Ranking based on optimal points multi-criteria decision-making method

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Abstract
Purpose – The purpose of this paper is to propose a new MCDM method called ranking based on optimal points (RBOP).
Design/methodology/approach – By employing two abstract groups of alternatives as the optimum alternatives and an optimal alternative, in order to offer the most desirable alternative, RBOP imitates human behavior in the decision-making process. RBOP policy is to find the best alternative through measuring alternatives distances from optimum alternatives and optimal alternative, thus, the best alternative must be sitting on the closest distance to its optimum points and the closest distance to the optimal points simultaneously.
Findings – In this paper, the author introduced a ten-step gray form of RBOP which is applied in a case of buying running shoes and results compared to the existing MCDM methods. Results showed the considerable differences.
Originality/value – Generally, in order to select the best alternative(s), and to aid decision makers (DMs) to make better decisions for the real-world problems, MCDM methods evaluate a number of alternatives via a number of criteria through the proposed mathematical algorithms. Frequently, for the direct impact of the DMs on the decision-making process, MCDM methods have inflexible algorithms. They only allow DMs to make an impact on the criteria analysis. The inflexibility emerges as a problem when perfect information is available for DMs and MCDM final results are not desirable. The process of the new method completely depends on DMs’ decisions, their interpretation of the periphery and their personal impressions. Hence, the output of RBOP is not necessarily the best alternative, but it offers the most desirable alternative to DM.
Keywords Grey models for decision making, Pairwise comparison, Human behaviour pattern, Multi-criteria decision making, Weighted geometric mean
Paper type Research paper

1. Introduction
According to Harris’ (1998) view, decision making is the study of identifying and choosing alternatives according to the values and preferences of the decision maker (DM). It also implies that there are alternative choices that must be considered, and in such a case we want, not only to identify as many of these alternatives as possible, but to choose the one that best fits with our goals, objectives, desires, values and so on. The classic steps of a decision-making process are: problem definition; requirements determination; targets and goals establishment; alternatives identifications; criteria definition; decision-making technique selection (depending on the modality, nature and context of the decision-making problem and its goals); alternatives evaluation; and solution validation.

Decision-making problems may not always have certain solutions and techniques. With respect to nature, circumstance and effective criteria in the decision-making process, there are various types of technique categories. For example, one of the proposed categories includes qualitative or subjective decision making, which mentions judgment, experience and insight-based decision making, such as satisfactory-level decision making, rational[1] (i.e. classic or economic man) decision making (Simon, 1977), clustering decision making, the organizational procedures view’s “program model or avoidance mode[2]” (March, 1988), the multiple perspectives approach (Mitroff and Linstone, 1995) and math-based algorithms,
which are extracted from the second subcategory as quantitative or objective decision making. Another category of decision-making technique types consists of two categories: single-criterion decision making, such as integer programming and linear and nonlinear programming; and multiple-criteria decision making.

In general, when we talk about MCDM, or multi-criteria decision analysis, or even multi-attribute decision making (MADM) problems, we refer to a matrix-form problem with a set of alternatives which are supposed to be prioritized; that is, a set of criteria that is used to analyze alternatives (as shown in the below equation), and a set of weights which demonstrate the importance of each criterion in the problem ($\sum_{i=1}^{n} w_j = 1$):

$$
\begin{pmatrix}
C_1 & C_2 & \cdots & C_n \\
A_1 & x_{11} & x_{12} & \cdots & x_{1n} \\
A_2 & x_{21} & x_{22} & \cdots & x_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & x_{m1} & x_{m2} & \cdots & x_{mn}
\end{pmatrix}
$$

In a typical decision matrix, ($A_i = \{A_1, A_2, \ldots, A_m\}$, $i = 1, 2, \ldots, m$), ($C_j = \{C_1, C_2, \ldots, C_n\}$, $j = 1, 2, \ldots, n$) and ($w_j = \{w_1, w_2, \ldots, w_n\}$, $j = 1, 2, \ldots, n$) refer to the set of feasible alternatives, the decision-making criteria and the weights of criteria, respectively. Furthermore, ($x_{mn}$) is the score of ($A_i$) according to the ($C_j$).

There are various available mathematical solutions that represent different algorithms for the analysis of a decision matrix. They rank the alternatives and give the best choice, which is the alternative that takes the first place as the first rank. Recent developments in MCDM methodologies have included step-wise weight assessment ratio analysis (Keršulienė and Turskis, 2011), the subjective weighting method using a continuous interval scale (Toloie-Eshlaghy et al., 2011), the superiority and inferiority ranking method (Xu, 2001), the IMP or multi-attribute evaluation using imprecise weight estimates method (Jessop, 2014) and the BWM or best-worst method introduced by Rezaei (2015). The most popular MCDM methods that are widely utilized in research and studies are the following: the technique for order of preference by similarity to ideal solution (TOPSIS) introduced by Hwang and Yoon (1981); the analytic hierarchy process first introduced by Saaty (1971, 1988); the analytical network process proposed by Saaty (1996); simple additive weighted (SAW) proposed by MacCrimmon (1968); data envelopment analysis proposed by Charnes et al. (1978); the visekriterijumska optimizacija i kompromisno resenje (VIKOR) method (Opricovic, 1998; Opricovic and Tzeng, 2002); the decision-making trial and evaluation laboratory (Fontela and Gabus, 1976); the preference ranking organization method for enrichment evaluations (Mareschal et al., 1984); and the elimination et choix traduisant la réalité or elimination and choice expressing reality method first introduced by Roy (1968, 1971, 1977) and Roy and Bertier (1972). A comparison of MCDM methods can be found in Wallenius et al. (2008), Opricovic and Tzeng (2003, 2007), Thor et al. (2013), Zamani-Sabzi et al. (2016) and Kralik et al. (2016).

