory communication, particularly how important it was to work with both colloquial and mathematical language.

### Discussion

We have traced the teachers’ planning, teaching, reflection, revising, reteaching and further reflection on the definition and meaning of the exterior angle of a triangle. We have evidenced substantial change in word use from an initial description in LP1B “an angle formed after extending any of the interior angles” and asserted by the teacher as the angle he was “interested in”, to the communication of more elaborate meaning both in LP2 and the teaching of lesson 2, that is, an exterior angle is the *adjacent angle* formed by *extending a side* of a triangle. This lexical elaboration and full descriptive sentence is critical to communicating meaning. We note too, that it was not only the words, but also their association to an accompanying diagram.

The shift from asserting to communicating meaning is an important step towards language responsive teaching. As discussed earlier, words are inevitably polysemic, with meanings encoded in their use (Planas, 2018) – here the meaning of “external” in an “external angle of a triangle” in mathematics. We say “a step towards” as richer word use needs to expand to include reasoning and justifying of mathematical ideas. For this to be visible in teachers’ collaborative reflection, different kinds of tasks that elicit such reasoning would need to be included in a lesson, hence the importance of both exemplifying and explanatory communication in the MTF. Other studies (e.g. Prediger, 2019) have shown how in the context of PD and their own teaching, teachers’ take-up at the level of word use comes first, before they are able to extend their reflection and action to more discursively rich reasoning. This work lies ahead in future LS cycles.

The teachers’ learning about explanatory communication and specifically word use, and their appreciation of this in the focus group discussion needs to be understood in relation to the functioning of a LS (our answer to research question 1). First, the role of the KO – the researcher-teacher educator – was evident in explicitly suggesting ideas related to the definition in particular and explanatory communication in the MTF more generally. KO further suggested ideas for learner tasks in attending to the diagrammatic and discursive aspects in geometry. Second was drawing teachers’ attention to the importance of predicting student thinking in their planning (Fujii, 2014) – a critical idea in doing LS. We thus see that teachers’ learning is a function of their collaboration, being challenged by KO’s ideas (Fauskanger *et al.*, 2021) and *their mediation*.

What then of the role of a framework (derived from networking theories), in general, and specifically the MTF (our research question 2)? We have evidence that teachers learned about word use, a mediational means in the MTF, a function of the LS being theory-informed, adding support to similar claims elsewhere (e.g. Huang *et al.*, 2019). Clearly there was explicit communication of the importance of language in their teaching, and MTF draws teachers’ attention to both word use and justification as important for language responsive teaching.

There are limitations to what we have evidenced and argued. We have focused on what some might consider a relatively “minor” mathematical object. However, it was this “minor” object and its definition that created opportunities for learning about defining, using words clearly, particularly when these were also words used in everyday life; as well as learning how defining in geometry is a function of both words and diagrams. That, in our view, is the essence of LS – the creation of opportunities to learn key teaching practices that in turn enhance opportunities for learning beyond the specifics of the LS. With respect to the MTF, as a framework for theory-informed LS, the focus on word use without extension to justification and criteria for what counts as mathematics is a limitation of the data. Previous work (Adler, 2021), however, has shown the significance of the MTF as a transparent resource, enabling the learning of exemplification (and the theoretical resources embedded in this) through LS thus extending our argument for the role of such a framework in teachers’ learning in and through LS.

## Conclusion

Through our study of participating teachers’ first enactment of a LS cycle we have built two inter-connected arguments. Firstly, teachers learned about their own and students’ word use with a diagram as connected resources for describing, defining and ultimately being able to recognise and use the exterior angle of a triangle. Secondly, the LS was theory-informed. Teachers’ planning, with a conceptual framework derived from networking theories, met student productions that provoked reflection and replanning, creating conditions and possibilities for learning language responsive teaching practices.

By focusing on the defining of the exterior angle of a triangle, we have given only a glimpse into the complex unfolding of the LS. Many additional aspects are out of view – the collaboration amongst teachers; the functioning of a model that includes PD workshops and more limited collaborative planning and reflection with a KO playing the role(s) described here. Learning to do LS, maximising its potential benefits for teacher learning, and advancing understanding of mathematics teaching and learning is a process. We hope that by zooming in on initial unfamiliar practices with teachers in a LS, and in a new context, we have brought new insights into the potentialities of LS across contexts, and the critical role of networked theoretical resources and their mediation by KOs in this process. While the content in focus could be interpreted as limited, it was precisely this focus on how to name and lexically elaborate what counts as an exterior angle of a triangle in mathematics that brought to light the importance of language responsive teaching – in any context – and its entailments, and how the LS process enabled learning towards this critical teaching practice.

## Notes

1. Huang *et al.* refer to “theory-driven” LS. We prefer “theory-informed”, as the goal of collaborative professional learning drives LS.
2. LP1A had the word “external”. Interestingly, the KOs did not notice this initially, but the teachers did, and changed this to exterior (following brief discussion) as they replanned the lesson.

## References

- Adler, J. (2001), *Teaching Mathematics in Multilingual Classrooms*, Kluwer Academic Publishers, Dordrecht.
- Adler, J. (2021), “Levering change: the contributory role of a mathematics teaching framework”, *ZDM–Mathematics Education*, Vol. 53 No. 6, pp. 1207-1220.

