Unsteady natural convection in a partially porous cavity having a heat-generating source using local thermal non-equilibrium model

Marina S. Astanina
Laboratory on Convective Heat and Mass Transfer, Tomsk State University, Tomsk, Russian Federation

Mikhail Sheremet
Department of Theoretical Mechanics, Tomsk State University, Tomsk, Russian Federation and Institute of Power Engineering, Tomsk Polytechnic University, Tomsk, Russian Federation, and

C. Jawali Umavathi
Department of Mathematics, Gulbarga University, Gulbarga, India

Abstract
Purpose – The purpose of this study is a numerical analysis of transient natural convection in a square partially porous cavity with a heat-generating and heat-conducting element using the local thermal non-equilibrium model under the effect of cooling from the vertical walls. It should be noted that this research deals with a development of passive cooling system for the electronic devices.

Design/methodology/approach – The domain of interest is a square cavity with a porous layer and a heat-generating element. The vertical walls of the cavity are kept at constant cooling temperature, while the horizontal walls are adiabatic. The heat-generating solid element is located on the bottom wall. A porous layer is placed under the clear fluid layer. The governing equations, formulated in dimensionless stream function, vorticity and temperature variables with corresponding initial and boundary conditions, are solved using implicit finite difference schemes of the second order accuracy. The governing parameters are the Darcy number, viscosity variation parameter, porous layer height and dimensionless time. The effects of varying these parameters on the average total Nusselt number along the heat source surface, the average temperature of the heater, the fluid flow rate inside the cavity and on the streamlines and isotherms are analyzed.

Findings – The results show that in the case of local thermal non-equilibrium the total average Nusselt number is an increasing function of the interphase heat transfer coefficient and the porous layer thickness, while the average heat source temperature decreases with the Darcy number and viscosity variation parameter.

Originality/value – An efficient numerical technique has been developed to solve this problem. The originality of this work is to analyze unsteady natural convection within a partially porous cavity using the local thermal non-equilibrium model in the presence of a local heat-generating solid element. The results would benefit scientists and engineers to become familiar with the analysis of convective heat transfer in...
enclosures with local heat-generating heaters and porous layers, and the way to predict the heat transfer rate in advanced technical systems, in industrial sectors including transportation, power generation, chemical sectors and electronics.

**Keywords** – Natural convection, Finite difference method, Unsteady regimes, Porous layer, Heat-generating solid element, Local thermal non-equilibrium model

**Paper type** Research paper

1. **Introduction**

Natural convection inside porous media has widespread applications in different engineering fields such as crude oil production, cooling of electronic devices and ground water pollution. Studies in this field can be found in books (Nield and Bejan, 2013; Ingham and Pop, 1998; Ingham and Pop, 2002; Ingham and Pop, 2005; Pop and Ingham, 2001; Vafai, 2005; Vafai, 2010; Vadasz, 2008) and in papers (Xu et al., 2017; Pop et al., 2016; Wang et al., 2016; Baytas et al., 2009; Chen et al., 2016; Montienthong et al., 2017; Sheremet et al., 2015; Sabour et al., 2017). It is known that different physical properties of working fluid and solid matrix of the porous medium have a great influence on the heat transfer process. A description of the flow and heat transfer inside a porous medium can be modeled using local thermal equilibrium (LTE) (Beckermann et al., 1987; Ghalambaz et al., 2015) or local thermal non-equilibrium (LTNE) (Buonomo et al., 2014; Ouarzazi et al., 2017; Zargartalebi et al., 2017; Tahmasebi et al., 2018; Zargartalebi et al., 2016; Zargartalebi et al., 2017; Mehrryan et al., 2018). Thus, the LTNE model is a physically approved approach for problems with an essential difference in the thermophysical properties of working liquid and solid porous matrix, while LTE approach has many limitations (Harzallah et al., 2014). As a result, using LTE approach can lead to some non-physical results.