The main focus of this paper is to propose a new multi-criteria decision-making method that ranks alternatives directly based on DMs’ decisions about the ideal alternatives. TOPSIS and VIKOR are the most popular MCDM methods that rank alternatives, according to the ideal alternatives. For example, with the identification of the two abstract alternatives as the positive and negative ideal solutions, TOPSIS philosophy is to select the alternatives that are at the closest distance to the positive ideal, and the farthest distance from the negative ideal alternative. The ideal alternatives are extracted from the decision matrix in both the TOPSIS and VIKOR methods. The following equations show the positive and negative ideal sets in the TOPSIS algorithm where ($v_{ij}$),
(A^+) and (A^-) show a weighted normalized value preference for the weighted normalized decision matrix, the set of positive ideals and the set of negative ideals, respectively:

\[ v_{ij} = w_j x_{mn} \left( \sum_{i=1}^{m} x_{mn}^2 \right)^{-1/2}, \]  

(2)

\[ A^+ = \left\{ \left( \max_i v_{ij} \mid j \in J_1 \right), \left( \min_i v_{ij} \mid j \in J_2 \right) \mid i = 1, 2, \ldots, m \right\}, \]

(3)

\[ A^- = \left\{ \left( \min_i v_{ij} \mid j \in J_1 \right), \left( \max_i v_{ij} \mid j \in J_2 \right) \mid i = 1, 2, \ldots, m \right\}, \]

(4)

\[ A^+ = \{ v_1^+, v_2^+, \ldots, v_n^+ \}, \]

(5)

\[ A^- = \{ v_1^-, v_2^-, \ldots, v_n^- \}. \]

(6)

As has been mentioned before, these ideal alternatives are derived from the decision matrix, while they are not listed as the problem's actual alternative. VIKOR also follows the same order. We think the ideal alternatives are not in the actual alternative list of the decision-making problem as those MCDM techniques mentioned extract them from the problem decision matrix. DMs are always seeking the alternative that is at the closest distance to an abstract ideal alternative, or in other words, the alternative that has more similarity to the mentioned abstract ideal alternative. These ideal alternatives are not necessarily derived from the decision matrix, but they are made by the DM's mind. We naturally think that DMs first investigate alternatives separately, then form a big picture of a problem, investigate it and finally make their decisions. The first part of the above procedure is the investigation of each alternative via the criteria, along with the problem goal, to illustrate a comprehensive picture of the problem. Then, the DM investigates the picture by comparing it with his/her mental abstract solution to find and select the best alternative. This paper aims to formulate a DM's mindset with the proposition of a new multi-criteria decision-making method that is based on a DM's decision-making behavioral pattern and also gives a reliable result for any real-world problem.

The remainder of this paper is organized as follows. In Section 2, ranking based on optimal points (RBOP) is introduced as a new MDCM method. In Section 3, the gray form of RBOP is proposed. In Section 4, RBOP is employed to solve a real-world decision-making problem and the results are compared with the results of the applied SAW, gray SAW, TOPSIS and TOPSIS-G methods. And finally, the conclusions and suggestions for future research are given in Section 5.

2. Ranking based on optimal points (RBOP)

The term “decision making” has been defined as a “process” by other scientific scholars in the following ways: “decision making is a process that chooses a preferred option or a course of actions from among a set of alternatives on the basis of given criteria or strategies” (Wang and Ruhe, 2007); and also as “the process of reducing the gap between the existing situation and the desired situation through solving problems and making use of opportunities” (Saroj, 2009). As discussed earlier, from our standpoint, when a DM is faced with a decision-making problem, he/she makes the final decision using a two-step process.
This behavior is the inspiration source, and it also made the main platform of our proposed MCDM method named “RBOP.”

For a better understanding of our view, let us outline a simple decision-making problem. Suppose we want to buy a tree for our home’s front yard, but we have incomplete information and do not know what is really appropriate. In that circumstance, we need to choose the best tree from the trees available at the tree shop. There are three options of tree provided to select the best tree from, while with respect to the theory of choice (EBA[3]), other trees were eliminated in accordance with our main criteria or goals. The trees have been illustrated in Figure 1 as Alternative 1, Alternative 2 and Alternative 3.

Consistent with our view, in the next step, the DM starts to investigate each alternative individually; then she/he creates an abstract alternative for each alternative, where the numbers of the new alternatives are equal to the problem alternatives (Figure 2). These abstract alternatives are called optimum alternatives. In the RBOP algorithm, in this section, the distance between each alternative and its optimum alternative will be calculated, as displayed in Figure 2.

![Figure 1. The three options of the decision-making problem](image1)

![Figure 2. The optimum alternatives for each tree](image2)

**Note:** The optimum alternative for each alternative of the problem based on the DM’s decision where $D_i = \{D_{1i}, D_{2i}, D_{3i}\}$ is the set of distances between each alternative from its optimum alternative and $i=1, 2, \ldots, m$ is the number of alternatives.
Distances can be a crisp number or demonstrated as a gray numerical interval or fuzzy membership function. Generally, to complete the following sentence, the optimum alternative will fill the blank, so “if it was [...] it would be better or [etc.]” In actual fact, each optimum alternative compares with the overall score of its corresponding alternative in the decision matrix. Moreover, \( D_i \) represents the distance between the optimum alternative’s overall score and the actual alternative’s overall score. In accordance with RBOP philosophy, three conditions must be met in this step. Let’s assume \( S_m \) is the overall score of the actual \( m \)th alternative and \( S'_m \) is the overall score of its corresponding optimum alternative, then \((S'_m < 2S_m), ((S_m/S'_m) < 1)\) and \((D_m < S_m)\). Furthermore, the DM may compare each alternative with two or more abstract alternatives, as illustrated in Figure 3.

As is shown in Figure 3, the DM defined two optimum alternatives in comparison with an actual alternative. As was mentioned before, there can be more than just two optimum alternatives for each alternative, though they must meet the same conditions as they expressed earlier; also, other conditions must be met. Before giving an interpretation of the conditions, we intend to present the calculation process for the distances in RBOP. To calculate a distance, all optimum alternatives need to be coagulated into the one optimum alternative. For the coagulation, the classic RBOP algorithm utilizes a weighted geometric mean \((\prod_{i=1}^{n} w_i x_i)^{-1}\) method that is shown in the following equation, where \( A'_i \) stands for the weighted geometric mean of the multiple optimum alternatives, \( w_i \) refers to the weight of each optimum alternative and \( x_i \) denotes each optimum alternative rate via each criterion:

\[
\overline{A'_i} = \left( \prod_{i=1}^{n} w_i x_i \right)^{-1} \quad i = 1, 2, \ldots, m.
\]

In the RBOP method, the pairwise comparison philosophy is employed to compute the weight of each optimum alternative. Table I demonstrates pairwise comparisons in RBOP, where \( A'_m \) is the \( m \)th optimum alternative equal to \( A'_n \) which is \( n \)th optimum alternative in the \((m \times n)\) pairwise comparison matrix.