- Adler, J. and Alshwaikh, J. (2019), "A case of lesson study in South Africa", in Huang, R., Takahashi, A. and da Ponte, J. (Eds), *Theory and Practice of Lesson Study in Mathematics Advances in Mathematics Education*, Springer, pp. 317-342.
- Adler, J. and Ronda, E. (2015), "A framework for describing mathematics discourse in instruction and interpreting differences in teaching", *African Journal of Research in Mathematics, Science and Technology Education*, Vol. 19 No. 3, pp. 237-254.
- Bussi, M.G., Bertolini, C., Ramploud, A. and Sun, X. (2017), "Cultural transposition of Chinese lesson study to Italy", *International Journal for Lesson and Learning Studies*, Vol. 6 No. 4, pp. 380-395.
- Chiphambo, S.M. and Feza, N.N. (2020), "Exploring geometry teaching model: polygon pieces and dictionary tools for the model", *EURASIA Journal of Mathematics, Science and Technology Education*, Vol. 16 No. 9, pp. 1-11.
- Fauskanger, J., Helgevold, N., Kazima, M. and Jakobsen, A. (2021), "Challenging Malawian primary teachers' views on mathematics teaching and learning through lesson study", *International Journal for Lesson and Learning Studies*, pp. 2046-8253.
- Fischbein, E. (1993), "The theory of figural concepts", *Educational Studies in Mathematics*, Vol. 24, pp. 139-162.
- Franke, M.L., Kazemi, E. and Turrou, A.C. (Eds) (2018), *Choral Counting and Counting Collections: Transforming the PreK-5 Math Classroom*, Stenhouse Publishers, Portsmouth.
- Fujii, T. (2014), "Implementing Japanese lesson study in foreign countries: misconceptions revealed", *Mathematics Teacher Education and Development*, Vol. 16 No. 1, pp. 65-83.
- Fujii, T. (2016), "Designing and adapting tasks in lesson planning: a critical process of lesson study", *ZDM Mathematics Education*, Vol. 48 No. 4, pp. 411-423.
- Halliday, M.A. (1985), *Spoken and Written Language*, Deakin University, Waurn Ponds, Victoria.
- Huang, R., Gong, Z. and Han, X. (2019a), "Implementing mathematics teaching that promotes students' understanding through theory-driven lesson study", in Huang, R., Takahashi, A. and da Ponte, J. (Eds), *Theory and Practice of Lesson Study in Mathematics Advances in Mathematics Education*, Springer, pp. 605-631.
- Huang, R., Takahashi, A. and da Ponte, J.P. (2019b), "Theory and practice of lesson study in mathematics around the world", in Huang, R., Takahashi, A. and da Ponte, J. (Eds), *Theory and Practice of Lesson Study in Mathematics Advances in Mathematics Education*, Springer, pp. 3-12.
- Kozulin, A. (2003), "Psychological tools and mediated learning", *Vygotsky's Educational Theory in Cultural Context*, Vol. 4 No. 6, pp. 15-38.
- Lave, J. and Wenger, E. (1991), *Situated Learning: Legitimate Peripheral Participation*, Cambridge University Press, Cambridge.
- Lewis, C., Perry, R. and Murata, A. (2006), "How should research contribute to instructional improvement? The case of lesson study", *Educational Researcher*, Vol. 35 No. 3, pp. 3-14.
- Marton, F. and Tsui, A. (2004), *Classroom Discourse and the Space for Learning*, Lawrence Erlbaum Associates, New Jersey.
- MoEST (2020), *National Education Sector Investment Plan: Education Sector Analysis*, Ministry of Education, Lilongwe, Malawi.
- Mwadzaangati, L., Takker, S. and Adler, J. (2022), "Teacher learning about exemplification in geometry through lesson study", *Proceedings of CERME12*.
- Planas, N. (2018), "Language as resource: a key notion for understanding the complexity of mathematics learning", *Educational Studies in Mathematics*, Vol. 98, pp. 215-229.
- Planas, N. (2021), "How specific can language as resource become for the teaching of algebraic concepts?", *ZDM-Mathematics Education*, Vol. 53 No. 2, pp. 277-288.

- 
- Prediger, S. (2019), "Investigating and promoting teachers' expertise for language-responsive mathematics teaching", *Mathematics Education Research Journal*, Vol. 31, pp. 367-392.
- Prediger, S., Bikner-Ahsbals, A. and Arzarello, F. (2008), "Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework", *ZDM Mathematics Education*, Vol. 40 No. 2, pp. 165-178.
- Sfard, A. (1998), "On two metaphors for learning and the dangers of choosing just one", *Educational Researcher*, Vol. 27 No. 2, pp. 4-13.
- Stigler, J.W. and Hiebert, J. (1999), *The Teaching Gap*, Free Press, New York, NY.
- Vygotsky, L.S. (1978), *Mind in Society: The development of Higher Psychological Processes*, Harvard University Press, Cambridge.
- Watson, A. and Mason, J. (2006), "Seeing an exercise as a single mathematical object: using variation to structure sense-making", *Mathematical Thinking and Learning*, Vol. 8 No. 2, pp. 91-111, doi: [10.1207/s15327833mtl0802\\_1](https://doi.org/10.1207/s15327833mtl0802_1).
- Wertsch, J. (2007), "Mediation", in Daniels, H., Cole, M. and Wertsch, J. (Eds), *The Cambridge Companion to Vygotsky*, Cambridge University Press, Cambridge, pp. 178-192.
- Willems, I. and Van den Bossche, P. (2019), "Lesson study effectiveness for teachers' professional learning: a best evidence synthesis", *International Journal for Lesson and Learning Studies*, Vol. 8 No. 4, pp. 257-271.
- Wood, K. (2018), "The many faces of lesson study and learning study", *International Journal for Lesson and Learning Studies*, Vol. 7 No. 1, pp. 2-7.
- Xu, H. and Pedder, D. (2014), "Lesson study: an international review of the research", in Dudley, P. (Ed.), *Lesson Study: Professional Learning for Our Time*, Routledge.

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