Recently, many studies have focused on the analysis of the convective flow and heat transfer inside porous or partially porous cavities using the LTNE model. For example, Zargartalebi et al. (2017) have analyzed numerically time-dependent natural convection in a porous cavity with heat-conducting solid walls using LTNE approach. They have stated that the absolute Nusselt numbers magnitude is in reverse proportion with the thickness of the vertical walls. (Tahmasebi et al., 2018) have examined computationally natural convection within a differentially heated square cavity with a vertical heat-conducting solid wall, vertical porous layer and vertical nanofluid layer. Analysis has been performed using LTNE approach and Buongiorno’s nanofluid model (Pop et al., 2016; Zargartalebi et al., 2016; Mehrryan et al., 2018). It has been ascertained that a growth of the porous layer thickness reduces the average Nusselt number for both phases of the nanofluid and porous matrix within the porous layer. Baytas and Baytas (2017) have studied thermal non-equilibrium natural convection in a square enclosure with a heat-generating porous layer. The results have shown that the thermal non-equilibrium model is needed for small values of the porosity-scaled thermal conductivity ratio and the solid/liquid-scaled heat transfer coefficient. Gangapatnam et al. (2018) have presented numerical simulations of heat transfer in metal foams. They have concluded that the LTE model highly underpredicts the heat transfer in these foams, while the LTNE model predicts the Nusselt number accurately. Torabi et al. (2015) have carried out numerical simulations of heat transfer and entropy generation in a channel partially filled with porous media using the LTNE model. Strong analytical solutions have been obtained for the velocity and temperature fields. Dickson et al. (2016) have investigated the entropy generation and heat transfer in nanofluid forced convection through a partially filled porous channel. The problem includes a fully developed flow in a channel with a central porous insert and under constant heat flux boundary...
The results have shown that the analysis presented in this paper highlighted the importance of interface models as the key element reflecting the thermal and entropic behavior of the system. The effect of LTNE approach on the onset of convection in a porous medium consisting of two horizontal layers has been studied analytically by Nield et al. (2015). It has been shown that diversity of interphase heat-transfer coefficient and porosity has the lesser effect, while diversity of solid thermal conductivity and source strength in the solid phase is relatively inconsiderable. Torabi et al. (2016) have considered forced convection of copper–water nanofluid through a channel partially filled by a centrally located porous insert. Two interface models have been used to specify the thermal boundary conditions at the interface of the porous insert and the nanofluid flow. Mahmoudi and Karimi (2014) have examined numerically the heat transfer enhancement in a pipe partially filled with a porous medium. Different values of $Nu$ number have been received from the two interface models. Forooghi et al. (2011) have studied the flow and heat transfer in a channel with two porous layers. They have appreciated the effects of local solid-to-fluid heat transfer (a criterion indicating the departure from the LTE condition) and the effects of LTNE condition and the thermal conductivity ratio in pulsatile flow.

This brief review of the papers considered LTE and LTNE conditions showed that now there are no papers devoted to analysis of natural convection heat transfer in a partially porous cavity under the effect of local heat-generating solid element using the differences between the temperatures inside porous solid matrix and fluid phase. Therefore, the objective of the present study is a numerical analysis of transient natural convection in a square partially porous cavity with a heat-generating and heat-conducting element using the LTNE model under the effect of cooling from the vertical walls. It should be noted that this research deals with a development of passive cooling system for the electronic devices.

2. Basic equations

The physics of the considered free convection in a cavity with a porous layer and heat-generating element is schematically demonstrated in Figure 1. The vertical walls are isothermal with a constant temperature $T_0$, while the horizontal boundaries are thermally insulated. A heat-conducting source of constant volumetric heat flux $Q$ is located on the bottom wall. The internal part of the cavity includes a horizontal porous layer of height $h$ and a horizontal clear fluid layer of height $L-h$.

![Figure 1. Physical model and coordinate system (a) and considered grid within the numerical region (b)](image)
It is supposed that the viscosity of the fluid varies with temperature (Astanina et al., 2015; Astanina et al., 2018), and the flow is laminar. The fluid is heat-conducting and Newtonian and the Boussinesq approximation is valid. Further, it is assumed that the temperature of the fluid phase is not necessarily equal to the temperature of the solid structure in the porous region, and the LTNE model is applicable in the present investigation. The porous matrix is considered to be isotropic and homogeneous. The fluid–porous interface is assumed to be permeable so that the fluid can penetrate into the porous layer. All the sidewalls are assumed to be impermeable. In the present investigation, the transient Brinkman-extended Darcy model has been adopted in the governing equations of the problem.