The core of the pairwise comparison is a “win, draw, lose” method. This type of comparison does not consider the details of optimum alternatives (comparison criteria), because when a DM considers an alternative as the optimum alternative for each problem.
alternative, the complete information (full information) about each alternative is available. Thus, the DM can decide the result of a pairwise comparison. In this method, \( W, D \) and \( L \) are assigned as win, draw and lose, respectively, and their scores in the computation process are 3, 1 and 0 correspondingly. The mentioned process is shown in Figures 4–6.

Consequently, the computation procedure for weighing each optimum alternative is as follows:

\[
W'_i = \sum_{j=1}^{n} w^*_j.
\] (8)

In the next step of the analysis procedure, the DM compares each optimal alternative against another abstract alternative which is the best alternative, the best answer or the best abstract alternative that meets all the particular decision-making object needs. In RBOP, this alternative is the “optimal alternative.” There is one and only one optimal alternative, which sits in the most preferable situation. The optimal alternative has been portrayed as a golden tree in Figure 7.

In this section, \( A^* \) is assigned as the optimal alternative. The optimal alternative must meet the following criteria: first, the optimal alternative always possesses a larger value than the problem alternatives, then \( (A^* > A_i) \); and second, \( (A^* \geq A'_i) \). This means the optimal alternative is greater than or equal to the optimum alternative. In other words, each optimum alternative of each alternative can stand as the optimal alternative on the stage; in the event that it is greater than other optimum alternatives. In Figure 7, \( D_i = \{D_1, D_2, D_3\} \) is the distance between the optimum alternatives and the optimal alternative. The distances (distances between each alternative from their corresponding optimal alternatives, and distances between each optimal alternative from the optimum alternative) make a paradigm

<table>
<thead>
<tr>
<th>( C_n )</th>
<th>( A'_1 )</th>
<th>( A'_1 )</th>
<th>( A'_1 )</th>
<th>( A'_n-1 )</th>
<th>( A' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A'_1 )</td>
<td>D</td>
<td>( w_{12}^* )</td>
<td>D</td>
<td>( w_{2(n-1)}^* )</td>
<td>D</td>
</tr>
<tr>
<td>( A'_2 )</td>
<td>( w_{12}^* )</td>
<td>D</td>
<td>( w_{2(n-1)}^* )</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>( A'_{m-1} )</td>
<td>( w_{1m}^* )</td>
<td>( w_{2m}^* )</td>
<td>( w_{3m}^* )</td>
<td>( w_{(n-1)m}^* )</td>
<td>D</td>
</tr>
</tbody>
</table>

Table I.
An overview of the optimum alternatives comparison in the RBOP method

Figure 4.
The comparison: winning

\[ A'_1 \leftarrow A'_2 \]

Note: When the DM prefers \( A'_1 \), it is win and the score is 3

Figure 5.
The comparison: drawing

\[ A'_1 \leftarrow A'_2 \]

Note: When \( A'_1 \) and \( A'_2 \) are equal, then it is draw and the score is 1

Figure 6.
The comparison: losing

\[ A'_1 \rightarrow A'_2 \]

Note: When the DM prefers \( A'_2 \), it is lose and the score is 0 for \( A'_2 \)
upon which the DM makes his/her decision. Then, the DM selects the tree which is closer to the positive ideal solutions.

All human-based activities contend with uncertainty and vagueness. Human judgments include incomplete information, partial ignorance and nonobtainable information. Therefore, to deal with these created uncertainties, two efficient systems have been recommended: the fuzzy system introduced by Zadeh (1965) and the gray systems theory (GST) developed by Deng (1982, 1985). As stated by Mierzwiak et al. (2018), the gray information refers to the partial knowledge and incomplete information in a three-part information box consisting of the complete and known information, the incomplete information and the unknown information, which are referred to as the white, gray and black information categories, respectively. In RBOP, due to the human judgments operations (the DM's decisions-based operations), gray systems are employed to handle the uncertainty. RBOP steps in gray environments are represented in the next section.

3. Gray RBOP (RBOP-G) steps

As discussed earlier, GST was proposed by Professor Deng in 1982. It focuses on the problems of small samples and poor information (Wang et al., 2018). According to Salookolaei et al. (2018), GST focuses on the study of such uncertain systems with partially known and partially unknown information, whereby matters of the characteristics of poor information are seen as its research subjects. There are four possibilities for the emergence of gray information (Zakeri and Keramati, 2015; Lin et al., 2004): the information about elements is gray; the structural information is gray; the boundary information is gray; and the behavior information of motion is gray.

The principal of gray operations is established on numerical intervals with two higher and lower bounds. It connotes that the real number is within a specific range. The relationship between the gray number and GST is analogous with the relationship between a fuzzy number and fuzzy mathematics (Xie and Liu, 2010). Zakeri and Keramati (2015) alluded to the difference between fuzzy and gray numbers: gray numbers are similar to fuzzy numbers, but their fundamental difference is that while the precise values of gray
numbers are vague, their intervals are known; in other words, the precise values of the left and right bounds of a sequence are known. The following equations address the gray number operations.

Let’s suppose $\square G_1 = \left[ G_{1l}, G_{1u} \right]$, $\square G_2 = \left[ G_{2l}, G_{2u} \right]$ are two gray numbers and $\square G_1 > \square G_1$ and $\square G_2 > \square G_2$ then:

$$- \square G_1 = \left[ -G_{1l}, -G_{1u} \right].$$

(9)

Gray number addition:

$$\square G_1 + \square G_2 = \left[ G_{1l} + G_{2l}, G_{1u} + G_{2u} \right].$$

(10)

Gray number subtraction:

$$\square G_1 - \square G_2 = \square G_1 + (- \square G_2) = \left[ G_{1l} - G_{2l}, G_{1u} - G_{2u} \right].$$

(11)

Gray number multiplication:

$$\square G_1 \times \square G_2 = \min \left\{ \frac{G_{1l} G_{2l}}{G_{1u} G_{2u}}, \frac{G_{1l} G_{2u}}{G_{1u} G_{2l}}, \frac{G_{1u} G_{2l}}{G_{1l} G_{2u}} \right\},$$

$$\max \left\{ \frac{G_{1l} G_{2l}}{G_{1u} G_{2u}}, \frac{G_{1l} G_{2u}}{G_{1u} G_{2l}}, \frac{G_{1u} G_{2l}}{G_{1l} G_{2u}} \right\},$$

(12)

$$r \times \square G_1 = \left[ r G_{1l}, r G_{1u} \right].$$

(13)