Under these assumptions the governing equations can be written as follows:

- for the clear fluid layer (Astanina et al., 2015; Astanina et al., 2018):

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0,
\]

\[
\rho \left( \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \left( \mu(T) \frac{\partial \bar{u}}{\partial x} \right) + \frac{\partial}{\partial y} \left[ \mu(T) \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \right] - \frac{\bar{u}}{K}
\]

\[
\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} = -\frac{\partial p}{\partial y} + 2 \frac{\partial}{\partial y} \left( \mu(T) \frac{\partial \bar{v}}{\partial y} \right) + \frac{\partial}{\partial x} \left[ \mu(T) \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) \right] - \frac{\bar{v}}{K} + \rho g \beta (T_f - T_i),
\]

- for the porous layer:

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0,
\]

\[
\rho \left( \frac{1}{e} \frac{\partial \bar{u}}{\partial t} + \frac{1}{e^3} \frac{\partial \bar{u}}{\partial x} + \frac{1}{e^3} \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \left( \mu_f(T_f) \frac{\partial \bar{u}}{\partial x} \right) + \frac{1}{e} \frac{\partial}{\partial y} \left[ \mu_f(T_f) \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right) \right] - \frac{\bar{u}}{K} \frac{\bar{u}}{\bar{u}}
\]

\[
\rho \left( \frac{1}{e} \frac{\partial \bar{v}}{\partial t} + \frac{1}{e^3} \frac{\partial \bar{v}}{\partial x} + \frac{1}{e^3} \frac{\partial \bar{v}}{\partial y} \right) = -\frac{\partial p}{\partial y} + 2 \frac{\partial}{\partial y} \left( \mu_f(T_f) \frac{\partial \bar{v}}{\partial y} \right) + \frac{1}{e} \frac{\partial}{\partial x} \left[ \mu_f(T_f) \left( \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} \right) \right] - \frac{\bar{v}}{K} \frac{\bar{v}}{\bar{v}} + \rho g \beta (T_f - T_i),
\]
\[ \varepsilon (\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \left( \bar{u} \frac{\partial T_f}{\partial \bar{x}} + \bar{v} \frac{\partial T_f}{\partial \bar{y}} \right) = \varepsilon \lambda_f \left( \frac{\partial^2 T_f}{\partial \bar{x}^2} + \frac{\partial^2 T_f}{\partial \bar{y}^2} \right) + \tilde{h} (T_s - T_f), \]  

(8)

\[ (1 - \varepsilon)(\rho c)_s \frac{\partial T_s}{\partial t} = (1 - \varepsilon) \lambda_s \left( \frac{\partial^2 T_s}{\partial \bar{x}^2} + \frac{\partial^2 T_s}{\partial \bar{y}^2} \right) + \tilde{h} (T_f - T_s), \]  

(9)

where \( \bar{x} \) and \( \bar{y} \) are the dimensional Cartesian coordinates; \( \bar{u} \) and \( \bar{v} \) are the velocity components along \( \bar{x} \) and \( \bar{y} \) directions, respectively; \( \rho \) is the fluid density; \( t \) is the dimensional time; \( p \) is the pressure; \( \bar{\rho}(T) = \mu_0 \cdot \exp(-\varepsilon \frac{T - T_f}{\Delta T}) \) is the temperature-dependent dimensional dynamic viscosity; \( T \) is the dimensional temperature; \( \mu_0 \) is the dynamic viscosity at initial temperature \( T = T_0 \); \( K \) is the porous medium permeability; \( g \) is the gravitational acceleration; \( \beta \) is the thermal expansion coefficient; \( c \) is the specific heat; \( \lambda \) is the thermal conductivity of the porous medium; and \( \tilde{h} \) is the solid/fluid heat transfer coefficient.

Introducing the stream function \( \psi = \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} \), vorticity \( \omega = \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} \) and the following dimensionless variables:

\[ x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L}, \quad \tau = t \sqrt{g \beta \Delta T/L}, \quad \theta_f = (T_f - T_0)/\Delta T, \quad \theta_s = (T_s - T_0)/\Delta T, \quad \mu = \bar{\rho}/\mu_0 \]

\[ u = \frac{\bar{u}}{\sqrt{g \beta \Delta T}} L, \quad v = \frac{\bar{v}}{\sqrt{g \beta \Delta T}}, \quad \psi = \frac{\bar{\psi}}{\sqrt{g \beta \Delta T}}, \quad \omega = \frac{\bar{\omega}}{\sqrt{L/g \beta \Delta T}} \]

the governing equations of convective heat transfer in dimensionless variables stream function – vorticity become:

- for the clear fluid layer (Astanina et al., 2015; Astanina et al., 2018):

\[ \frac{\partial^2 \psi}{\partial \xi^2} + \frac{\partial^2 \psi}{\partial \eta^2} = -\omega, \]  

(11)

\[ \frac{\partial \omega}{\partial \tau} + u \frac{\partial \omega}{\partial \xi} + v \frac{\partial \omega}{\partial \eta} = \sqrt{\Pr} \left( \frac{\partial^2 (\mu \omega)}{\partial \xi^2} + \frac{\partial^2 (\mu \omega)}{\partial \eta^2} \right) + \frac{\partial \theta}{\partial \xi} \]

\[ + 2 \sqrt{\Pr} \left[ \frac{\partial^2 \mu}{\partial \xi^2} \frac{\partial u}{\partial \eta} - \frac{\partial^2 \mu}{\partial \eta^2} \frac{\partial u}{\partial \xi} + \frac{\partial^2 \mu}{\partial \xi \partial \eta} \left( \frac{\partial v}{\partial \eta} - \frac{\partial u}{\partial \xi} \right) \right], \]  

(12)

\[ \frac{\partial \theta}{\partial \tau} + u \frac{\partial \theta}{\partial \xi} + v \frac{\partial \theta}{\partial \eta} = \frac{1}{\sqrt{\mathrm{Ra} \cdot \mathrm{Pr}}} \left( \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \eta^2} \right), \]  

(13)
for the porous layer:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega, \quad (14)$$

$$\epsilon \frac{\partial \omega}{\partial \tau} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \epsilon \sqrt{\frac{\text{Pr}}{\text{Ra}}} \left( \frac{\partial^2 (\mu \omega)}{\partial x^2} + \frac{\partial^2 (\mu \omega)}{\partial y^2} - \epsilon \frac{\mu \omega}{\text{Da}} \right) + \epsilon \frac{\partial \theta_f}{\partial x} +$$

$$+ 2\epsilon \sqrt{\frac{\text{Pr}}{\text{Ra}}} \left[ \epsilon u \frac{\partial \mu}{\partial y} + \epsilon v \frac{\partial \mu}{\partial x} + \frac{\partial^2 \mu}{\partial x^2} \frac{\partial \omega}{\partial y} - \frac{\partial^2 \mu}{\partial y^2} \frac{\partial \omega}{\partial x} + \frac{\partial^2 \mu}{\partial x \partial y} \frac{\partial \omega}{\partial x} \right], \quad (15)$$

$$\epsilon \frac{\partial \theta_f}{\partial \tau} + \frac{\partial \psi}{\partial y} \frac{\partial \theta_f}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta_f}{\partial y} = \frac{\epsilon}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left( \frac{\partial^2 \theta_f}{\partial x^2} + \frac{\partial^2 \theta_f}{\partial y^2} \right) + \frac{\xi}{\sqrt{\text{Ra} \cdot \text{Pr}}} (\theta_s - \theta_f), \quad (16)$$

$$(1 - \epsilon) \frac{\partial \theta_s}{\partial \tau} = \frac{1 - \epsilon}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left( \frac{\partial^2 \theta_s}{\partial x^2} + \frac{\partial^2 \theta_s}{\partial y^2} \right) + \frac{\xi \gamma}{\sqrt{\text{Ra} \cdot \text{Pr}}} (\theta_f - \theta_s), \quad (17)$$

for the heat-generating source:

$$\frac{\partial \theta}{\partial \tau} = \frac{\alpha_{hs} / \alpha_f}{\sqrt{\text{Ra} \cdot \text{Pr}}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \text{Os} \right). \quad (18)$$

Here $Ra = \rho g \beta \Delta T L^3 / (\alpha \mu_0)$ is the Rayleigh number; $\text{Pr} = \mu_0 / (\rho c)$ is the Prandtl number; $Da = K / L^2$ is the Darcy number; $Os = Q L^2 / (\lambda_{hs} \Delta T)$ is the Ostrogradsky number; $\mu = \exp (\xi \theta)$ is the temperature-dependent dimensionless dynamic viscosity; $\xi = \nu L^2 / \lambda_f$ is the Nield number for the fluid/solid matrix interface (fluid/solid matrix interface parameter); $\gamma = (\rho c_f) / (\rho c_s)$ is the heat capacitance ratio (between the liquid and the solid matrix); and $\Lambda = \lambda_s / \lambda_f$ is the thermal conductivity ratio (between the liquid and the solid matrix).