Gray number division:

$$\square G_1 / \square G_2 = \left[ \frac{G_{1l}}{G_{1u}}, \frac{G_{1u}}{G_{1l}} \right] \times \left[ \frac{1}{G_{2l}}, \frac{1}{G_{2u}} \right] = \left[ G_{1l}, G_{1u} \right] \times \left[ \frac{G_{2l}^{-1}}{G_{2u}^{-1}}, \frac{G_{2u}^{-1}}{G_{2l}^{-1}} \right]$$

$$= \min \left\{ \frac{G_{1l} G_{2l}^{-1}}{G_{1u} G_{2u}^{-1}}, \frac{G_{1l} G_{2u}^{-1}}{G_{1u} G_{2l}^{-1}}, \frac{G_{1u} G_{2l}^{-1}}{G_{1l} G_{2u}^{-1}} \right\},$$

$$\max \left\{ \frac{G_{1l} G_{2l}^{-1}}{G_{1u} G_{2u}^{-1}}, \frac{G_{1l} G_{2u}^{-1}}{G_{1u} G_{2l}^{-1}}, \frac{G_{1u} G_{2l}^{-1}}{G_{1l} G_{2u}^{-1}} \right\},$$

(14)

$$\square G_1 \div a = \left[ \frac{G_{1l}}{a}, \frac{G_{1u}}{a} \right],$$

(15)

$$a \div \square G_1 = \left[ \frac{a}{G_{1l}}, \frac{a}{G_{1u}} \right].$$

(16)

The possibility degree of $\square G_1 \leq \square G_2$:

$$p(\square G_1 \leq \square G_2) = \frac{\max \left( 0, L^* - \max \left( 0, G_{1l} - G_{2l} \right) \right)}{L^*},$$

(17)

where $L^* = L(\square G_1) + L(\square G_2)$.

And the gray linguistic variables are as shown in Table II.
Characteristically, RBOP uses Shannon’s (2001) entropy in its algorithm. Shannon’s entropy is an important notion in information sciences that measures the uncertainty and complexity of dynamic systems. In the MCDM methods, entropy plays an important role as a tool for calculating the weights of decision-making problem criteria. In line with Sachdeva et al. (2009) and Das et al. (2014), the gray entropy is calculated with respect to the following equations, where $e_{G^j}$ and $e_{\overline{G^j}}$ express the entropy of each criteria, and $w_{G^j}$ and $w_{\overline{G^j}}$ define the weight of each criteria in an interval:

$$e_{G^j} = -\frac{1}{\ln m} \sum_{i=1}^{m} G_{ij} \ln G_{ij},$$  
(18)

$$e_{\overline{G^j}} = -\frac{1}{\ln m} \sum_{i=1}^{m} \overline{G_{ij}} \ln \overline{G_{ij}},$$  
(19)

With respect to Equations (11) and (12), the computation of the weight of $j$th criterion is as follows:

$$w_{G^j} = \left(1-e_{G^j}\right) \cdot \left(\sum_{j=1}^{n} \left(1-e_{G^j}\right)\right)^{-1},$$  
(20)

$$w_{\overline{G^j}} = \left(1-e_{\overline{G^j}}\right) \cdot \left(\sum_{j=1}^{n} \left(1-e_{\overline{G^j}}\right)\right)^{-1}.$$  
(21)

Where $\otimes G_j = \left[G_{ij}, \overline{G_{ij}}\right]$ is a normalized form of $\otimes G_j = \left[G_{ij}, \overline{G_{ij}}\right]$.

$$G_{ij} = \left(\sum_{i=1}^{m} G_{ij}\right)^{-1} G_{ij},$$  
(22)

$$\overline{G_{ij}} = \left(\sum_{i=1}^{m} \overline{G_{ij}}\right)^{-1} \overline{G_{ij}}.$$  
(23)

Let us assume ($\lambda_j$) is an added value such as a DM’s decisions about the weight of each criteria, thus, entropy computes as per the following equations. ($\lambda_j$) is a crisp number, then:

$$w_{G^j} = \lambda_j \left(1-e_{G^j}\right) \cdot \left(\sum_{j=1}^{n} \lambda_j \left(1-e_{G^j}\right)\right)^{-1},$$  
(24)

<table>
<thead>
<tr>
<th>Scale</th>
<th>Very poor (VP)</th>
<th>Poor (P)</th>
<th>Medium poor (MP)</th>
<th>Fair (F)</th>
<th>Medium good (MG)</th>
<th>Good (G)</th>
<th>Very good (VG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray</td>
<td>[0, 1]</td>
<td>[1, 3]</td>
<td>[3, 4]</td>
<td>[4, 5]</td>
<td>[5, 7]</td>
<td>[7, 9]</td>
<td>[9, 10]</td>
</tr>
</tbody>
</table>
\[ w_{G_j} = \lambda_j \left(1 - e_{G_j}\right) \left(\sum_{j=1}^{n} \lambda_j \left(1 - e_{G_j}\right)\right)^{-1}. \]  
\( (\lambda_j) \) is a gray number \((\lambda_j = [\lambda_j, \bar{\lambda}_j])\), then:

\[ w_{G_j} = \lambda_j \left(1 - e_{G_j}\right) \left(\sum_{j=1}^{n} \lambda_j \left(1 - e_{G_j}\right)\right)^{-1}. \]  

The entropy algorithm for the crisp numbers with the complete and known information is as follows:

\[ er_j = \frac{1}{\ln m} \sum_{i=1}^{m} r_{ij} \ln r_{ij}, \]  

\[ w_{r_j} = (1 - e_{r_j}) \left(\sum_{j=1}^{n} (1 - e_{r_j})\right)^{-1}. \]  

The normalization process is in accordance with the following equation:

\[ r_j = \left(\sum_{i=1}^{m} x_{ij}\right)^{-1}. \]  

The ten-step RBOP-G is presented as follows.

Step 1. Constructing the gray decision matrix:

\[ \otimes X_{mn} = \begin{bmatrix} A_1 & C_1 \ G_{11} & G_{12} & \ldots & G_{1j} \\ A_2 & C_2 \ G_{21} & G_{22} & \ldots & G_{2j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & C_n \ G_{m1} & G_{m2} & \ldots & G_{mj} \end{bmatrix}. \]  

Step 2. Making the optimum decision matrix as the following matrix:

\[ \otimes X'_{mn} = \begin{bmatrix} \ A'_1 & C_1 \ G'_{11} & G'_{12} & \ldots & G'_{1j} \\ \ A'_2 & C_2 \ G'_{21} & G'_{22} & \ldots & G'_{2j} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \ A'_m & C_n \ G'_{m1} & G'_{m2} & \ldots & G'_{mj} \end{bmatrix}. \]  

where \( \otimes X_{mn} \) is the decision matrix, \( \otimes X'_{mn} \) refers to the optimum decision matrix according to the DMs decisions for each alternative and \( G_{mn} \) and \( G'_{mn} \) are the gray numbers.