The initial and boundary conditions for the formulated problems (11)-(18) are as follows:

$$\tau = 0: \quad \psi = \omega = \theta = 0 \quad \text{at} \ 0 \leq x \leq 1 \quad \text{and} \ 0 \leq y \leq 1$$

$$\tau > 0:\quad \psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2}, \quad \theta_f = \theta_s = 0 \quad \text{at} \ x = 0 \quad \text{and} \ 0 \leq y \leq 1$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial x^2}, \quad \theta_f = \theta_s = 0 \quad \text{at} \ x = 1 \quad \text{and} \ 0 \leq y \leq 1$$

$$\psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial y^2}, \quad \frac{\partial \theta_f}{\partial y} = \frac{\partial \theta_s}{\partial y} = 0 \quad \text{at} \ y = 0, \ 1 \quad \text{and} \ 0 < x < 1$$

(19a)
\[
\tau > 0: \quad \psi = 0, \quad \omega = -\frac{\partial^2 \psi}{\partial n^2}, \quad \begin{cases}
\frac{\partial \theta_f}{\partial n} |_{\text{porous}} = \frac{\lambda_f}{\lambda_s} \frac{\partial \theta_s}{\partial n} \\
\frac{\partial \theta_f}{\partial n} |_{\text{porous}} = \frac{\lambda_f}{\lambda_s} \frac{\partial \theta_s}{\partial n}
\end{cases}
\text{ at heat source surface}
\]

\[
\tau > 0:
\begin{align*}
\psi_{\text{porous}} &= \psi_f, \\
\frac{\partial \psi_{\text{porous}}}{\partial y} &= \frac{\partial \psi_f}{\partial y}, \\
\omega_{\text{porous}} &= \omega_f, \\
\frac{\partial \omega_{\text{porous}}}{\partial y} &= \frac{\partial \omega_f}{\partial y}, \\
\frac{\partial \theta_f}{\partial y} |_{\text{clear fluid}} &= \frac{\partial \theta_f}{\partial y} |_{\text{porous}} \\
\frac{\partial \theta_f}{\partial y} |_{\text{clear fluid}} &= \Lambda \frac{\partial \theta_s}{\partial y} |_{\text{porous}}
\end{align*}
\text{ at internal fluid – porous interface}
\]

(19b)

(19c)

The control parameters are the local Nusselt number \( Nu \) along the heat source surface and the average Nusselt number \( \overline{Nu} \):

\[
\overline{Nu} = \frac{1}{l} \int_0^l \overline{Nu} \, d\zeta
\]

3. Numerical method

The partial differential equations (11)-(18) with corresponding initial and boundary conditions (19a-c) have been solved by the finite difference method using a uniform grid (Pop et al., 2016; Sheremet et al., 2015; Astanina et al., 2015; Bondareva et al., 2015). For the approximation of the convective and diffusive terms, we used the difference scheme of the second order accuracy. The parabolic equations have been solved on the basis of Samarskii locally one-dimensional scheme. The discretized equations have been solved by Thomas algorithm. The equations for the stream function (11) and (14) have been discretized using the five-point difference scheme on the basis of central differences for the second derivatives. The obtained difference equations have been solved by the successive over relaxation method. Optimum value of the relaxation parameter has been chosen on the basis of computing experiments.

The complicated validation analysis has been performed earlier (Pop et al., 2016; Sheremet et al., 2015; Astanina et al., 2015; Bondareva et al., 2015). First, the developed computational code was verified using the free convection problem in a square porous cavity at \( Ra = 1000 \) for different values of the solid/fluid-scaled heat transfer coefficients \( \xi \) and porosity-scaled conductivity ratios \( \delta_s = \frac{1}{N1-\xi} \). Figure 2 shows the values of the average Nusselt number computed for various solid/fluid-scaled heat transfer coefficients.
and porosity-scaled conductivity ratios in comparison with the data of Baytas and Pop (2002). This comparison shows an excellent agreement.

Second, the developed computational code was verified using the numerical results of Singh and Thorpe (1995) for natural convection in a partially porous differentially heated square cavity. Figures 3 and 4 show a good agreement between the obtained streamlines and isotherms at different Rayleigh numbers and the results of Singh and Thorpe (1995) for the case when the height of the porous layer is equal to the height of the fluid layer. It should be noted that Singh and Thorpe (1995) used the Beavers–Joseph empirical boundary conditions at the fluid-porous interface when the Darcy model is used. Results on the basis of the Brinkman model are (–) and on the basis of the Darcy model are (–) in Figures 4a and 5a.

For the purpose of obtaining grid independent solutions, a grid sensitivity analysis was performed. Four cases are tested: a coarse grid of 100 × 100 points, fine grids of 200 × 200 points and 300 × 300 points and a much finer grid of 400 × 400 points. Figure 5 shows the time variations of average temperature inside the heat-generating element for these considered grids at $Ra = 10^5$, $Pr = 7.0$, $Os = 5.0$, $Da = 10^{-3}$, $\xi = 0.0$ and $\delta = h/L = 0.5$.

It can be seen clearly that the grid independence is achieved when the grid size is up to 200 × 200 points and the results do not have significant change with the improvement of finer grid. Thus, consideration of the accuracy required and the computation time, the grid system of 200 × 200 points is chosen for all calculations reported in this study. The considered uniform mesh is shown in Figure 1b.

4. Results and discussion
The part is devoted to analysis of natural convective fluid flow and heat transfer within the considered cavity. Numerical study has been conducted at the following values of the control parameters: Rayleigh number ($Ra = 10^5$), Darcy number ($10^{-4} \leq Da \leq 10^{-3}$), Prandtl number ($Pr = 7.0$), Ostrogradsky number ($Os = 5.0$), viscosity variation parameter ($\xi = 0, 1$), porous layer height ($0.0 \leq \delta \leq 1.0$), interphase heat transfer coefficient ($10 \leq \xi \leq 1000$) and dimensionless time ($0 \leq \tau \leq 100$). Particular efforts have been focused on the effects of these parameters on the fluid flow and heat transfer. Streamlines, isotherms,
average Nusselt number at the heater surface, fluid flow rate and average temperature inside the heat-generating element for different values of governing parameters mentioned above are illustrated in Figures 6-11.

Figure 6 shows streamlines, isotherms inside the fluid phase and isotherms inside the solid matrix of the porous medium for $\delta = 0.5$, $\xi = 100$ and different values of the viscosity variation parameter and the Darcy number. Regardless of the values of viscosity variation parameter and Darcy number, two symmetric convective cells are appeared in the cavity. Cores of these convective cells are found in a layer with a clear fluid, above the fluid-porous interface. The appearance of convective cells shows the presence of one ascending flow in the central part and two descending flows near the vertical isothermal walls.

A thermal plume is formed in the cavity and it expands in the region with a clear fluid near the upper wall. It should be noted that inside the porous layer warm-up occurs evenly because heat conduction is a major heat transfer mechanism. In addition, the heater is a heat-generated and heat-conducted element. A growth of the viscosity parameter from 0.0 till 1.0 leads to small changes inside the porous structure, special for isotherms. Inside the fluid phase, these changes are essentially both for isolines of stream function and isotherms. Small discrepancies inside the solid phase can be explained by low motion velocities. In the fluid phase, discrepancies are more significant because of temperature difference between these phases. Moreover, a rise of the porous structure Darcy number from $10^{-4}$ to $10^{-3}$ enhances the flow intensity, while the temperature within the cavity reduces.
Figure 4. Comparison of streamlines $\psi$ and isotherms $\theta$ at $Da = 10^{-5}$, $Ra = 10^6$: numerical results of Singh and Thorpe (1995) – $a$ and results of the present study – $b$.