Step 3. Normalization of the decision matrices according the following equations.
If \( j \) is a set of benefit attributes, then the normalized value of \( r_{ij} \) computes as the following equations, where \( r_{ij} \) is the normalized value of \( G_{ij} \) and \( r_{ij}^{0} \) is the normalized value of \( G_{ij}^{0} \):

\[
r_{ij}^{+} = \left[ \frac{G_{ij}}{G_{ij}^{\text{max}}} \right]
\]

\[
r_{ij}^{0+} = \left[ \frac{G_{ij}^{0}}{G_{ij}^{0\text{max}}} \right]
\]

where \( \otimes G_{ij} = [G_{ij}, \overline{G_{ij}}] \) and \( \otimes G_{ij}^{\text{max}} = \max_{1 \leq i \leq m} (G_{ij}) \).

If \( j \) is a set of cost attributes, then the normalized value of \( r_{ij} \) is as shown in the following equations, where \( r_{ij} \) is the normalized value of \( G_{ij} \) and \( r_{ij}^{0} \) is the normalized value of \( G_{ij}^{0} \):

\[
r_{ij}^{-} = \left[ \frac{G_{ij}^{\text{min}}}{G_{ij}} \right]
\]

\[
r_{ij}^{0-} = \left[ \frac{G_{ij}^{0\text{min}}}{G_{ij}^{0}} \right]
\]

where \( \otimes G_{ij}^{\text{max}} = \min_{1 \leq i \leq m} (G_{ij}) \).

Step 4. Computation of the optimum point by calculating the distance between two of the normalized decision matrices (the alternatives and optimum alternatives) with respect to the following procedures.

To calculate optimum points, RBOP uses the importance weights of each alternative and optimum alternative, where the mentioned weights are the constant coefficients in the whole calculation process for each attribute. The following process shows the classic optimum calculation in RBOP.

In accordance with the RBOP policy, optimum points tend to the optimum alternative; hence, let us assume \( x \) and \( x' \) are the two points of the one-dimensional space (Figure 8) \( (x' > x) \), and \( w_{x} \) refers to the importance weight of \( x \); as such, \( w_{x'} \) denotes the importance weight of \( x' \), then the following condition must be met: \( w_{x'} \geq w_{x} \); and \( w_{x'} + w_{x} = 10 \). The importance weight can be a crisp number, then its impact on the procedure follows Equation (13), or it can be an interval as a gray number \( (w = [\underline{w}, \overline{w}]) \), then it sticks to Equation (12) to impact on the calculation process of the optimum point. The calculation of the importance weight is not a numerical procedure; the DM decides its value with the commitment of the two mentioned conditions.

The first condition indicates that the optimum points are naturally closer to the optimum alternative. Also, the second condition can be translated as the following equations.

Note: Distance between two points in a one dimensional space and the importance weights according to the two mentioned conditions.

![Figure 8. Distance between two points in a one-dimensional space](image-url)
If \((w_x'' + w_x' \neq 10)\) then:
\[
w_x' = \frac{w_x''}{w_x'' + w_x'} \times 10,
\]
(35)
\[
w_x = \frac{w_x'}{w_x'' + w_x'} \times 10.
\]
(36)

When \((w_x'' > w_x')\) and its attributes benefit, then the optimum point computes as the following equation where \(O^+\) expresses the benefit attributes’ optimum point:
\[
O^+ = x' - \left(\frac{x' - x}{10 \times w_x}\right).
\]
(37)

And if \((w_x' = w_x'')\), then the optimum point computes as follows:
\[
O^+ = x' - \left(\frac{x' - x}{2}\right).
\]
(38)

When an attribute is a nonbeneficial (cost) attribute, then the calculation of the optimum can be found in the following equations. As the cost attribute optimum point, the calculation process of \(O^-\) is of the same order as \(O^+\) with Equations (37) and (38):
\[
x = (x + x')^{1/2} - (x)^{1/2},
\]
(39)
\[
x' = (x + x')^{1/2} - (x')^{1/2}.
\]
(40)

Hence, the calculation of optimum points in RBOP-G is as per the following equations.

For the cost attribute (see Equations (33) and (34)), where \(r_{ij}^- = [r_{ij}'', r_{ij}']\) and \(r_{ij}' = [r_{ij}'', r_{ij}']\):
\[
+r_{ij}'\left[+r_{ij}'', +r_{ij}'\right] = \begin{cases} +r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2},
\end{cases}
\]
(41)
\[
+r_{ij}'\left[+r_{ij}'', +r_{ij}'\right] = \begin{cases} +r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2}, \\
+r_{ij}' = \sqrt{\left(r_{ij}' - r_{ij}''\right)^2},
\end{cases}
\]
(42)

\[
O_{ij}^- = \begin{cases} \lambda = \left(+\frac{r_{ij}'' + r_{ij}'}{10} \times w_x'\right) + (+r_{ij}''), \\
v = \left(+\frac{r_{ij}'' + r_{ij}'}{10} \times w_x'\right) + (+r_{ij}').
\end{cases}
\]
(43)

Then \(O^- = [\lambda, v]\) where \(\lambda\) is the lower bound and \(v\) is the upper bound.
And for the benefit attributes (see Equations (31) and (32)), where \( r_{ij}^+ = [r_{ij}^{+1}, r_{ij}^{+2}] \) and \( r_{ij} = [r_{ij}^{-1}, r_{ij}^{+2}] \):

\[
O_{ij}^+ = \begin{cases} 
\lambda = \left( \frac{r_{ij}^+ - r_{ij}^-}{10} \times w_\lambda \right) + (r_{ij}^+) \\
v = \left( \frac{r_{ij}^+ + r_{ij}^-}{10} \times w_v \right) + (r_{ij}^+) 
\end{cases}.
\] (44)

Thus, \( O^+ = [\lambda, v] \) where \( \lambda \) is the lower bound and \( v \) is the upper bound.

**Definition 1.** In RBOP-G, \((O)\) stands for an interval, which includes the optimum point. In an \(n\)-dimensional space, \((n)\) is the optimum on the distance lines between two points where \(n\) is the number of criteria.