Figure 5. Variation of the average temperature inside the heat-generating element versus the dimensionless time and mesh parameters.
Figure 6.
Isolines of stream function $\psi$, fluid phase temperature $\theta_f$ and solid phase temperature $\theta_s$ for $\delta = 0.5$, $\xi = 100$, $\zeta = 0.0$ (solid lines), $\zeta = 1.0$ (dashed lines): $Da = 10^{-4} - a$, $Da = 10^{-3} - b$

Figure 7.
Dependences of the average Nusselt number (a), fluid flow rate (b) and average temperature inside the heater (c) on time, Darcy number and viscosity variation parameter for $\delta = 0.5$, $\xi = 100$

Figure 7 demonstrates the effects of the viscosity variation parameter $\zeta$, the Darcy number and dimensionless time $\tau$ on the total average Nusselt number at the heater surface, fluid flow rate inside the cavity and average temperature inside the heat-generating element. The initial time level for all studied parameters shows the instability of the process, especially for fluid flow rate, because of two heat transfer modes (thermal conduction and convection) develop at the beginning of the process, but at time $\tau = 100$ one can find a formation of steady state. A growth of $\zeta$ leads to the following: a decrease in the heat transfer rate from
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Figure 8. Isolines of stream function $\psi$, fluid phase temperature $\theta_f$, and solid phase temperature $\theta_s$ for $Da = 10^{-4}$, $\xi = 100$, $\zeta = 0.0$, $\delta = 0.0 - a$, $\delta = 0.25 - b$, $\delta = 0.5 - c$, $\delta = 0.75 - d$, $\delta = 1.0 - e$.
the heater, a small growth of the fluid flow rate and a diminution of the average temperature inside the heater.

It is interesting to note that $\zeta = 1$ characterizes a reduction of viscosity with the growth of temperature, and as a result, one can find more intensive circulation inside the cavity and diminution of the average heater temperature. The latter is more essential for cooling of the heat-generating elements in electronic devices. At the same time, a raise of the Darcy number reduces the total average Nusselt number and average temperature inside the heater, while fluid flow rate increases. This behavior can be explained by an increase in the permeability of the porous layer. As has been mentioned above, an intensification of fluid flow is associated with a decrease in the dynamic viscosity of the liquid with time because of the adopted temperature-dependent viscosity $\mu = \exp(-\zeta \theta)$. It should be noted that minimum value of $\theta_{avg}$ is attained for $Da = 10^{-4}$ and $\zeta = 1$. We can say that most effective cooling of the heater can be achieved by using working liquids with temperature dependent viscosity.

The effect of the porous layer thickness on fluid flow and heat transfer inside the liquid and solid phases is presented in Figure 8 for $Da = 10^{-4}$, $\xi = 100$, $\zeta = 0.0$. In a cavity without a porous layer (Figure 8a), two convective cells and a stable thermal plume over the heater are formed. Convective flow in the cavity occurs very strong. When porous layer of a thickness $\delta = 0.25$ (see Figure 8b) is inserted, the cores of the cells slightly change their spatial orientation, and the thermal plume is expanding near the upper wall; the flow in the cavity intensifies. It should be noted that the heater is a heat-generated and heat-conducted element; therefore, one can see isotherms inside the heater too. In the enclosure with a porous layer the heating takes place evenly. Further increase in the porous layer thickness ($\delta = 0.5$ in Figure 8c) leads to the displacement of the convective cells cores in the vertical direction to the upper adiabatic wall because of the resistance effect from the porous layer. The convection flow in the clear liquid zone weakens; the thermal plume over the heater acquires the shape of a triangle because of uniform heating in the region with the porous layer with the gravity force influence. In the cases of $\delta = 0.75$ (see Figure 8d) and $\delta = 1$ (see Figure 8e), the convective flow inside the cavity loosens. The cores of the convective cells move to the zones near the heater in the case of whole porous cavity (see Figure 8e). Moreover, the fluid heating is more intensive with high thickness of the porous layer; the heat transfer from the heater increases and the temperature inside the enclosure rises also. This can be explained by the difference in temperatures of the solid and fluid phases. Thus, the porous layer within LTNE illustrates the heat transfer enhancement from the heat-generating element. It is worth noting that inside the solid matrix of porous medium the main heat transfer mechanism is a heat conduction where heating rates from this element to

![Figure 9](image-url)

Dependences of the average Nusselt number (a), fluid flow rate (b) and average temperature inside the heater (c) on time and porous layer thickness for $Da = 10^{-4}$, $\xi = 100$, $\zeta = 0.0$.
the vertical walls are equal, while in vertical direction, this heating is more intensive because of cooling effect from the isothermal vertical walls.