**Step 5. Defining the optimal points:**

\[
\otimes X^* = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \begin{bmatrix} R_1^* \\ R_2^* \\ \vdots \\ R_n^* \end{bmatrix}.
\]

**Step 6. Construction of the distance decision matrix between optimum point \((O)\) and the optimal point \((R)\) according to the following procedure.**

In the calculation process for the midpoint, due to the involvement of the normalized values of the optimum points, the optimal points must be normalized by respecting the following equations:

\[
NR_{ij} = \frac{R_{ij}}{\max_{1 \leq i \leq m} O_{ij}},
\]

\[
\Omega_{ij} = \frac{d(NR_{ij}-O)}{2} + O.
\] (46)

**Definition 2.** \((\Omega)\) refers to the midpoint on the distance line between the optimal point and the optimum point, where \(O = [\lambda_{ij}, v_{ij}]\) and \(\Omega = [\Lambda_{ij}, \Upsilon_{ij}]\) then.

**Step 7. Prioritizing alternatives in a new decision matrix in accordance with the following equation, where \(P(A_i)\) is the ranking value of each alternative:**

\[
P(A_i) = \prod_{j=1}^{n} (\Lambda_{ij} + Y_{ij})^{W_j}, \quad i = 1, 2, 3, \ldots, m.
\] (47)

In this step, as the weight of each criterion, attributable to the uncertainty of the environment, for calculating \(W_j\), there are two provided weights: the first weight which is derived by the gray entropy (Equations (18)–(21)), and the second weight leads by the DM’s decisions (Equations (24)–(30)).

Note: if \((\Lambda_{ij} + Y_{ij} = 0)\), the impact on the equation process must be neglected.

**Step 8.** Making the weighted normalized decision matrix of \(\otimes X_{mn}^w\).

Then, the weighted normalized matrix computes as per the following equation:

\[
\otimes G_{ij}^w = \otimes r_{ij} \times W_j,
\] (48)
where $\otimes r_{ij} = [r_{ij}, r_{ij}^c]$ is the normalized form of each member of the decision matrix, which is calculated with respect to Equations (31) and (33).

Step 9. Computation of the distance between each alternative from the largest values of the midpoints:

$$\Omega_{ij}^\text{max} = 1 - \left( \max_i \Omega_{ij} | j \in \{1, \ldots, n\}, \right) \tag{49}$$

$$\otimes \Phi_j = \sqrt{\sum_{j=1}^{n} \left( \Omega_{ij}^\text{max} - \otimes C_{ij}^* \right)^2}, \tag{50}$$

$$\Phi_j = \Phi_j + \overline{\Phi}; \quad \otimes \Phi_j = \left[ \Phi_j, \overline{\Phi} \right] . \tag{51}$$

$\otimes \Phi_j$ and $\Phi_j$ stand for the gray distance and crisp form distance of each alternative from the set of highest values of midpoints, respectively.

Note: $\max_i \Omega_{ij}$ is the highest value of the $i$th alternative in the $j$th criterion.

Step 10. The final RBOP step is the prioritization of alternatives with respect to the following equation, where $n$ is the number of criteria:

$$R_j = P(A_j) \times (2n - \Phi_j). \tag{52}$$

When the value of $R_j$ is higher, the ranking order of the alternative is better. Otherwise, the ranking is worse. The RBOP algorithm is illustrated in Figure 9.

4. Real-world application and results

In this section, the application of ARPASS on real-world decision-making issue is discussed, and the results are presented. To achieve this, we provide a simple decision-making problem that most people have to deal with, that is, buying running shoes. We consider a problem where a person intends to buy “running shoes” using various criteria including price, brand, color and design.

4.1 Data collection

In the market, the best available running shoes are Reebok Floatide Run and the buyer recognizes them as the best shoes that can be bought (optimal point). Therefore, unconsciously, the buyer compares other shoes with the Reeboks and he/she will choose the shoes, which are the closest to Reebok, because at the time, Reeboks are unavailable (unattainable ideal). As well as that, for other available ones, there are optimum points that are not related directly to the optimal point (Reeboks). For instance, it is like when a buyer wants to buy goods and the buying criteria are their appearance, brand and features (performance). He/she would think that if it was cheaper, or had a darker color, or had specific performance metrics, it would be better. That is the optimum point. Goods that are close to the optimum point will be the best decision for the buyer. To show RBOP performance in respect of this problem, decision matrix analysis has been provided step by step pursuant to Equations (7)–(45).

4.2 Results and comparison

We applied gray SAW and TOPSIS-G to the data we collected to compare their results with the RBOP-G results. As declared before, to buy running shoes, the DM investigates alternatives with their appearance, brand and features (performance) as the problem criteria.
In this case, appearance includes the color (blue color range) and design, and the features consist of pronation, over pronation, supination and barefoot/minimalist running[4] so that the DM’s scoring is in accordance with these properties. The running shoes must be selected from among pairs of Nike Men’s Revolution 3 Running Shoes, ASICS Men’s Gel Vanisher Running Shoes, and Under Armour Men’s Micro G Assert 7 Sneakers that have been briefly referenced as “Nike,” “ASICS” and “Under Armour” in the RBOP running process.

Constructing the running shoes selection decision matrix is the first step of the procedure. A decision matrix is provided in Table III.

As mentioned earlier, the features (performance) include pronation, over pronation, supination and barefoot/minimalist running. To make the right decision for the features, and the right number subsequently in the decision matrix (Table III), Equations (7) and (8) and Table I have been employed. For this example, the following procedure shows the investigation of features in the decision matrix which is applied in the case of Nike.

The main section of Equation (7) is the rate of each criterion, which is denoted originally as the optimum alternatives rate against each criterion in the RBOP algorithm. In Table IV,

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nike</td>
<td>[7, 9]</td>
<td>[5, 7]</td>
</tr>
<tr>
<td>ASICS</td>
<td>[7, 9]</td>
<td>[7, 9]</td>
</tr>
<tr>
<td>Under Armour</td>
<td>[5, 7]</td>
<td>[7, 9]</td>
</tr>
</tbody>
</table>
In contrast with quantitative values, the rating of the features subcriteria deals with qualitative values according to the gray linguistic variables with respect to Table II.

As declared in Table I and Equation (9), the pairwise comparison process is as shown in Table V.

Hence, the features rating in the decision matrix for Nike is [14, 21]. The rates for ASICS and Under Armour are [14, 21] and [14, 21], respectively. Their calculations are represented in Tables AI–AIV.

The second step is constructing an optimum decision matrix, which is constructed on the optimum alternatives. For the case of running shoes, for each alternative, there is one optimum alternative. As argued previously, the optimum alternative is not necessarily the real-world existing alternative, it can be an abstract alternative, or it can be an abstract alternative which is inspired by a real-world alternative. The optimum decision matrix is demonstrated in Table VI.

The third step is the normalization of the matrices. Since the running shoes selection decision matrix contains the only benefit criteria, the normalization process for the two matrices follows Equation (31). The results of normalization process are displayed in Table VII.

And the normalized optimum decision matrix is shown in Table VIII.

### Table IV.
Gray linguistic variables for the rating process of the features subcriteria for Nike

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>[7, 9]</td>
<td>[7, 9]</td>
<td>[4, 5]</td>
</tr>
</tbody>
</table>

### Table V.
Application of the pairwise comparison for the subcriteria of features in Nike running shoes

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pronation</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Supination</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Barefoot/minimalist running</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table VI.
Optimum running shoes selection decision matrix

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_Nike</td>
<td>[9, 10]</td>
<td>[20, 26]</td>
</tr>
<tr>
<td>A_ASICS</td>
<td>[7, 9]</td>
<td>[25, 29]</td>
</tr>
<tr>
<td>A_Under Armour</td>
<td>[7, 9]</td>
<td>[25, 30]</td>
</tr>
</tbody>
</table>

**Note:** Optimum running shoes selection decision matrix, where A\_Nike, A\_ASICS and A\_Under Armour are the optimum alternatives for Nike, ASICS and Under Armour running shoes.

### Table VII.
The normalized running shoes selection decision matrix

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nike</td>
<td>[1.0, 1.0]</td>
<td>[0.58, 0.72]</td>
</tr>
<tr>
<td>ASICS</td>
<td>[0.9, 1]</td>
<td>[0.96, 1]</td>
</tr>
<tr>
<td>Under Armour</td>
<td>[0.6, 0.78]</td>
<td>[1.0, 1.0]</td>
</tr>
</tbody>
</table>
The next step is computing the optimum points. The optimum point is between the problem alternative and the optimum alternative. Indeed, the optimum point is on the distance line of the alternative and the optimum alternative. In order to reach the optimum points, RBOP deals with Equations (35) and (36) for the cost and benefit criteria, respectively. Therefore, according to Equation (36), the results are shown in Table IX as per the distance matrix.

During the above process, first, the distance has been computed, and then to find the optimum point, RBOP assumes that the optimum point is right on the midpoint. The alternatives, optimum alternatives and optimum points are portrayed in Figures 10–12.

In the RBOP algorithm, after determining the optimum points, the optimal alternative must be defined. The optimal alternative includes optimal points according to the criteria. Regularly, the DM has a clear image of the optimal alternative in his/her mind that compares it with the other decision-making problem alternatives. It is a real-world alternative. As such, sometimes, this alternative is built by a mental process. Therefore, what the DM considers to be an optimal alternative is a mental image. The mental image may include the wide range of other alternatives’ dimensions, which is generated as a hybrid alternative, or as mentioned earlier, it may be a real-world alternative. In our case, it is a real-world alternative: Reebok Floatide Run.

Table X shows the optimal points according to the problem’ criteria.

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nike</td>
<td>[0.000, 0.000]</td>
<td>[0.145, 0.110]</td>
</tr>
<tr>
<td>ASICS</td>
<td>[0.345, 0.050]</td>
<td>[0.000, 0.000]</td>
</tr>
<tr>
<td>Under Armour</td>
<td>[0.360, 0.060]</td>
<td>[0.000, 0.000]</td>
</tr>
</tbody>
</table>

Table VIII. The normalized running shoes selection decision matrix for the optimum alternatives

Table IX. The distance matrix of optimum alternatives and alternatives

**Note:** The red lines denote the distances of optimum alternatives and the alternative, and the black dots are the optimum points.

Figure 10. Nike, the optimum alternative of Nike and its corresponding optimum point.
According to Equation (45), the next step is the calculation of the midpoint of optimal points and the optimum points, which sits on the distances between them (see the example of a one-dimensional space in Figure 13). As discussed in the second section of this paper, in an $n$-dimensional space where ($n$) is the number of criteria, in some cases, the midpoint sits on the value of the optimum alternative in a specific criterion of the problem. It happens when the optimal alternative value and the value of the optimum alternative are equal in a criterion (see Figure 14).

In our case, as the optimal alternative, the Reebok brand is equal to the optimum Nike brand. Therefore, with respect to Equations (45) and (46), the normalized Reebok matrix and the midpoints are demonstrated in Tables XI and XII.
The seventh step for RBOP is the first ranking process of alternatives. The evaluation of running shoes by the three criteria is distributed in Table XIII, in line with Equation (47).

Finally, in accordance with Equations (48)–(51), the ranking of the running shoes is shown in Table XIV.

### Table XI. Normalized optimal alternative matrix

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reebok</td>
<td>[1, 1]</td>
<td>[1.286, 1.111]</td>
</tr>
</tbody>
</table>

**Note:** As the normalized optimal alternative matrix: the normalized matrix of Reebok Floatide Run’s ratings for brand, appearance and features

### Table XII. The normalized running shoes selection decision matrix

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nike</td>
<td>[0.000, 0.000]</td>
<td>[0.215, 0.111]</td>
</tr>
<tr>
<td>ASICS</td>
<td>[0.283, 0.025]</td>
<td>[0.143, 0.056]</td>
</tr>
<tr>
<td>Under Armour</td>
<td>[0.290, 0.080]</td>
<td>[0.143, 0.056]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Brand</th>
<th>Appearance</th>
<th>Features</th>
<th>$P(A_j)$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nike</td>
<td>[0.000, 0.000]</td>
<td>[0.215, 0.111]</td>
<td>[0.138, 0.103]</td>
<td>0.44</td>
</tr>
<tr>
<td>ASICS</td>
<td>[0.283, 0.025]</td>
<td>[0.143, 0.056]</td>
<td>[0.020, 0.008]</td>
<td>0.11</td>
</tr>
<tr>
<td>Under Armour</td>
<td>[0.290, 0.080]</td>
<td>[0.143, 0.056]</td>
<td>[0.000, 0.000]</td>
<td>0.55</td>
</tr>
</tbody>
</table>

**Note:** Ranking process of running shoes where $(w'_j)$, $(\lambda_j)$ and $(w_j)$ are the extracted weight from entropy, the weight according to the DM’s decision and the final importance weight, respectively.
As a gray number \((\otimes w_j = [\bar{w}_j, \bar{w}_j])\), then, to calculate the crisp number of \(\otimes w_j\), we used the simple Equation (52) to use \(\bar{w}_j\) as the midpoint of the numerical interval of \(\otimes w_j\):

\[
w_j = \frac{\bar{w}_j + \bar{w}_j}{2}.
\] (53)

As can be seen in Table XIV, Under Armour is standing in the first place where ASICS is the last option for the DM to buy. However, the results of the SAW and TOPSIS applications are completely different. The ranking of running shoes is displayed in Figure 15.