An influence of the porous layer thickness $\delta$ and dimensionless time $\tau$ on the total average Nusselt number at the heater surface, fluid flow rate inside the cavity and average temperature inside the heat-generating element is presented in Figure 9 for $Da = 10^{-4}$, $\xi = 0.5$, $\xi = 0.0$: $\xi = 10 - a$, $\xi = 100 - b$, $\xi = 1000 - c$.

Figure 10. Isolines of stream function $\psi$, fluid phase temperature $\theta_f$ and solid phase temperature $\theta_s$, for $Da = 10^{-4}$, $\delta = 0.5$, $\xi = 0.0$: $\xi = 10 - a$, $\xi = 100 - b$, $\xi = 1000 - c$.
the heat element surface and the liquid-porous interface where more intensive heat transport inside the porous layer can affect the temperature gradient at this interface. A rise of $\delta$ results in a growth of the average temperature inside the heater, especially for $\delta$ between 0.25 and 0.5 because of low temperature gradient at the heater surface and high thermal conductivity of the solid matrix of porous medium. It should be noted that high values of the porous layer thickness reflect more delayed formation of steady state for the average Nusselt number and more quick achievement of this steady state for the average temperature inside the heat-generating element.

Figure 10 displays streamlines, isotherms for fluid phase and solid matrix at different values of the interphase heat transfer coefficient while $Da = 10^{-4}$, $\delta = 0.5$, $\zeta = 0.0$. It should be noted that the interphase heat transfer coefficient or this coefficient is known as the Nield number, characterizes the interaction between the fluid phase and solid phase. A growth of this parameter results in an intensification of the abovementioned interaction and as a result one can find an intensification of convective flow inside the clear fluid layer and more intensive heating of the cavity from the heat-generating element. Low values of $\zeta$ (see Figure, 10a) reflect a development of the heat conduction regime inside the porous layer, but inside the clear fluid layer convective heat transfer dominates. The temperature is the maximum near the heat source. A growth of the Nield number leads to the temperature increase inside the cavity, an intensification of convective flow over the porous layer with a decrease in the thermal boundary layers thicknesses near the isothermal vertical walls. Moreover, the porous layer warms more evenly than a clear fluid layer taking into account the isotherms shape.

The combined effect of the Darcy number, Nield number and porous layer thickness on the total average Nusselt number, fluid flow rate and the average temperature inside the heat-generated element is presented in Figure 11 for $\zeta = 0.0$. Regardless of the Nield and Darcy numbers values a growth of the porous layer thickness leads to a rise of the average Nusselt number and a reduction of the fluid flow rate. An increase in $\delta$ results in a rise of the average temperature inside the heater for all considered cases except for one case. Namely, for $Da = 10^{-3}$ and $\zeta = 1000$, a growth of the porous layer thickness leads to an increase in the fluid flow rate for $\delta$ between 0.75 and 1.0 and diminution in the average temperature inside the heater. Such behavior can be explained by high permeability of the porous layer and high interphase heat transfer coefficient. As a result, one can conclude that porous layer thickness and the Nield number are the parameters that can control the heat transfer within the cavity.

![Figure 11](image.png)

**Figure 11.** Dependences of the average Nusselt number (a), fluid flow rate (b) and average temperature inside the heater (c) on porous layer thickness, Darcy and Nield numbers for $\zeta = 0.0$. 
5. Conclusions
Natural convective fluid flow and heat transfer within a square cavity having local heat-generated element under the effects of temperature-dependent viscosity and porous layer insertion has been studied. The LTNE model and transient Brinkman-extended Darcy model are applicable in the present investigation. The cavity has the cold vertical walls, adiabatic horizontal walls and a heat-generating element located on the bottom wall. Taking into account the obtained results, we conclude the following:

- A growth of the Darcy number leads to an increase in the heat transfer inside the cavity with a reduction of the average temperature inside the heat-generating solid element.
- Changes in the viscosity variation parameter from 0.0 till 1.0 leads to the better cooling of the heat-generating element because of more intensive circulation with decreasing viscosity of the liquid affected by the increasing temperature.
- Varying the porous layer thickness has a strong effect on the flow and heat transfer within the cavity.
- A growth of the interphase heat transfer coefficient intensifies the convective flow and heat exchange inside the cavity.

References


**Corresponding author**

Mikhail Sheremet can be contacted at: Michael-sher@yandex.ru

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