As portrayed in Figure 15, the result shows a significant difference between the outputs of TOPSIS-G and gray-SAW when compared with RBOP-G. In this paper, the RBOP-G result is also compared with the application results of SAW and TOPSIS in a certain environment with crisp numbers (see Table XV) to show the similarity between RBOP-G and the other MCDM methods mentioned. To compute the similarity between the results of the applied MCDM methods, we utilized Spearman’s rank correlation coefficient. Spearman’s rank correlation coefficient is a measurement tool for computing the similarity between two sets of rankings. The average similarities between the \(k\)th MCDM method and other MCDM methods is calculated as per Equation (54), where \((q)\) is the
number of MCDM methods, and as Spearman’s rank correlation coefficient between the $k$th and $i$th MCDM methods. ($\rho_{ki}$) can be obtained from Equation (55), where $(m)$ stands for the number of alternatives and $(d_i)$ is the difference between two MCDM methods (Kou et al., 2012):

$$\rho_k = \frac{1}{q-1} \sum_{i=1, i \neq k}^{q} \rho_{ki}, \ k = 1, 2, \ldots, q, \quad (54)$$

where:

$$\rho_{ki} = 1 - \left( \frac{\sum (d_i)^2 (m(m^2-1))^{-1}}{} \right) -1 \leq \rho_{ki} \leq 1. \quad (55)$$

A large value for ($\rho_{ki}$) of an MCDM method shows its good agreement with other MCDM methods.

As displayed in Table XV, the only description for the differences is the impact of the optimum and optimal alternatives. By contrast with RBOP-G, TOPSIS, TOPSIS-G, gray SAW and SAW considered ASICS as the best running shoes, while Nike is the last option that they offer for buying. As discussed before, RBOP is developed based on human behavioral patterns and human decision-making processes. Like other conventional MCDM methods, SAW and TOPSIS extract the best alternative through the analysis of a decision matrix with various operations and processes, while the RBOP algorithm is thoroughly dependent on a DM’s orientation. In fact, RBOP not only provides a mathematical framework for decision making, it also predicts a DM’s orientation toward the problem alternatives. Indeed, RBOP does not focus on finding the best alternative for a decision-making problem, instead it offers the DM’s desired result as given through decision matrix analysis. RBOP is a complex MCDM technique. As Chatterjee et al. (2011) stated, the complexity of an MCDM method and its transparency are directly related. Due to the structural similarity between a DM’s mindset and the RBOP algorithm, RBOP has low transparency if compared with TOPSIS and SAW, while a DM can track each step of the algorithm to identify any mistake.

5. Conclusion and future research

In this paper, we introduced an MCDM methodology named “RBOP”. Amongst MCDM methods, RBOP has the closest structure to human behavior patterns. RBOP is developed based on the human decision-making process. The foundation of the RBOP procedure is to deal with the optimal alternatives as “the better options” and the optimum alternative as “the best option.” In real-world human decision making, there are ideals that are benchmarks for the selection of the best decision alternatives; these ideals are always unattainable while they have a direct impact on the decision-making process and its results.

In $(m \times n)$ dimensional space, with respect to RBOP policy, the best alternative has the closest distance to its optimum points and has the closest distance to the optimal points simultaneously. The RBOP procedure directly depends on human decisions and human imaginings about the ideals, alternatives and the optimal and optimum alternatives. Due to the bold role of a DM’s decisions in fundamental elements of the RBOP algorithm, and due to the uncertainty associated with human perception as well, to handle the uncertainty, we proposed an approximate reasoning mechanism for the RBOP algorithm called “RBOP-G,” which deals with the GST and the gray numbers operations. A ten-step process was employed for the prioritization of the alternative. To present the applicability of RBOP-G, we posed a simple real-world decision-making problem, that of buying running shoes, utilizing the three brands of Nike, Under Armour and ASICS as alternatives and also giving Reebok as the optimal alternative. Moreover, we compared the results of RBOP-G with TOPSIS-G and gray SAW. The results showed significant differences between the RBOP-G results and the results of the other methods mentioned. The most important indicator that made the
difference was the impact of optimum and optimal alternatives on the decision-making process. Due to the impressive effect of a DM’s decisions on the RBOP procedure, the result of this new method should seem more desirable to a DM than other MCDM methods’ results. Furthermore, RBOP uses a proposed weighted geometric mean and a new proposed pairwise comparison method in its algorithm.

Despite the prediction of human decision orientations, RBOP imitates human behavior. Thus, we are developing its application for artificial intelligence. As an interesting future research, we suggest involving group decision making to extend the proposed method. In addition, its execution in the fuzzy environment and its integration with the MADM and MODM methods can be considered as a future research topic. We suggest its development, and application for real-world business problems such as supply chain management, market selection, customers’ segmentation and customer behavior such as the analysis and prediction of consumer’s emotional, mental and behavioral responses and psychological experiments. Also, its application to other aspects of human life could be considered as an interesting topic for future research.

Notes
1. The four steps of the rational model, which are given by Simon (1979), are: intelligence; design; choice; and review.
2. Huber (1981) named the organizational “procedure view” as a “program model,” and Das and Teng (1999) referred to it as “avoidance mode.”

References


Further reading

Appendix

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>[9, 10]</td>
<td>[7, 9]</td>
<td>[5, 7]</td>
</tr>
</tbody>
</table>

Note: As declared in Table I and Equation (9), the pairwise comparison process is shown in Table V

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pronation</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Supination</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Barefoot/minimalist running</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table AII. Application of the pairwise comparison for the subcriteria features for Nike running shoes

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rating</td>
<td>[9, 10]</td>
<td>[9, 10]</td>
<td>[7, 9]</td>
</tr>
</tbody>
</table>

Note: As declared in Table I and Equation (9), the pairwise comparison process is as shown in Table V

<table>
<thead>
<tr>
<th></th>
<th>Pronation</th>
<th>Supination</th>
<th>Barefoot/minimalist running</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pronation</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Supination</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Barefoot/minimalist running</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table AIV. Application of the pairwise comparison for the subcriteria features for Under Armour running shoes

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